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PRODUCTION PERFORMANCE OF HORIZONTAL GAS WELLS ASSOCIATED WITH NON-DARCY FLOW

Jing Lu[1], Sima Li[1], Md Motiur Rahma[1] and Freddy H. Escobar[2]
[1]Department of Petroleum Engineering, The Petroleum Institute, Abu Dhabi, United Arab Emirates
[2]Universidad Surcolombiana/CENIGAA, Av. Pastrana-Cra. 1, Neiva, Huila, Colombia
E-Mail: jlu@pi.ac.ae

ABSTRACT

In the petroleum literature, non-Darcy flow is assumed to be a near wellbore phenomenon; consequently a gas reservoir could be divided into Darcy's flow domain and non-Darcy's flow domain. Assume only radial flow occurs in the near wellbore non-Darcy's flow domain, and assume the radius of this domain is integer multiple of wellbore radius, Lu et al. (2011) proposed binomial deliverability equations for partially penetrating vertical gas wells and horizontal gas wells. By solving a set of simultaneous equations with respect to non-Darcy's flow domain r_n and flow rate at standard conditions Q_{sc}, this paper presents new binomial deliverability equations for horizontal gas wells, which can account for the advantages of horizontal gas wells where non-Darcy effect is less pronounced than that in vertical gas wells. The calculation results show that non-Darcy flow domain radius is smaller than 15 times wellbore radius, which further proves turbulent effect only occurs in the vicinity of wellbore. The calculation results also show that the production rate loss of horizontal wells caused by the turbulent flow is small.

Keywords: non-darcy flow, productivity, reservoir area, stabilization.

1. INTRODUCTION

Borisov (1964) firstly studied the performance of a horizontal well and how it produced differently from the vertical well. The equation Borisov derived makes the horizontal well perform like an infinite-acting fracture. Joshi (1986) developed an equation for horizontal well productivity through dividing the 3-D model into two 2-D mathematic problems. Babu (1989) developed a horizontal well performance equation for semi-steady state flow and a uniform flux assumption at the wellbore instead of infinite conductivity. This equation is the first to attempt to deal with a changing wellbore pressure, by using the uniform flux assumption, i.e. constant influx rate. Lu (2001) proposed productivity formulae for horizontal wells based on the average potential and the point convergence pressure. As opposed to Joshi's productivity equation which is obtained from a simplified two-dimensional model, Lu's equations are derived from a three dimensional solution of the Laplace equation. Lu (2003) proposed productivity formulae for horizontal wells in steady state with circular cylinder drainage volume. Lu concluded that if the drainage volume is a circular cylinder with gas cap or bottom water, the fluid from the top or bottom boundary flows into a horizontal well approximately vertically. Gas cap drive or bottom water drive is the main drive mechanism. Even if there is edge water drive, it has little influence on the well flow rate. Escobar and Montealegre-M (2008) provided a solution for estimation the productivity index in horizontal wells and they proved to provide better results than Joshi's. Lu *et al.* (2010) pointed out that if the drainage volume is a circular cylinder without a gas cap or bottom water, the top and bottom boundaries are impermeable and only edge water drive is available, and the circular cylinder radius plays an important role in well productivity.

The so-called non-Darcy flow effect in a gas reservoir has been associated with high gas flow rates. The non-Darcy flow in porous media occurs if the flow velocity becomes large enough so that Darcy's law for pressure gradient is no longer sufficient. To describe the nonlinear flow situation, a quadratic term was included by Forchheimer to generalize the flow Equation (Lee and Wattenbarge, 1996):

$$\frac{dP}{dr} = \frac{\mu}{K} v + \beta \rho v^2 \tag{1}$$

where P is pressure, r is radial distance, ρ is gas density, β is gas turbulence factor, v is flow velocity, μ is gas viscosity, and K is permeability. Equation (1) is based on the assumption that only radial flow occurs in the near wellbore non-Darcy's flow domain.

Define pseudo pressure as below (Lee and Wattenbarge, 1996):

$$\Psi(P) = 2\int_{P_{ref}}^{P} \frac{P}{\mu z} dP \tag{2}$$

where P_{wf} is a reference pressure, z is gas deviation factor, ψ is pseudo pressure. Consequently, pseudo pressure gradient is expressed as follows (Lu *et al.*, 2011):

$$\frac{d\Psi}{dr} = \left(\frac{2P}{\mu z}\right)\left(\frac{\mu}{K} v + \beta \rho v^2\right) \tag{3}$$

and gas formation volume factor is (Guo and Ghalambor, 2005) :

$$B_g = \frac{P_{sc} zT}{PT_{sc}} \tag{4}$$

where T is reservoir temperature, P_{sc} and T_{sc} are pressure and temperature at standard conditions, respectively. Gas density can be calculated by (Guo and Ghalambor, 2005):

$$\rho = \frac{28.97\gamma_g P}{z\Re T} \tag{5}$$

where γ_g is gas specific gravity, $\Re$ is gas universal constant, $\Re = 8.314 J/(mol.K)$.

