VOL. 11, NO. 15, AUGUST 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

A NOVEL VAL: QUADROTOR CONTROL TECHNIQUE FOR TRAJECTORY TRACKING BASED ON VARYING THE ARM'S LENGTH

Yasameen kamil N.[1], D. Hazry[1], Khairunizam Wan[1, 2] and Zuradzman M. Razlan[1,2]
[1]Center of Excellence for Unmanned Aerial system (COEUAS), University Malaysia perlis (UniMAP), Malaysia
[2]School of Mechatronic Engineering, University Malaysia perlis (UniMAP), Arau, perlis, Malaysia
E-Mail: [1]moonom2002@gmail.com

ABSTRACT

This paper presents the design and analysis of quadrotor with a novel mathematical model based on varying the arm's length instead of varying the motors' speed. This model is named Variable Arms Lengths quadrotor (VAL-quadrotor). The objective of this design is to tackle the problem of nonlinear system model in attitude movement. This accomplished by exploiting the moments of arms to create variable torque around the center of gravity,through increasing or decreasing the length of VAL-quadrotor arms while fixing the motors speed. Thus, the model is converted to a linear controlling system. Since the controlling input to the attitude system is the arms length which is a function of a first order system. The effect of changing the design configuration on the moments of inertia is derived. The stability of the altitude, attitude, and position are achieved by adding a PID controller. The performance of VAL-quadrotor is evaluated by simulation which based on MATLAB code to perform the trajectory tracking.

Keyword: arm's length, PID controller, quadrotor, trajectory tracking, UAV, VAL-quadrotor.

INTRODUCTION

Quadrotor has developed thanks to the current research during this decade, it is presenting a wide application in civilian, military and commercial [1].Where, it is used in a broad field of relief, inspection, and surveillance. Quadrotor has happened revolution in the both fields of robotics and aeronautics, due to its simple design, small rotors, good manoeuvrability and small size [2]. The quadrotor considered a nonlinear characteristic system, strongly coupled, and multivariable system [3]. These limitations make the quadrotor not easy to control and must be used a strict controller to stabilize the performance of the system. A number of projects have been tackled the nonlinearity and control problem of quadrotor, based on varying the speed of motors. In [4], feedback linearization was adopted to design an attitude and position controller, the under-actuated system divided into two fully actuated systems and separately designed the attitude and position controller. Unfortunately, error steady state was presented between the actual and the desired response during the trajectory tracking. Feedback linearization with PD controller for the transitional system are used in [3] and compared the results with PID controller. Since a nonlinear controller is used, the complexity of the system was increased where the backstepping needed some iteration in derived besides adding some theories to examine its stability like Lyapunov and pole placement methods. The quadrotor is a complex system; choosing appropriate control algorithm was a very important issue. Since adopting a complex nonlinear controller increased the complexity of the design [5]. Besides, it increased the time consumption and the computational time. Then, the most common linear controller used is PID controller[6]. Where In [7], two control techniques were presented in attitude, the classical PID controller, and LQ modern technique. The authors concluded that the PID technique more efficient, more stable and easy to control than LQR technique under ideal conditions. In [8], Iterative Feedback Tuning (IFT) is used for PID controller tuning to specify appropriate gain for PID controller that will be effective to stabilize and control the quadrotor performance. Nevertheless, it's complicated method to calculate the gain and required a time for iteration. Due to the nonlinear dynamic model of quadrotor which is a function of square angular velocity of the motor input [9]. This paper presents a new design and derivation of a mathematical model for the quadrotor. VAL-quadrotor depends on varying the arms' lengths of quadrotor which has the ability of overcomes the nonlinearity limitation of quadrotor system. Then, the attitude movements are achieved by exploiting the variation in the overall torque caused by varying arms length and fixing the motors' speed, instead of that adopted in standard quadrotor achieved by varying motors speed and fixing the arms' length. The attitude movement is employed when VAL- quadrotor reached the desired altitude. Therefore, the VAL-quadrotor is considered a linear system due to the control torque which is a function of first order system depended on arm's length variation. The arm' length variation is performed by fixing stepper motor in each arm for increasing or decreasing the length of arms. A mathematical expression is adopted for the optimal choice of the arm's length. The stability performance is achieved by using PID controller which is used in altitude, attitude, and position. The simulation model is implemented using MATLAB code to perform a trajectory tracking and verify the behaviour of VAL-quadrotor.

The rest of this paper is organized as follows. A VAL-quadrotor design is presented in section 2, which contains motion control of VAL-quadrotor and aerodynamic force and moments acting on VAL-quadrotor. VAL-quadrotor dynamic model is presented in section 3 which containsvariation in the moment of inertia. In section 4 system controller design is presented which contains altitude, attitude, and position controller design,section 5 refers toevaluation scenario for arm selection, The simulation results are discussed in section 6 which consiste of simulation results for evaluation the arms length selection and simulation results for VAL-quadrotor performance, and the conclusions are presented in section 7.

VAL-QUADROTOR DESIGN

VAL-quadrotor design consists of four motors fixed on the end of four cross arms; each arm consists of two parts as shown in Figure-1. The First part is fixed with VAL- quadrotor body while the second part is sliding inside the fixed arm to increase or decrease the total length of the quadrotor's arm. Stepper motors are fixed inside the fixed arms of VAL-quadrotor. Therefore, instantaneously the sliding arm slides out the fixed arm when the stepper motor rotates in C.W with constant speed and the total arm length increases. Conversely, the total arm length decreases when the stepper motor rotates in C.C.W and the sliding arm slides in the fixed arm. Therefore, VAL-quadrotor has the ability to perform two flight modalities at a short time which essentially performs the movement as a standard quadrotor and perform the movement as VAL-quadrotor.

