SIMULATION MODEL PERFORMANCE EVALUATION OF REPAIR-DIAGNOSTIC COMPLEX

Abu-Abed F. N.[1], Martynov D. V. [1], Ivanova A. V.[2], Dopira R. V.[2] and Kordyukov R. Yu.[3]
[1] Tver State Technical University, Nikitin Quay, Tver, Russian Federation
[2]NPO RusBITex, Kalininaavenue, Tver, Russian Federation
[3]Main Department of Scientific and Research Activities and Technological Support of the Advanced Technologies of the Ministry of Defense of the Russian Federation, Profsoyuznaya, Moscow, Russian Federation

ABSTRACT

A promising way to improve the repair of complex geographically distributed technical systems (GDTS), organized on the industrial production, resource extraction bases, is the creation of repair and complex (RDC).RDC generally includes in its membership sites operational monitoring, diagnostics of components used in GDTS, as well as sites equipped with the necessary technological equipment, measuring tools, instruments and tools for maintenance replacement elements (RE). To ensure the repair, RDF equipped with a means of power supply, spare equipment and accessories (SEA) and operational documentation. It is assumed that personnel carrying out repairs have the appropriate expertise.

Analysis of the functioning of such RDC is actual problem for many technological spheres.

The most frequently RDC can be represented by a network of queuing network (QN). The study of such system with distribution functions of service time other than exponential complicates the use of analytical models, so the most effective is the use of simulation methods.

The article presents basic principles of creating models of complex systems and discusses aspects of a structured approach to the simulation of such systems.

The developed software implementation two RDC models allows us to simulate the process of its functioning with followed assessment of characteristics, the most important of which is the average time to service requests in the system, and determine the number of channels necessary on each of the sites. The model also provides the ability to use beta-distribution as a priory to set the values of duration of work and some other random distributions. The developed model can be aggregated into the overall research model processes operating GDTS.

The software tool is implemented in an object oriented programming language C# with using an integrated development environment (IDE) software Microsoft Visual Studio 2010.

Keywords: repair and diagnostic equipment, simulation, queuing system, geographically distributed technical systems, replacement components.

INTRODUCTION

An important problem in the development and continued operation RDC in the GDTS is to maximize the effectiveness of their work to ensure high production and economic indicators that cannot be done without making a careful selection and configuration parameters, and not finding the optimal settings. The process of eliminating failures and malfunctions of complex technical systems includes pre-operation check, revealing the fact of failure (refusal) – controls, the localization of fault – diagnostics, and elimination of failure – repair. Coming for repair RE (requests) if they are real inoperability consistently pass functional testing, diagnostics to localize faults and repair with the appropriate equipment and spare replacement part (SRP) [1].In some cases, the first stages – monitoring and diagnostics – are expedient to consolidate and secure their passage in one subsystem. This is due to the logical relationship between these processes and their co-localization in technical systems. Past repair RE re-tested functional testing, and failure detection process is repeated. If the repair is carried out successfully, RE returned to replenish expended sets of operational SRP.

Such RDC models widely distributed and used in technical systems of various industries, so in what follows we consider RDC with such a structure and behavior. When creating data models for their subsequent analysis for the purpose of maximizing the efficiency inevitably raises issues of distribution of service time intervals for requests in different sites, the arrival time interval units of load flows. All this, starting immediately with an adequate real object model allows to determine the methods of computer simulation.

STRUCTURE RDC MODELS

Based on the above features of the functioning of the RDC, it can be represented as QN with the relevant block diagrams (Figure-1 and 2).

Coming from GDTS RE represent the input flow for RDC. At the same time some models need to ensure separation of the input load into two flows with different intensity. Working RE, got to the input of RDC on false registration failure (probability Pf), go directly into the output flow. The other RE consistently pass the separate or combined control, diagnostics, and further repairs on the appropriate sites.

Each of the sites consists of several lines. Past repair RE with probability Prep at the input after the re-test go to the output flow, and with probability

$P_{rep} = 1 - \overline{P_{rep}}$ on a site of diagnostics and, in the following, on the site of repair, etc.

QN consist of several units arranged in series model, that is consists of several queuing systems (QS) [2].