In a radial flow domain, the flow velocity at a distance r from the center of a circular drainage area is given by (Lu *et al.*, 2011):

$$v = \frac{Q}{A} = \frac{B_g Q_{sc}}{A} = \frac{P_{sc} TzQ_{sc}}{PT_{sc}(2\pi r L_p)} \tag{6}$$

where A is cylinder lateral area, Q is gas flow rate at reservoir conditions, L_P is well producing length, Q_{sc} is well flow rate at standard conditions. Substituting Equations (5) and (6) in Equation (3), we obtain (Lu *et al.*, 2011):

$$\frac{d\Psi}{dr} = \frac{P_{sc} TQ_{sc}}{\pi T_{sc} KL_p r} + \frac{28.97\beta\gamma_g P_{sc}^2 TQ_{sc}^2}{2\pi^2 \mu RT_{sc}^2 L_p^2 r^2} \tag{7}$$

As shown in Figure-1, assume only radial flow occurs in the near wellbore non-Darcy's flow domain, and assume non-Darcy's flow domain is a circular cylinder with height L_p and radius r_n, there holds (Lu *et al.*, 2011):

$$\Psi_n - \Psi_w = \left(\frac{P_{sc} TQ_{sc}}{\pi T_{sc} KL_p}\right) \ln\left[\frac{r_n}{r_w}\right] + \left(\frac{28.97\beta\gamma_g P_{sc}^2 TQ_{sc}^2}{2\pi^2 \mu RT_{sc}^2 L_p^2}\right)\left[\frac{1}{r_w} - \frac{1}{r_n}\right] \tag{8}$$

where Ψ_n is pseudo pressure at the outer boundary of non-Darcy's flow domain, Ψ_w is pseudopressure at wellbore.

Cook (1973) defined the Reynolds number as below:

$$R_{\text{eynolds}} = \frac{v\rho K^{1/2}}{15120\mu\phi^{3/2}} \tag{9}$$

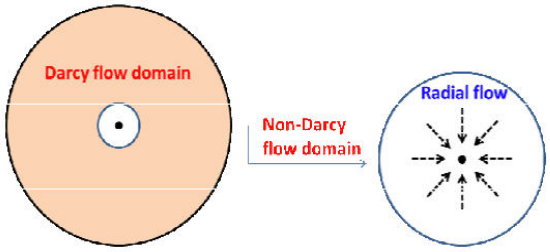

Figure-1. Schematic of non-darcy flow domain and darcy flow domain.

Cook (1973) defined the Reynolds number as below:

$$R_{\text{eynolds}} = \frac{v\rho K^{1/2}}{15120\mu\phi^{3/2}} \tag{9}$$

where $R_{eynolds}$ is the Reynolds number, v is the fluid velocity (m/d); K is permeability (μm^2); ρ is fluid density (g/cm^3); μ is fluid viscosity (mPa.s); ϕ is porosity.

If using field units, Equation (9) should be written as:

$$R_{\text{eynolds}} = \frac{1.014\times10^{-8} v\rho K^{1/2}}{\mu\phi^{3/2}} \tag{10}$$

where v is the fluid velocity (ft/d); K is permeability (mD); ρ is fluid density (lbm/ft^3); μ is fluid viscosity (cp); ϕ is porosity.

If the non-Darcy flow domain is a circular cylinder with height L and radius r_n, the fluid velocity on the lateral surface of the circular cylinder can be expressed as:

$$v = \frac{Q}{A} = \frac{Q}{2\pi r_n L} \tag{11}$$

where Q is the production rate, A is the lateral area. In field units, Equation (11) can be expressed as below:

$$v = \frac{159154.94Q}{r_n L} \tag{12}$$

Substitute Equation (12) into Equation (10), the Reynolds number can be written as:

$$R_{\text{eynolds}} = \frac{1.614\times10^{-3} Q\rho K^{1/2}}{Lr_n\mu\phi^{3/2}} \tag{13}$$

where Q is the production rate (MMscf/d), K is permeability (mD); ρ is fluid density (lbm/ft^3); μ is fluid viscosity (cp); ϕ is porosity. L is the length of the circular

cylinder (ft); r_n is the radius of non-Darcy flow circular cylinder (ft).