Motion control of VAL-quadrotor

In this paper, controlling the movement of quadrotor depends on varying its arm's length instead of the angular velocity of the motors while fixing the motors speed. This produces a variable moment around the quadrotor centre of mass which directly controls the moment of the rotational movement in pitch, roll, and yaw while indirectly controls the position in x and y to accomplish the required manoeuvrability. Therefore, to model the VAL-quadrotor some important assumptions must be considered as

- The angular velocity of each motor fixed to be constant after reach desired altitude.
- The arm's lengths are varying according to the desired movement during the flight.
- The thrust coefficients for all motors are constant

Figure-2 (a) shows the pitch angle Θ about the y-axis that can be controlled by increasing the arm's length related to M_3 and decreasing the arm's length related to M_1 and fixed all motors speed with the indirect control of the motion along the x- axis. Figure-2(c) shows the roll angle ϕ about x-axis which is the same as pitch angle except that the roll angle is increasing the arm's length related to

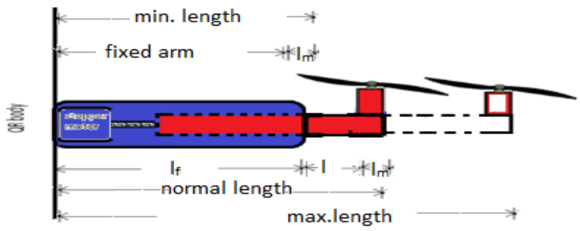


Figure-1. The arm's length design.

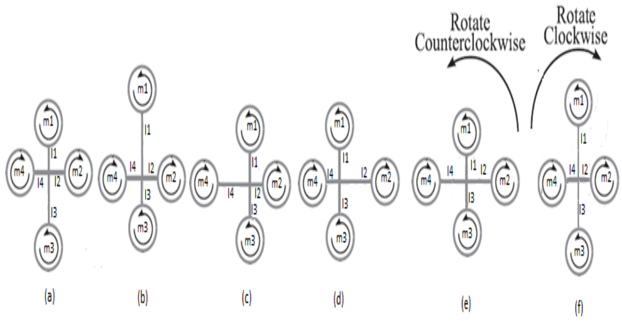


Figure-2. VAL-Quadrotor's movements.

M_4 and decreasing the arm's length related to M_2 with the indirect control of the motion along y- axis.

Finally, yaw angle ψ about the z-axis can be controlled by increasing the arms length of two related motors which rotate clockwise (M_2 and M_4), and decreasing the arms length of two related motors which rotate counter clockwise (M_1 and M_3). Therefore, quadrotor mismatches the balance in a moment and rotated in a direction of the greatest moment according to the direction of the propeller rotation as shown in Figure-2 (e).

Aerodynamic force and moments acting on VAL-quadrotor

The aerodynamic force and moment are extracted from a combination of the moment and blade element theory [10]. As mentioned VAL- quadrotor has four motors with propellers. The applied power to the rotor shaft generate a net torque which result in a thrust, the forward velocity causes a drag force in the direction opposite the quadrotor motion. Where the thrust (f) and drag (D) forces are defined as:

$$f_i = C_T \rho A r^2 \omega^2 \tag{1}$$

$$D_i = C_d \rho A r^2 \omega^2 \tag{2}$$

Where *A* represents the blade area, ρ represents the air density, *r* represents the radius of blade, ω represents the angular velocity of the propeller and C_T, C_d represent aerodynamic coefficients. The total forces acting on the VAL-quadrotor body are given by:

$$F_{total} = -\frac{1}{2} C_{x,y,z}\, \rho A\, (\dot{x}^2, \dot{y}^2, \dot{z}^2) - Mgz + \sum_{i=1}^{4} f_i - \sum_{i=1}^{4} D_i(xy) \tag{3}$$

Where the first term of equation (3) represents the frictional force on quadrotor along x, y and z axes and $C_{x,y,z}$ represents the drag coefficients and ($\dot{x}, \dot{y}, \dot{z}$) represent the speed in (x, y and z) axes respectively, M represent the total mass of the quadrotor, g represent gravity, z represent the vertical motion. The total torque in quadrotor design acting on the quadrotor body as in [3].

$$\begin{aligned} T_{total} &= (-1)^i \sum_{i=1}^{4} Q_i z \\ &+ (-1)^{i+1} \sum_{i=1}^{4} R_{mi}(xy) + h.\sum_{i=1}^{4} D_i(-yx) + (f_4 - f_2) l_x \\ &+ (f_3 - f_1) l_y + \{[(D_2 - D_4) + (D_3 - D_1)] l\} z \end{aligned} \tag{4}$$

Where

$$Q = C_Q \rho \mathrm{A} r^2 \omega^2 r \tag{5}$$

$$R_m = C_R \rho \mathrm{A} r^2 \omega^2 r \tag{6}$$

Where R_m represents the rolling moment, h represents the vertical distance between propeller centre and gravity centre of the aircraft and Q represents the torque from each motor. According to VAL-quadrotor design since fixing the angular velocity, the thrust and drag forces are equalled for all motors ($f_1 = f_2 = f_3 = f_4 = f$) and ($D_1=D_2=D_3=D_4=D$) respectively. The total torque in Equation (8) expressed as:

$$\begin{aligned} T_{total} &= (-1)^{i+1} \sum_{i=1}^{4} R_{mi}(xy) + h.\sum_{i=1}^{4} D_i(-yx) + \\ & f(l_{m4} - l_{m2}) + f(l_{m3} - l_{m1}) + (-1)^i \sum_{i=1}^{4} D l_i \end{aligned} \tag{7}$$

Where l_{mi} represents the arm's length from the pivot centre to the related motor as shown in Figure-3. When comparing equation (4) with equation (7), the first term of equation (4) will be cancelled due to fixing the speed of motors to be constant for VAL-quadrotor.

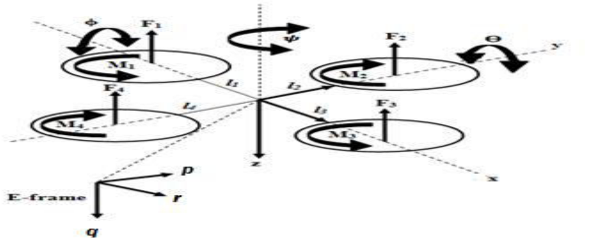


Figure-3. Architecture and coordinate of VAL- quadrotor.