By solving a set of simultaneous equations with respect to non-Darcy's flow domain r_n and flow rate at standard conditions Q_{sc}, this paper presents new binomial deliverability equations for horizontal gas wells, which can account for the advantages of horizontal gas wells where non-Darcy effect is less pronounced than that in vertical gas wells.

2. EQUATIONS AND SOLUTIONS

Figure-2 shows the schematic of a circular cylinder reservoir model. In this paper, well model, reservoir model, reservoir initial condition and boundary conditions are the same as those given in Lu *et al*. (2011).

For a point sink at (x', 0, z') on the horizontal wellbore, there holds,

$$\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)= \frac{\partial(\phi\rho)}{\partial t}+q^*\delta(x-x')\delta(y)\delta(z-z') \tag{14}$$

where q^* is point sink mass flow rate, SI unit for q^* is *kg/s*, ρ is density, SI unit for ρ is kg/m^3. Note that,

$$\rho=\frac{PM}{z\Re T}$$
$$v_x=\frac{K_x}{\mu}\frac{\partial P}{\partial x};\ v_y=\frac{K_y}{\mu}\frac{\partial P}{\partial y};\ v_z=\frac{K_z}{\mu}\frac{\partial P}{\partial z}$$
$$\Psi=2\int_0^P\left(\frac{P}{\mu z}\right)dP \tag{15}$$

Define average permeability as below:

$$K_a=\left(K_xK_yK_z\right)^{1/3}=K_h^{2/3}K_v^{1/3} \tag{16}$$

and define the following dimensionless parameters:

$$x_D=\left(\frac{x}{L}\right)\left(\frac{K_a}{K_x}\right)^{1/2};\ y_D=\left(\frac{y}{L}\right)\left(\frac{K_a}{K_y}\right)^{1/2};\ z_D=\left(\frac{z}{L}\right)\left(\frac{K_a}{K_z}\right)^{1/2} \tag{17}$$

$$t_D=\frac{K_at}{\phi\mu c_tL^2};\quad \Psi_D=\frac{K_aML(\Psi_i-\Psi)}{2q\Re T} \tag{18}$$

Consequently, Equation (14) is changed to,

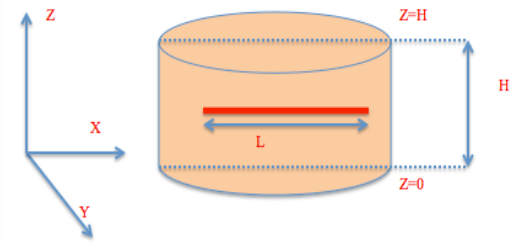

Figure-2. Schematic of a horizontal well in a circular cylinder reservoir.

$$\frac{\partial^2\Psi_D}{\partial x_D^2}+\frac{\partial^2\Psi_D}{\partial y_D^2}+\frac{\partial^2\Psi_D}{\partial z_D^2}= \frac{\partial\Psi_D}{\partial t_D}-\delta\left(x_D-x_D'\right)\delta\left(y_D\right)\delta\left(z_D-z_D'\right) \tag{19}$$

In steady state,

$$\frac{\partial^2\Psi_D}{\partial x_D^2}+\frac{\partial^2\Psi_D}{\partial y_D^2}+\frac{\partial^2\Psi_D}{\partial z_D^2}= -\delta\left(x_D-x_D'\right)\delta\left(y_D\right)\delta\left(z_D-z_D'\right) \tag{20}$$

Case 1: The lateral boundary pressure is constant, $P_e=P_i$, the top and bottom boundaries are impermeable, we have,

$$\Psi_D\big|_{r_D=r_{eD}}=\Psi_{eD}=\Psi_{iD} \tag{21}$$

$$\frac{\partial\Psi_D}{\partial z_D}\Big|_{z_D=0}=\frac{\partial\Psi_D}{\partial z_D}\Big|_{z_D=H_D}=0 \tag{22}$$

$$\frac{\ \ _D}{r_D}\Big|_{r_D=r_{eD}}=0,\quad \frac{\ \ _D}{z_D}\Big|_{z_D=H_D}=0 \tag{23}$$

$$\Psi_D\big|_{Z_D=0}=\Psi_{eD}=\Psi_{iD} \tag{24}$$

Solving the diffusivity equation of a horizontal gas well in a circular cylinder drainage volume with impermeable top and bottom boundaries, and constant lateral boundary, we obtain (Lu, 2003):

$$\Psi_D=\frac{\Lambda}{3\pi H_D}=\frac{\Lambda L}{3\pi H} \tag{25(a)}$$

where

$$\Lambda = \left[\ln\left(\frac{2R_e}{L}\right)+1\right] - \left(\frac{5H}{4L}\right)\ln\left[4\sin\left(\frac{\pi Z_w}{H}\right)\sin\left(\frac{\pi r_w}{H}\right)\right] + \frac{R_e^2}{L^2}\ln\left(\frac{4R_e^2+L^2}{4R_e^2-L^2}\right) + \frac{1}{4}\ln\left(\frac{R_e^4}{L^4}-\frac{1}{16}\right) - \frac{1}{2}\ln\left(\frac{R_e}{L}\right) \quad 25(b)$$