VAL-QUADROTOR DYNAMIC MODEL

VAL-quadrotor consists of two coordinate frames which are earth inertia frame and body- fixed frame. The earth inertia frame coordinates are $\varepsilon_E = [r\ p\ q]$ and the body-fixed frame coordinates are $x_B = [x\ y\ z]$. The orientation of quadrotor is represented by the attitude Euler angles which are roll, pitch and yaw angles (ϕ, Θ, and ψ) around the x, y and z axis respectively. The transformation from body frame to the earth frame is represented by transformation matrix E as in [6] under the assumption of:

- The quadrotor structure is supposed to be rigid and symmetric.
- The centre of the mass and the origin of the body frame are coincide

Where, E is defined as in the matrix below as in [11].

$$= \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\psi + s_\theta c_\phi c_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi - c_\phi c_\psi & s_\theta s_\psi c_\phi - c_\psi s_\phi \\ -s_\theta & s_\phi c_\theta & c_\theta c_\phi \end{bmatrix} \tag{8}$$

Where $c_{(\bullet)}, s_{(\bullet)}$ represent cos (.) and sin (.) respectively. The lift forces and the rotational moments in the body frame will be expressed as:

$$f_{\mathrm{B}} = \begin{bmatrix} f_{\mathrm{B}x} \\ f_{\mathrm{B}y} \\ f_{bz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} f_i \end{bmatrix},\ \tau_{\mathrm{B}} = \begin{bmatrix} \tau_\theta \\ \tau_\phi \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} f(l_3 - l_1) \\ f(l_4 - l_2) \\ D(l_2 + l_4 - l_1 - l_3) \end{bmatrix} \tag{9}$$

VAL- quadrotor is 6-DOF. The equation of motion of VAL-quadrotor subjected to the force f_{B} and moment τ_{B} described using Newton-Euler method as in [12]:

$$M\dot{v} = f_{\mathrm{B}} + \Omega \times Mv \tag{10}$$

$$I\dot{\Omega} = \tau_{\mathrm{B}} + \Omega \times I\Omega \tag{11}$$

Where Ω represents the angular velocity in body frame I represents the moment of inertia. The forces in the earth inertia frame can be expressed as:

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = f_{\mathrm{B}} E = \left(\sum_{i=1}^{4} f_i\right) \begin{bmatrix} s_\phi s_\psi + s_\theta c_\phi c_\psi \\ s_\theta s_\psi c_\phi - c_\psi s_\phi \\ c_\theta c_\phi \end{bmatrix} \tag{12}$$

By substitute equation (12) in (10) the dynamic model of the transitional motion of VAL-quadrotor in (x, y, z) axes in the earth inertia frame will be described as in [13].

VOL. 11, NO. 15, AUGUST 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$$M\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix}=\begin{bmatrix}f_x-k_1\dot{x}\\ f_y-k_2\dot{y}\\ f_z-k_3\dot{z}\end{bmatrix}=\begin{bmatrix}\sum_{i=1}^{4}f_i(s_\phi s_\psi+s_\theta c_\phi c_\psi)-k_1\dot{x}\\ \sum_{i=1}^{4}f_i(s_\theta s_\psi c_\phi-c_\psi s_\phi)\quad -k_2\dot{y}\\ \sum_{i=1}^{4}f_i(c_\theta c_\phi)-Mg-k_3\dot{z}\end{bmatrix} \qquad (13)$$

Where k_1, k_2, k_3 represent the drag coefficients.

The rotational movement in pitch, roll, and yaw angles can be described by using Euler form by substitute equation (7) in equation(11) and adding the gyroscope action caused by the rotation of rigid body, the gyroscope action caused by the rotation of the propeller [10] and the aerodynamic drag which opposite the motors torque [14]. VAL-quadrotor dynamic model of rotational movements will be:

$$\left.\begin{array}{c} I_{xx}\ddot{\phi}=\dot{\theta}\dot{\psi}(I_{yy}-I_{zz})+u_2+\sum_{i=1}^{4}(-1)^{i+1}R_{yi}+h\sum_{i=1}^{4}D_{xi}+I_{xx}\dot{\phi}\omega-k_4\dot{\theta}\\ I_{yy}\ddot{\theta}=\dot{\phi}\dot{\psi}(I_{zz}-I_{xx})+u_3+\sum_{i=1}^{4}(-1)^{i+1}R_{xi}-h\sum_{i=1}^{4}D_{yi}-I_{yy}\dot{\theta}\omega-k_5\dot{\phi}\\ I_{zz}\ddot{\psi}=\dot{\theta}\dot{\phi}(I_{xx}-I_{yy})+u_4-k_6\dot{\psi} \end{array}\right\} \qquad (14)$$

Where k_i represent the drag coefficients and u_1, $u_{2,}, u_3 and u_4$ the inputs to the system and defined as:

$$\left.\begin{array}{c} u_1=b(\omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2)=\sum_{i=1}^{4}f_i\\ u_2=b.\omega^2(-l_2+l_4)\\ u_3=b.\omega^2(-l_1+l_3)\\ u_4=d.\omega^2(-l_1-l_3+l_2+l_4) \end{array}\right\} \qquad (15)$$

Where b and d represent the thrust and drag coefficients. According to equation (15), u_2, u_3, and $u_4$represent the control torques those impact the attitude and position movements. The control torque depends on varying the arms' lengths where represent a first order system controlling function. VAL-quadrotor system is considered as a linear system. As compared with the angular velocities which are second order system controlling function in standard quadrotor. These control inputs substituted in equations (13) and (14)

Variation in the moment of inertia

The moment of inertia also calculated due to VAL-quadrotor design and it will be the same for standard quadrotor that chosen for simulation. Therefore, the moments of inertia for VAL-quadrotor divided into two parts, the first part around the x and y - axes for roll and pitch respectively, and second part around z-axis for yaw. For roll moment of inertia also contains two parts, the first part for motor 2 and 4. The second part for motors 1 and 3. As consideration, the motors and the body of VAL-quadrotor have a cylindrical shape [9]. Then, the moment of inertia around x-axis calculated as

$$I_{xx}=\frac{m_m r^2}{2}+\frac{m_m h^2}{6}+m_m l_2^2+m_m l_4^2+\frac{m_b R^2}{4}+\frac{m_b H^2}{12} \qquad (16)$$