From Equation (25a), Q_{sc} can be solved,

$$Q_{sc} = \frac{3\pi KHT_{sc}\Delta\Psi}{2TP_{sc}\Lambda} \quad (26)$$

In the Darcy flow domain:

$$\Psi_e - \Psi_n = \frac{2TP_{sc}\Lambda Q_{sc}}{3\pi KHT_{sc}} \quad (27)$$

If there exists non-Darcy's flow area near wellbore, combine Equation (27) and Equation (8), we obtain:

$$\Psi_e - \Psi_n = \frac{2TP_{sc}\Lambda Q_{sc}}{3\pi KHT_{sc}} \quad 28(a)$$

where:

$$a = \frac{P_{sc}T}{\pi KLT_{sc}}\ln\left(\frac{r_n}{r_w}\right) + \frac{2TP_{sc}\Pi}{3\pi KHT_{sc}} \quad 28(b)$$

$$b = \frac{28.97\beta\gamma_g P_{sc}^2 T}{2\pi^2\mu\Re T_{sc}^2 L^2}\left(\frac{1}{r_w}-\frac{1}{r_n}\right) \quad 28(c)$$

$$\Pi = \left[\ln\left(\frac{2R_e}{L}\right)+1\right] - \left(\frac{5H}{4L}\right)\ln\left[4\sin\left(\frac{\pi Z_w}{H}\right)\sin\left(\frac{\pi r_w}{H}\right)\right] + \frac{R_e^2}{L^2}\ln\left(\frac{4R_e^2+L^2}{4R_e^2-L^2}\right) + \frac{1}{4}\ln\left(\frac{R_e^4}{L^4}-\frac{1}{16}\right) - \frac{1}{2}\ln\left(\frac{R_e}{L}\right) \quad 28(d)$$

For Case 1, the binomial deliverability equation in field units can be expressed as below:

$$\Psi_e - \Psi_w = \left[\frac{1.422T}{KL}\ln\left(\frac{r_n}{r_w}\right) + \frac{0.948T\Pi}{KH}\right]Q_{sc} + 4.48\times10^{-19}\frac{\beta\gamma_g T}{\mu L^2}\left(\frac{1}{r_w}-\frac{1}{r_n}\right)Q_{sc}^2 \quad (29)$$

If there does not exist non-Darcy flow near the wellbore, the deliverability equation for Case 1 is:

$$Q_{sc} = \frac{1.059KH\Delta\Psi}{T\Lambda} \quad (30)$$

For Case 2, the binomial deliverability equation in field units is expressed as below:

$$\Psi_e - \Psi_w = \left\{\frac{1.422T}{KL}\ln\left(\frac{r_n}{r_w}\right) + \frac{1.185T\{\ln[4H/(\pi r_n)] + \ln\tan[(\pi Z_w)/(2H)]\}}{KL}\right\}Q_{sc} + 4.480\times10^{-19}\left[\frac{\beta\gamma_g T}{\mu L^2}\left(\frac{1}{r_w}-\frac{1}{r_n}\right)\right]Q_{sc}^2 \quad (31)$$

If there does not exist non-Darcy flow near the wellbore, the deliverability equation for Case 2 is:

$$Q_{sc} = \frac{0.847KL\Delta\Psi}{T\{\ln[4H/(\pi r_w)]\} + \ln\tan[(\pi Z_w)/(2H)]} \quad (32)$$

3. SENSITIVITY ANALYSIS

The deliverability equations (considering non-Darcy flow near the wellbore) for the above two cases can be expressed in the following form:

$$\Psi_e - \Psi_w = aQ_{sc} + bQ_{sc}^2 \quad (33)$$

Coefficients a and b are actually functions of r_n , a and b can be expressed as: $a = f(r_n)$, $b = F(r_n)$. Consequently, there holds:

$$\Psi_e - \Psi_w = f(r_n)Q_{sc} + F(r_n)Q_{sc}^2 \quad (34)$$

If there is non-Darcy flow domain near the wellbore, there exists the following relationship between r_n and Q_{sc}:

$$R_{\text{eynolds}} = \frac{1.614\times10^{-3}Q_{sc}\rho K^{1/2}}{Lr_n\mu\phi^{3/2}} \quad (35)$$

where $R_{eynonlds}$ is the critical Reynolds number which is between 0.2-0.3. If all the other factors are known except for Q_{sc} and r_n, from Equation (35), we know that r_n is a function of Q_{sc}, and the relationship between Q_{sc} and r_n can be expressed as follows:

$$r_n = y(Q_{sc}) \quad (36)$$

If we combine Equation (36) and Equation (34), we can obtain the following set of simultaneous equations:

$$\begin{aligned}\Psi_e - \Psi_w &= f(r_n)Q_{sc} + F(r_n)Q_{sc}^2\\ r_n &= y(Q_{sc})\end{aligned} \quad (37)$$

in which only Q_{sc} and r_n are unknown. If all the other factors are given, Q_{sc} and r_n can be calculated with Matlab® codes.

Using the hypothetical data in Table-1 and the method of solving the set of simultaneous equations

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mentioned above, the following figures give the non-Darcy radius r_n, the production rate considering non-Darcy flow $Q_{sc\text{-}non\text{-}Darcy}$, and the ratio of $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ with various values of payzone thickness H and different values of permeability K and horizontal well length L.

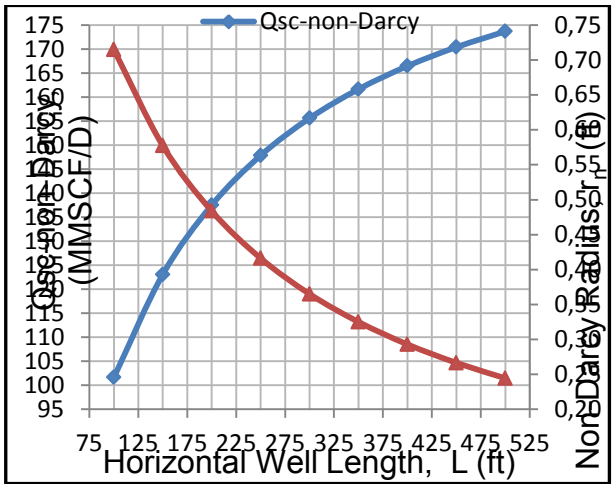

Figure-3. The effects of L on $Q_{sc\text{-}non\text{-}Darcy}$ and r_n (Case 1).

Table-1. Given Gas Reservoir/Well Data.

r_w (ft)	0.25	β (1/ft)	$2x10^8$
T (ºR)	590	K (mD)	20
r_e (ft)	3000	γ_g	0.65
μ (cp)	0.02	$\Delta\Psi$ ($MMpsi^2/cp$)	$2.55x10^2$
P_w (psi)	500	P_i (psi)	2200
Z_w (ft)	25	z	0.9
P_{SC} (psi)	14.7	T_{SC} (ºR)	520

3.1. Effects of horizontal well length

Figure-3 shows the effect of horizontal well length L on the non-Darcy radius r_n and the production rate $Q_{sc\text{-}non\text{-}Darcy}$ when all the other factors stay constant, and the reservoir is with impermeple top and bottom boundaries.

From Figure-3, the production rate $Q_{sc\text{-}non\text{-}Darcy}$ is an increasing function of the horizontal well length L. The non-Darcy flow radius r_n decreases with the horizontal well length L increasing. r_n decreases approximately 3 times as the well length L increases from 100 ft to 500 ft. From Equation (12), when L increases, fluid velocity v decreases, which introduces smaller r_n.

Figure-4 shows the effect of horizontal well length L on the ratio of $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ when all the other factors stay constant. Also, From Figure-4, it is noticed that as the horizontal well length L increases, the ratio of $Q_{sc\text{-}non\text{-}Darcy}$ to $Q_{sc\text{-}Darcy}$ also increases from 96.75% to 99.75%. As shown in Figure-3, the non-Darcy flow radius r_n decreases when L increases, which means the turbulent effect in the vicinity of the wellbore doesn't get more serious with L increasing. The increment of the ratio of $Q_{sc\text{-}non\text{-}Darcy}$ to $Q_{sc\text{-}Darcy}$ can be attributed to the fact that the negative effect of turbulent flow on the production rate is offset by the positive effect of the increasing well length.

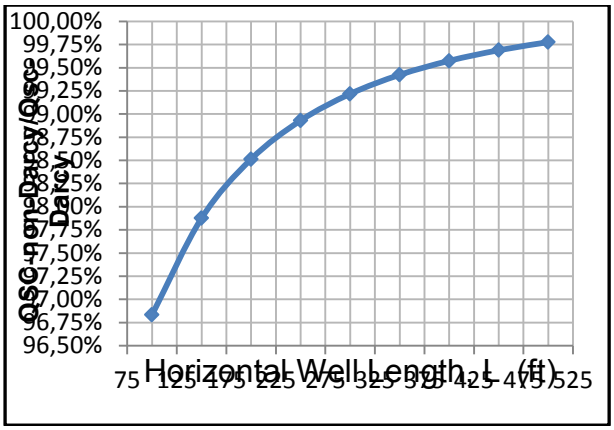

Figure-4. The effects of L on $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ (Case 1).