Where m_m, m_b represent the mass of motor and body respectively, r, R the radius of motor and body respectively, and h, H the height of the motor and the body. The same calculation for moments of inertia around y-axis as in equation

$$I_{yy}=\frac{m_m r^2}{2}+\frac{m_m h^2}{6}+m_m l_1^2+m_m l_3^2+\frac{m_b R^2}{4}+\frac{m_b H^2}{12} \qquad (17)$$

The moment s of inertia around the z-axis is calculated as follows:

$$I_{zz}=\frac{m_b R^2}{2}+m_m l_1^2+m_m l_3^2+m_m l_2^2+m_m l_4^2 \qquad (18)$$

As $l_1 = l_3$, $l_2 = l_4$ where, in yaw attitude the arms of VAL- quadrotor in the same axis have the same amount. Then equation (18) can be rewritten as

$$I_{zz}=\frac{m_b R^2}{2}+2m_m l_1^2+2m_m l_2^2 \qquad (19)$$

As an assumption, the moment of inertia is a diagonal matrix due to the symmetry of quad rotor's geometry [14]

$$I=\begin{bmatrix}I_{xx}&0&0\\0&I_{yy}&0\\0&0&I_{zz}\end{bmatrix} \qquad (20)$$

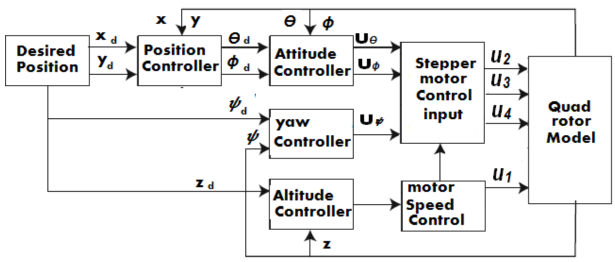


Figure-4. Block digram of VAL- quadrotor full control.

SYSTEM CONTROLLER DESIGN

Although VAL- quadrotor is an under-actuated system, it should be controlled easily by dividing the system into subsystems because its 6-DOF. That means four input forces should be control six output variables{x, y, z, θ, ϕ, ψ}

In this paper, the system is divided into altitude subsystem, attitude subsystem and position subsystem as in Figure-4.Two microcontrollers are used in VAL-

ARPN Journal of Engineering and Applied Sciences

quadrotor. One for controlling the speed of motors and the second for controlling the stepper motors

In attitude system, there are three control inputs (u_2, u_3, u_4) and three outputs that separately controlled. The altitude system is controlled using the control input (u_1). Whereas the position control only has one control input (u_1) and three outputs (x, y, z), so this controller cannot directly control by one of the system control input. Therefore, the position (x, y) dependently controlled through pitch and roll angle respectively [12]. This design ensures that the quadrotor is tracking the desired trajectory through altitude and attitude controller and navigation through the position controller [15].

Altitude controller design

PID controller is used to control the altitude of the quadrotor by controlling the speed of four motors to ensure the altitude required. This can be done by increasing all motors speed as in standard quadrotor and the lift force required for takeoff must be greater than quadrotor full weight[16]. The error signal for this controller will be expressed as

$e_z = z_d - z_{ref}$ (21)

The dynamic model of altitude with controller is

$$M\ddot{z} = k_p e_z + k_i \int e_z dt + k_d \dot{e}_z - Mg \tag{22}$$

Where k_p, k_i, k_d represent the proportional, integral, and derivative gain and must be positive values.

Attitude controller design

For designing the attitude of quadrotor, a PID controller is also used. Pitch attitude is performed by increasing the arm's length of the motor (l_3) and decreasing the arm's length of the motor (l_1) while fixed the other two arm's lengths and the motors speed are fixed. Finally, the pitch attitude controller error signal will be expressed in equation below.

$$e_\theta = \theta_d - \theta_{ref} \tag{23}$$

The PID controller equation for pitch attitude is

$$u_\theta = k_p e_\theta + k_i \int e_\theta dt + k_d \dot{e}_\theta \tag{24}$$

The equation of rotation for quadrotor around y-axis includes PID controller and neglect the gyroscope effect [7] will be:

$$\ddot{\theta} = \frac{u_3}{I_y} u_\theta \tag{25}$$

Similarly for other attitude angles (roll and yaw) the error signal will be:

$$e_\phi = \phi_d - \phi_{ref}$$
$$e_\psi = \psi_d - \psi_{ref}$$

The PID controller equations for roll and yaw attitude are:

$$\left. \begin{aligned} u_\phi &= k_p e_\phi + k_i \int e_\phi dt + k_d \dot{e}_\phi \\ u_\psi &= k_p e_\psi + k_i \int e_\psi dt + k_d \dot{e}_\psi \end{aligned} \right\} \tag{26}$$

The equations of roll and yaw attitude include PID controller will be:

$$\left. \begin{aligned} \ddot{\phi} &= \frac{u_2}{I_x} u_\phi \\ \ddot{\psi} &= \frac{u_4}{I_z} u_\psi \end{aligned} \right\} \tag{27}$$

Position controller design

The horizontal motion in x and y- axis can be accomplished by controlling the position of the quadrotor. Position control can be achieved by pitching or rolling the quadrotor, and track the desired waypoint from x and y-axis. PID controller with saturation function is used for position control. From the position controller calculating the desired acceleration ($\ddot{x}_d$, $\ddot{y}_d$), as in [15]

$$\left. \begin{aligned} e_x &= x_d - x_{ref} \\ e_y &= y_d - y_{ref} \end{aligned} \right\} \tag{28}$$

$$\left. \begin{aligned} \ddot{x}_d &= k_p e_x + k_i \int e_x dt + k_d \dot{e}_x \\ \ddot{y}_d &= k_p e_y + k_i \int e_y dt + k_d \dot{e}_y \end{aligned} \right\} \tag{29}$$