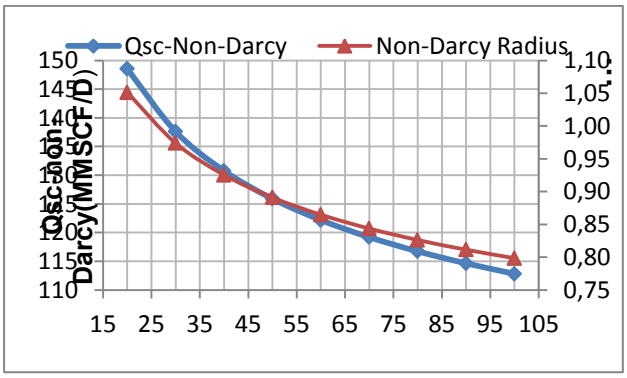

Figure-5. The effects of H on $Q_{sc\text{-}non\text{-}Darcy}$ and r_n (Case 2).

3.2. Effects of payzone thickness, *H*

Figure-5 shows the effect of payzone thickness H on the non-Darcy radius r_n and the production rate $Q_{sc\text{-}non\text{-}Darcy}$ when all the other factors stay constant, and the reservoir is with impermeable top boundary, and with bottom water drive.

From Figure-5, it is noticed that the production rate $Q_{sc\text{-}non\text{-}Darcy}$ is a decreasing function of the pay zone thickness H. The driving force is from bottom water, when the pseudo-pressure difference is fixed, the big H will introduce long distance between bottom water and wellbore, and will further introduce small pressure gradient and descending flow rate $Q_{sc\text{-}non\text{-}Darcy}$. Consequently, the flow velocity decreases, which means the turbulent effect weakens with increasing H. Then non-Darcy flow radius also decreases as the pay zone thickness increases. The non-Darcy radius ranges from 1.1 ft to 0.8 ft, approximately 4.4 times r_w to 3.2 times r_w.

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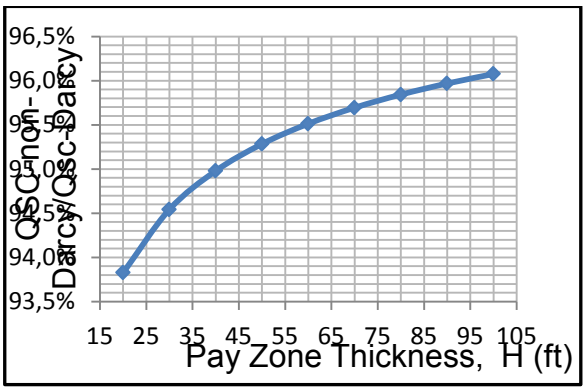

Figure-6. The effects of H on $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ (Case 2).

Figure-6 shows the effect of payzone thickness H on the ratio of $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ when all the other factors stay constant. From Figure-6, it is noticed that as the payzone thickness H increases, the ratio of $Q_{sc\text{-}non\text{-}Darcy}$ to $Q_{sc\text{-}Darcy}$ also increases from 93.8% to 96.1%. As is shown in Figure-5, the non-Darcy flow radius r_n decreases when H increases from 20 ft to 100 ft, which means the turbulent effect in the vicinity of the wellbore weakens with H increasing. The increment of the ratio of $Q_{sc\text{-}non\text{-}Darcy}$ to $Q_{sc\text{-}Darcy}$ can be attributed to the fact that the negative effect of turbulent flow on the production rate weakens with increasing pay zone thickness H.

3.3. Effects of permeability K

Figure-7 shows the effect of permeability K on the non-Darcy radius r_n and the production rate $Q_{sc\text{-}non\text{-}Darcy}$ when all the other factors stay constant, and the reservoir is with impermeable top boundary, and with bottom water drive.

From Figure-7, it is noticed that the $Q_{sc\text{-}non\text{-}Darcy}$ is an increasing function of the permeability K, when K increases from 20 mD to 100 mD, the $Q_{sc\text{-}non\text{-}Darcy}$ increases from 125 MMscf/D to 582 MMscf/D. When K increases, turbulent flow becomes more pronounced, consequently the non-Darcy flow radius increases from 0.9 ft to 9.2 ft (approximately 3.6 times r_w to 37 times r_w). Compared to payzone thickness H and horizontal well length L, the non-Darcy flow radius r_n is more sensitive to permeability K.