Where (x_d, y_d) represent the waypoints desired, (x_{ref}, y_{ref}) represent the reference positions and k_p, k_i, k_d represent the proportional, integral and derivative controller gains respectively. The desired pitch and roll angles are calculated from the desired acceleration $\ddot{x}$ and $\ddot{y}$ from equation (19).

$$\ddot{x}_d = -\frac{u_1}{m}\left[\sin\phi_d \sin\psi + \cos\phi_d \sin\theta_d \cos\psi\right] \quad (30)$$

$$\ddot{y}_d = \frac{u_1}{m}\left[\sin\phi_d \cos\psi - \cos\phi_d \sin\theta_d \sin\psi\right] \quad (31)$$

As considering the quadrotor, is under hover condition. Thus, $\sin\phi_d = \phi_d, \sin\theta_d = \theta_d$ and $\cos\phi_d = \cos\theta_d = 1$ then equations (30) and (31) can be simplified as

$$\left.\begin{aligned} \ddot{x}_d &= -\frac{u_1}{m}[\phi_d \sin\psi + \theta_d \cos\psi] \\ \ddot{y}_d &= \frac{u_1}{m}[\phi_d \cos\psi - \theta_d \sin\psi] \end{aligned}\right\} \quad (32)$$

Then, from equation (32) can find the desired roll (ϕ_d) and pitch (θ_d) angles as in [15].

$$\begin{bmatrix}\phi_d \\ \theta_d\end{bmatrix} = \begin{bmatrix} -\sin\psi & -\cos\psi \\ \cos\psi & -\sin\psi \end{bmatrix}^{-1} \frac{m}{u_1} \begin{bmatrix}\ddot{x}_d \\ \ddot{y}_d\end{bmatrix} \quad (33)$$

The desired angles ϕ_d, θ_d must be limited to the ranges that ensure the condition of small angles [15] above. This can be done by using saturation function which is.

$$\left.\begin{aligned} \phi_d &= sat(\phi_d) \\ \theta_d &= sat(\theta_d) \end{aligned}\right\} \quad (34)$$

Where

$$\text{Sat (k)} = \begin{cases} k & \|k\| \le k_{\max} \\ sign(k)k_{\max} & \|k\| > k_{\max} \end{cases} \quad (35)$$

And

$$\text{Sign(k)} = \begin{cases} -1 & k < 0 \\ 1 & k \ge 0 \end{cases} \quad (36)$$

EVALUATION SCENARIO FOR ARM SELECTION

For selection the appropriate arms' lengths that guarantee the stability in system design during the flight path and do not drift the quadrotor from its path. A mathematical expression is derived to choose the maximum arms length and minimum arms length as expressed in equations below.

$(l_f + l_m + L) + \Delta d \times (l_f + l_m)$ = maximum length (37)

$(l_f + l_m + L)$ - $\Delta d \times (l_f + l_m)$ = minimum length (38)

Where, l_f represents fixed arm length, l_m represents the diameter of the motor, L represents a specific length to ensure permittivity in sliding arm movement, Δd represents the rate of change in the arm to verify the maximum and minimum length and (l_f + l_m+ L) represent the normal length as shown in Figure-1.

The appropriate arm's length is evaluated based on the trade off in accuracy and fast response of the performance according to the scenario. Therefore, the fixed arm is designed in length equal to 17.5 cm and the motor diameter is 2.5 cm fixed at the end of sliding arm. Therefore, the minimum length for the total arm becomes equal to 20 cm while the maximum total length can reach is 30 cm. According to the equations (37) and (38) and by applying l_f =17.5cm, l_m =2.5cm, maximum total length equal to 30 cm, minimum total length equal to 20 cm and solve these equations for Δd, L.

Therefore, L, Δd will be equal to 5 cm and 0.25 respectively. This means that the range of Δd is ($0<\Delta d \le 0.25$). The range of Δd represents the key of movements controlling in the proposed quadrotor.

Therefore, Table -1 shows the effects of varying the rate of change of the length Δd which is measured to observe there effect on the arm's length. The optimal arms length is selected as in next section. Figure-5 shows the configuration of varying the arm's length according to the Table-1.

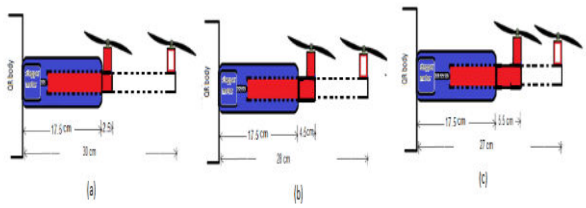


Figure- 5. The design of the arm's length due to Table-1.

Table-1. The effect of varying Δd on length of arm

l_f (cm)	l_m (cm)	**L** (cm)	**Δd**	**Max. length**(cm)	**Min. Length**(cm)
17.5	2.5	5	0.25	30	20
7.5	2.5	5	0.15	28	22
7.5	2.5	5	0.1	27	23

SIMULATION RESULTS AND DISCUSSIONS

Simulation results for evaluation the arms length selection

The parameters of system design that used in simulation are: M=2 Kg, gravity=9.81 m/s^2, b=3.31×10^{-5}, d=7.5×10^{-6}, the arm length is selected according to first choice of Table-1, $I_{xx}=I_{yy}$=1.25 Ns^2/rad, I_{zz}=2.5 Ns^2/rad, $k_1=k_2=k_3$=0.010 Ns/m, and $k_4=k_5=k_6$=0.12 Ns/m.

The appropriate arm length is selected according to the values which calculated due to the scenario in the previous section and represented in Table-1. Three

ARPN Journal of Engineering and Applied Sciences

evaluation values of Δd are performed in this section to show the effects of the different arm length on the roll, pitch and yaw attitude angles. These results are collected in three figures to explain the response of each individual angle with respects to three different lengths. Figure- 6, represents the response of pitch angle when the length of l_1 represented by the minimum length values and l_3 represented by the maximum length values in the Table-1. Then, the best pitch angle response in this figure when the maximum arm length (l_3=30 cm) and the minimum arm length (l_1=20 cm) while other arms are 25 cm (normal arm length). Figure- 7 shows the response of roll angle and explains that the best response occurs when the maximum arm length (l_4 =30 cm) and the minimum arm length (l_2=20 cm) while other arms are 25 cm (normal arm length).Figure-8 shows the response of yaw angle, then the best choice when the maximum arms length (l_2=l_4=30 cm) and the minimum arms length (l_1=l_3=20 cm). According to this evaluation, the VAL- quadrotor is considered the minimum arms' length equal to 20 cm and the maximum arms' length equal to 30 cm to perform more stability of the system.