Figure-8 shows the effect of permeablitity K on the ratio of $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ when all the other factors stay constant. From Figure-8, it is noticed that the ratio of $Q_{sc\text{-}non\text{-}Darcy}$ to $Q_{sc\text{-}Darcy}$ is a decreasing function of the permeability K. When K increases from 20 mD to 100 mD, the ratio of $Q_{sc\text{-}non\text{-}Darcy}$ to $Q_{sc\text{-}Darcy}$ decreases from 95.2% to 88.2%. The decrease of the ratio is because when permeability K increases, the negative effect of turbulent flow on production rate becomes more pronounced, which will cause an increasing difference between $Q_{sc\text{-}non\text{-}Darcy}$ and $Q_{sc\text{-}Darcy}$.

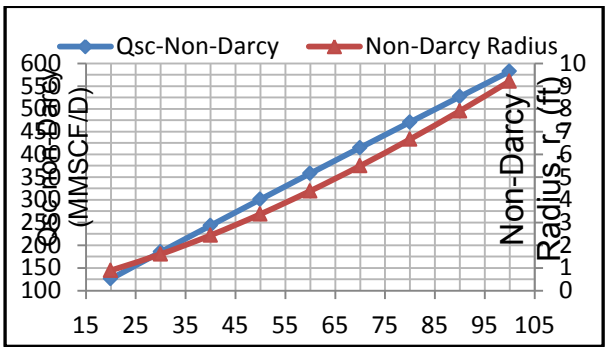

Figure-7. The effects of K on $Q_{sc\text{-}non\text{-}Darcy}$ and r_n (Case 2).

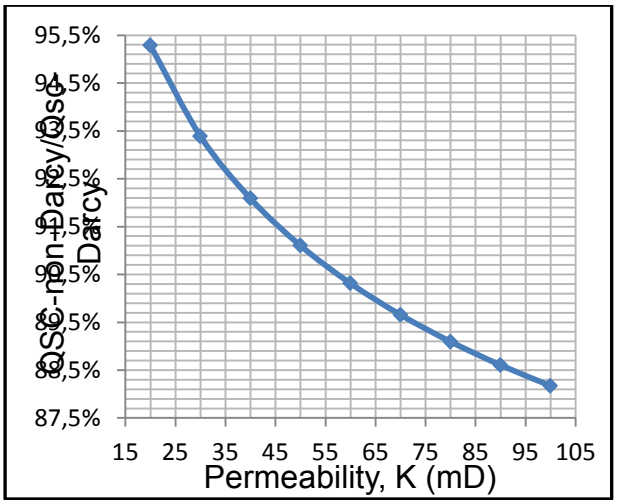

Figure-8. The effects of K on $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ (Case 2).

4. COMPARISON BETWEEN HORIZONTAL WELLS AND VERTICAL WELLS

Using the data in Table-1 and assuming the open interval of a vertical well to be 25 ft, the sensitivity analysis of the permeability K is conducted for both the vertical well and the horizontal well in a cylindrical drainage volume with bottom water.

Figure-9 shows the effect of permeability K on the production rate $Q_{sc\text{-}non\text{-}Darcy}$ when all the other factors stay constant. Note that the producing length of the horizontal well is 100 ft which is 4 times of the producing length of the vertical well.

From Figure-9, it is observed that when K increases from 20 mD to 100 mD, $Q_{sc\text{-}non\text{-}Darcy}$ of the vertical well increases 3 times (from 50 MMscf/D to 150 MMscf/D), meanwhile $Q_{sc\text{-}non\text{-}Darcy}$ of the horizontal well increases 4.6 times (from 125 MMscf/D to 575 MMscf/D). Thus, the non-Darcy negative effect on production rate for the vertical well is more pronounced than that for horizontal well. It is concluded that the bigger the permeability is, the more obvious advantage the horizontal well has over the vertical well in terms of $Q_{sc\text{-}non\text{-}Darcy}$.

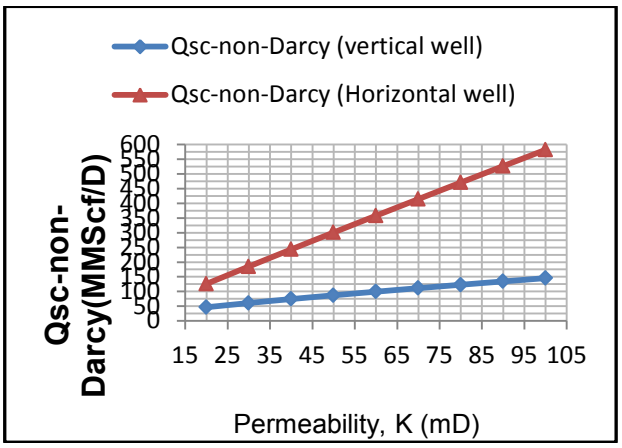

Figure-9. The effects of K on $Q_{sc\text{-}non\text{-}Darcy}$ of Vertical Well and Horizontal Well.