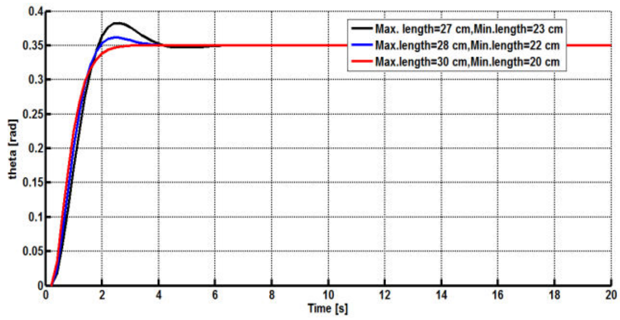


Figure-6. The pitch angle response according to Table-1 for arm's length selection.

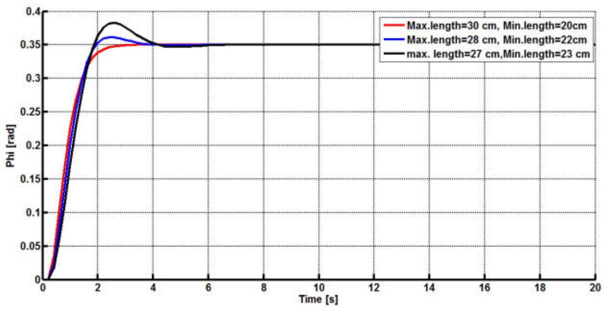


Figure-7. The roll angle response according to Table-1 for arm's length selection.

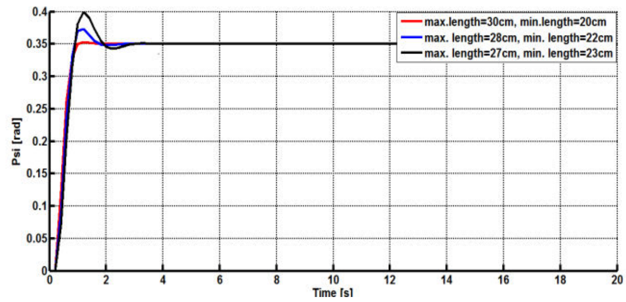


Figure-8. The yaw angle response according to Table-1 for arm's length selection.

Simulation results for VAL-quadrotor performance

This section presents the simulation results for the attitude, position, altitude and trajectory tracking of the Val-quadrotor design and compared the results with standard quadrotor response. After selecting the appropriated arm's length, the parameters of VAL-quadrotor are assumed as in previous section and the parameter of the standard quadrotor are: M=2Kg, gravity=9.81m/s^2, b=3.31×10^{-5}, d=7.5×10^{-7}, arm length=25cm, I_{xx}=I_{yy}=1.25 Ns^2/rad, I_{zz}=2.5Ns2/rad, k_1=k_2= k_3=0.010 Ns/m, and K_4= k_5= k_6=0.12 Ns/m.

The simulation results demonstrated the waypoint trajectory tracking in which the aircraft take-off from (0, 0, 0) waypoint to the altitude of 10 m, then it travels to the first waypoint (5, 0, 10), after that moves to waypoint (5, 5, 10) then moves to (3, 2, 10) before landing smoothly to desired waypoint (3, 2, 0). According to this trajectory Figure-9 shows the desired path in z, x and y-axes which the aircraft moved on from the starting point to the destination point as the waypoint scenario.Figure-10 shows the response of the standard quadrotor and VAL-quadrotor in altitude position when the aircraft take-off from the start point until landing point. This figure shows that VAL-quadrotor moved accurately according to desired path. Conversely, the standard quadrotor response drifts from the path.

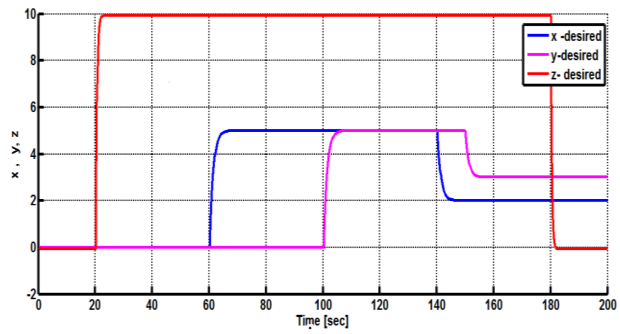


Figure-9. X,y,z desired due to desired waypoint.

ARPN Journal of Engineering and Applied Sciences

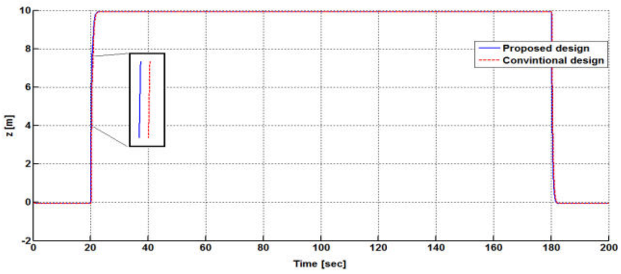


Figure-10. The altittude of the proposed and standardQuadrotor.