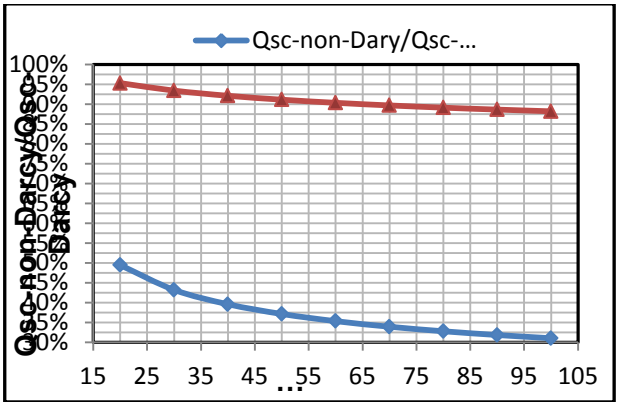

Figure-10. The effects of K on $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ of Vertical Well and Horizontal Well.

Figure-10 shows the effect of permeablitity K on the ratio of $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ when all the other factors stay constant. When K increases from 20 mD to 100 mD, $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ decreases from 95% to 87.5% for the horizontal well, meanwhile $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ decreases from 50% to 30% for the vertical well. $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ for the horizontal well is much higher than the vertical well, which means the negative effect on production rate caused by the turbulent flow for the vertical well is more pronounced. The reason for this phenomenon is the producing length of the vertical well is much smaller than that of the horizontal well, so even though the total production rate of the vertical well is low, the production rate of per unit length of the vertical well is still higher than that of the horizontal well, in other words, the fluid speed around the vertical wellbore is high, which causes a more pronounced non-Darcy effect.

CONCLUSIONS

Based on this study, the following conclusions are reached:

a) By the method of solving a set of simultaneous equations, the radius of non-Darcy's flow domain of a horizontal gas well can be calculated.
b) From sensitivity analysis, mostly the radius of non-Darcy's flow domain is smaller than 15 times r_w, which further proves turbulent flow only occurs in the vicinity of a horizontal well.
c) Of all the factors causing turbulent flow, permeability K is the most important one.
d) From the ratio of $Q_{sc\text{-}non\text{-}Darcy}/Q_{sc\text{-}Darcy}$ of a horizontal gas well, it is concluded that the production rate loss caused by the turbulent effect is usually no more than 15%.
e) As opposed to vertical wells, of which the production rate loss caused by the turbulent effect is usually up to more than 50% of $Q_{sc\text{-}Darcy}$, horizontal wells have a good capability of resisting the negative effect of turbulent flow.

Table-2. Units conversion factors.

Field units	SI units
bbl *0.1589873	m^3
*cp*0.001	*Pa·s*
*ft*0.3048	*m*
*psi*6894.7259	*Pa*
day/86400	*s*

β	gas turbulence factor, 1/m
ϕ	porosity, fraction
μ	gas viscosity, Pa.s
ρ	gas density, kg/m^3
γ_g	gas specific gravity
Λ	a function defined by Equation（25-b）
Π	a function defined by Equation（28-d）
Ω	drainage domain
Ψ	pseudo pressure, Pa/s

Nomenclature

A	cylinder lateral area, m^2
B_g	gas formation volume factor, Rm^3 / Sm^3
H	pay zone thickness, m
K	formation effective permeability, m^2
L	drilled well length, m
L_p	producing well length, m
L_1	z coordinate of the beginning wellbore point on producing well length, m
L_2	z coordinate of the end wellbore point on producing well length, m
P	pressure, *Pascal*
Q	gas flow rate at reservoir condition, Rm^3/s
Q_{sc}	gas flow rate at standard condition, Sm^3/s
$R_{eynolds}$	Reynolds number
$\Re$	gas universal constant, *J/mol.kelvin*
r_w	wellbore radius, m
r_n	non-Darcy flow domain radius, m
R_e	circular cylinder drainage radius, m
S	skin factor
v	flow velocity, *m/s*
T	gas temperature, *Kelvin*

Z	gas deviation factor, dimensionless.
Z_w	horizontal well location in vertical direction, *m*
e	external
i	initial
n	non-Darcy
p	producing
sc	standard condition
w	well

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