Whereas the angles of rotational subsystem independent on the transitional subsystem, while the transitional components depend on the rotation angles. The attitude control keeps the aircraft at the desired state. The pitch and roll angles are limited to a range of (-0.35) rad to (0.35) rad in position controller. Figure-11 and Figure-12 show the response of x-position and pitch angle. The quadrotor moved to the first point after taking –off operation. The position controller generate negative pitch angle due to start moved toward x position. When compared the response of the standard quadrotor with VAL-quadrotor, there is adrift from the desired for the response of the standard quadrotor due to the oscillation in pitch angle. Contrary, VAL-quadrotor reached the desired waypoint faster and at a time less than the time in standard quadrotor. This stable performance is due to the stability of the pitch angle which follows the desired accurately. Figure-13 and Figure-14 present the response of y-position and the dependent roll angle respectively. At time equal to100 sec, VAL- quadrotor moved in direction of positive y -axis, the roll angle responded to this change with negative angle. These figures illustrated the comparison between the standard quadrotor and VAL-quadrotor. In which, VAL-quadrotor is better than the standard quadrotor according to the stability of the roll angle and steady state time.

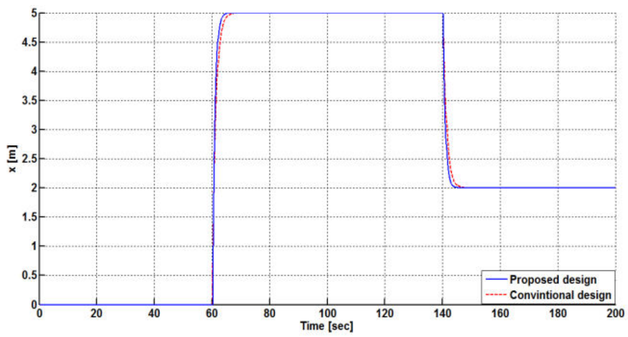


Figure-11. The response of x-position due to the proposed and standard quadrotor.

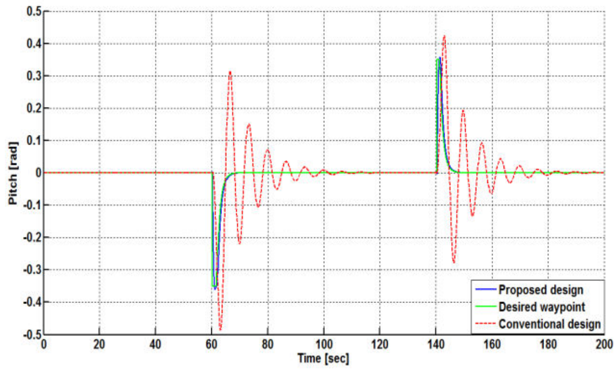


Figure-12. The pitch angle response for the desired, proposed and standard quadrotor.

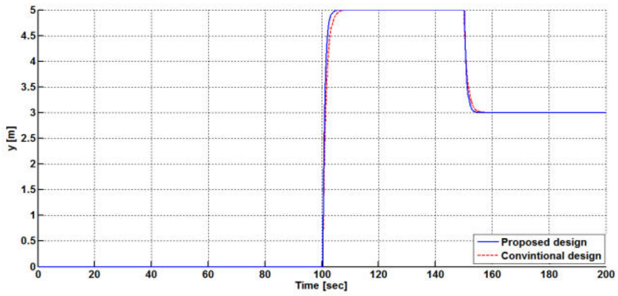


Figure-13. the response of y-position due to the proposed and standard quadrotor.

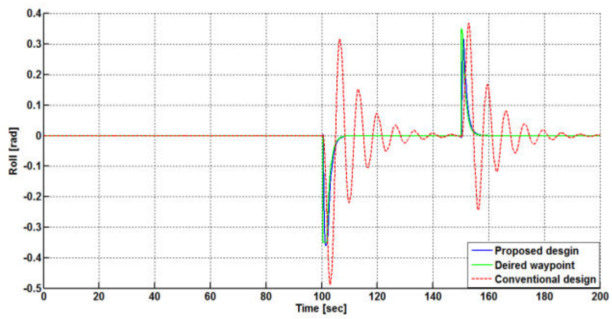


Figure-14. the roll angle response for the desired , the proposed and standard quadrotor.

Figure-15 shows yaw response angle in VAL-quadrotor and standard quadrotor which has oscillation in response and after (40 sec) of time it reaches steady state contrarily in VAL-quadrotor which is more stable and reached its steady state at less time with no oscillation.

Figure-16(a) shows the trajectory tracking for quadrotor performance under ideal conditions (without disturbance) with the desired trajectory tracking and the ability of the new model-based controller to accomplish take-off, hovering, trajectory tracking and landing.

Figure-16 (b) shows the response of trajectory tracking due to the standard quadrotor with the desired trajectory tracking. The results illustrate a little drift from the desired at the same parameters gain of (PID) controller used in the VAL- quadrotor which are (k_p=30, k_d=20 and k_i=20) for altitude and for attitude are (k_p=30, k_d=40 and k_i=20).

ARPN Journal of Engineering and Applied Sciences

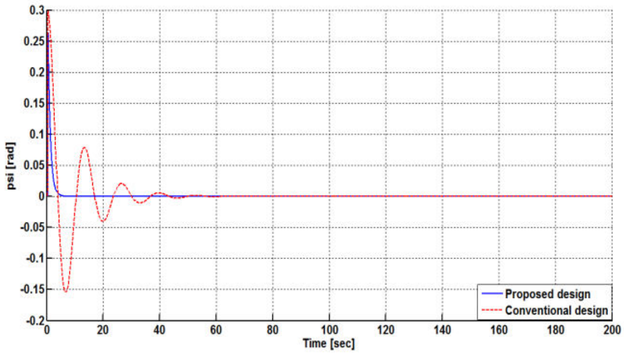
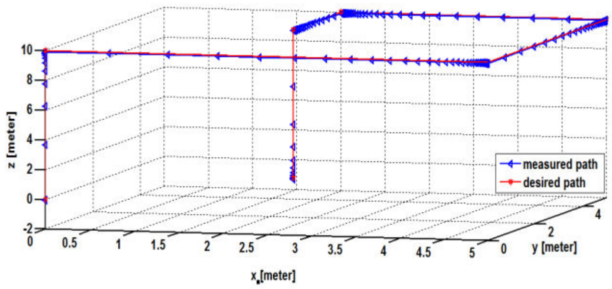
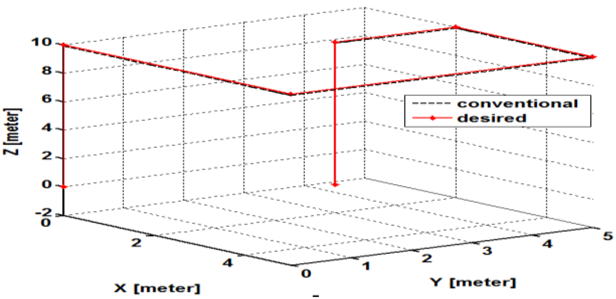


Figure-15. The yaw angle response for the proposed and standard quadrotor.



(a)



(b)

Figure-16. The trajectory tracking (a) for the VAL-quadrotor and (b) for standard quadrotor.

CONCLUSIONS

In this paper, a novel controlling technique based on varying the arm's length is presented, this design used to accomplish attitude and position movement. The mathematical model is developed to be compatible with the requirement for VAL-quadrotor, and derived using Newton- Euler method. New mathematical equation adopted to specify the amount of increasing and decreasing in the arm's length. The complexity of the standard quadrotor design due to varying the angular velocities of the motors which introduce a second order system beside the nonlinearity of the system model overall. Conversely, VAL-quadrotor is considered a linear system due to varying the arm's length of which is a function of a first order system. In addition, the best arms length selection is evaluated to mimic the real design environment of VAL-quadrotor. Therefore, the evaluation results proved that VAL-quadrotor capable of following the desired path and improving the response of the system. Thus, VAL-quadrotor considered a linear controlling system, simple and two flight modalities which make VAL-quadrotor compatible with the future requirement.

REFERENCES

[1] A. A. Mian, M. I. Ahmad and D. Wang. 2008.Backstepping based nonlinear flight control strategy for 6 DOF aerial robot.in Smart Manufacturing Application, 2008. ICSMA 2008. International Conference on. pp. 146-151.

[2] S. Bouabdallah, P. Murrieri and R. Siegwart. 2004. Design and control of an indoor micro quadrotor.in Robotics and Automation, 2004. Proceedings. ICRA'04. 2004 IEEE International Conference on. pp. 4393-4398.

[3] Y. Ming and X. Yunqing. 2013. Design of Nonlinear Aero Controller and Simulation for Quadrotor Unmanned Aerial Vehicles.in Software Engineering (WCSE), 2013 Fourth World Congress on. pp. 283-287.

[4] A. R. Patel, M. Patel and D. R. Vyas. 2012. Modeling and analysis of quadrotor using sliding mode control.in System Theory (SSST), 2012 44th Southeastern Symposium on. pp. 111-114.

[5] E. Reyes-Valeria, R. Enriquez-Caldera, S. Camacho-Lara and J. Guichard. 2013. LQR control for a quadrotor using unit quaternions: Modeling and simulation.in Electronics, Communications and Computing (CONIELECOMP), 2013 International Conference on. pp. 172-178.

[6] B. Samir, N. Andk and S. Roland. 2004.PIDvsLQ Control Techniques Applied to an Weight augmentation High energy conruniption Indoor Micro Quadrotor" Proceedings of 2004 1EEElRS. In J Int. Conference Intelligent Robots Syst. pp. 2451-2456.

[7] T. Pruckšakorn, K. Wachirarattanakornkul and I. Nilkhamhang. 2013. Unmanned aerial vehicle for observing landslide with iterative feedback tuning.in Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON), 2013 10th International Conference on. pp. 1-5.

[8] S.-h. Lee, S. H. Kang and Y. Kim. 2011. Trajectory tracking control of quadrotorUAV.in Control,

Automation and Systems (ICCAS), 2011 11th International Conference on. pp. 281-285.

[9] A. A. Mian and W. Daobo. 2008.Modeling and backstepping-based nonlinear control strategy for a 6 DOFquadrotor helicopter. Chinese Journal of Aeronautics. 21: 261-268.

[10] M. K. Joyo, D. Hazry, S. Faiz Ahmed, M. H. Tanveer, F. Warsi and A. Hussain. 2013. Altitude and horizontal motion control of quadrotorUAV in the presence of air turbulence. In Systems, Process and Control (ICSPC), 2013 IEEE Conference on. pp. 16-20.

[11] M. H. Tanveer, S. F. Ahmed, D. Hazry, M. K. Joyo, and F. Warsi. 2013. Disturbance and Noise Rejection Controller Design for Smooth Takeoff/Landing and Altitude Stabilization of Quad-rotor. Journal of Applied Sciences Research. 9: 3316-3327.

[12] D. Lee, H. J. Kim and S. Sastry. 2009. Feedback linearization vs. adaptive sliding mode control for a quadrotor helicopter. International Journal of control, Automation and systems. 7: 419-428.

[13] S. Bouabdallah and R. Siegwart. 2007. Full control of a quadrotor.in Intelligent robots and systems, 2007. IROS 2007. IEEE/RSJ international conference on. pp. 153-158.

[14] H. K. Kim, T. T. Nguyen, S. J. Oh, and S. B. Kim. 2014. Position Control of a Small Scale Quadrotor Using Block Feedback Linearization Control.inAETA 2013: Recent Advances in Electrical Engineering and Related Sciences, ed: Springer. pp. 525-534.

[15] F. Sharifi, M. Mirzaei, B. W. Gordon, and Y. Zhang. 2010. Fault tolerant control of a quadrotorUAV using sliding mode control.in Control and Fault-Tolerant Systems (SysTol), 2010 Conference on. pp. 239-244.

[16] L. R. G. Carrillo, A. E. D. López, R. Lozano and C. Pégard. 2013.Modeling the quad-rotor mini-rotorcraft.in Quad Rotorcraft Control, ed: Springer. pp. 23-34.

[17] A. Nagaty, S. Saeedi, C. Thibault, M. Seto, and H. Li. 2013. Control and navigation framework for quadrotor helicopters. Journal of Intelligent and Robotic Systems. 70: 1-12.

[18] M. H. Tanveer, D. Hazry, S. Faiz, M. Kamran, and F. A. Warsi. 2013. Design of overall Stabilized controller for Quad-rotor.KärntnerBotanikzentrum.