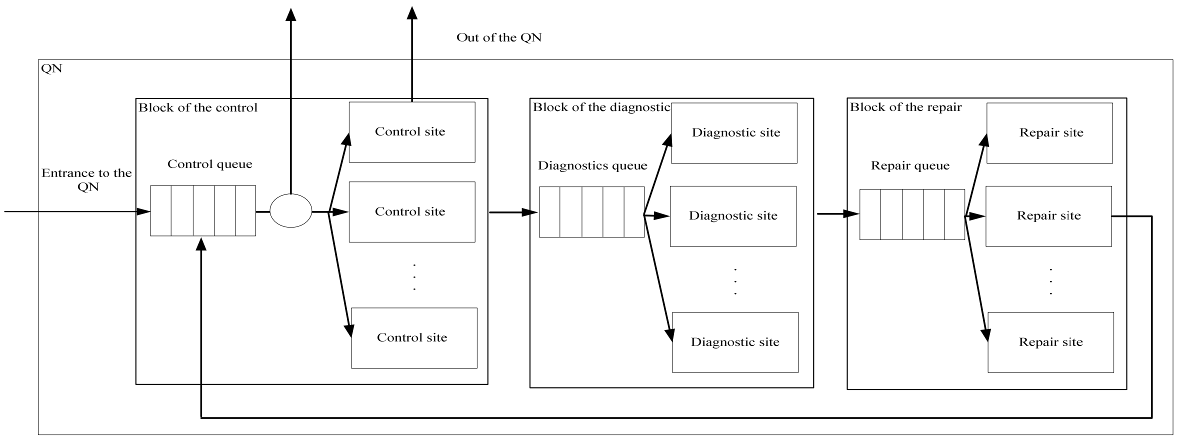

Figure-1. Representation RDCQN with differentiated stages control-diagnostics.

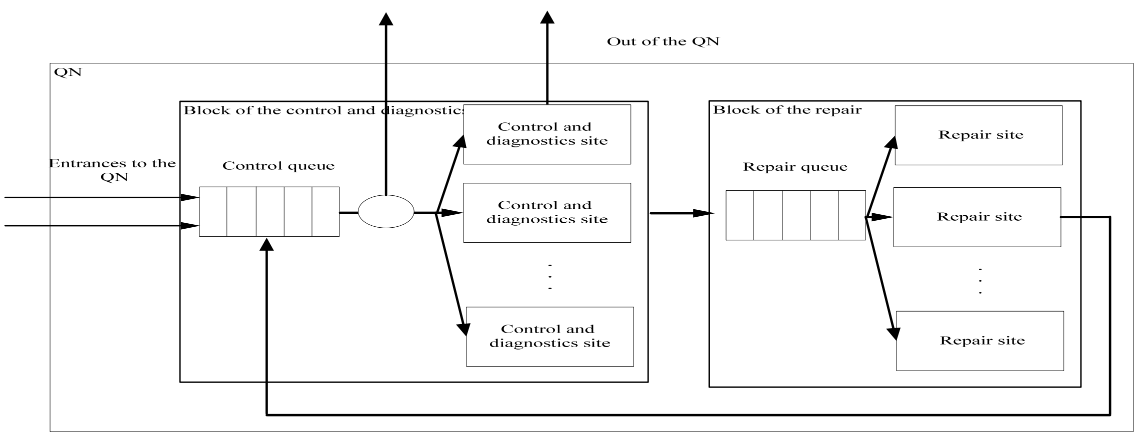

Figure-2. Representation RDC QN with two sources of load.

All requests served in the one node are sent to the next node. In other words, the output flow of one node QN is the input of the next [5].

The exact calculation of the characteristics of such QN is only possible if all flows of requests are Poisson, and all service time are random variables distributed according to an exponential law. In other cases, only an approximate calculation of the QN characteristics is possible.

When calculating the characteristics of multiphase QN consider the following aspects [4]:

- if the input QS receives multiple flows of requests, the total flow intensity of the requests in this QS is the sum of the individual flows intensities;
- if the input QS receives some of the requests from flow, the intensity of which is equal to λ, the input flow intensity in the QS can be defined by the formula: $\lambda_{in}(t) = P\lambda(t)$, where P is probability of getting a request in the input flow;
- the output flow intensity in the QS (that is the flow of serviced requests) is equal to the intensity of input flow.

Requests received for service in the QS form flow of requests. Serving requests elements of the QS create channels of service.

Block diagrams differences of the first and second RDC models are due to feature scope of their use.RDC, the structure of which provides separation of incoming flows for two, is more flexible and allows to fine-tune the receipt of requests process for service. On the other hand, models with a more careful differentiation phases of pre-repair service, separating it into two individual phases control and diagnostics can more accurately reflect the actual aspects of the system, if it is used divided inspection of RE.

In most cases the interval between the time of requests receipt and/or service time of requests in the QS is a random variable, it is unknown in advance when the next request arrive and how long it takes care. So the

VOL. 11, NO. 16, AUGUST 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

theory of queuing systems is based on the mathematical apparatus of the theory of probability and mathematical statistics.

The most accurate calculation of characteristics possible for QS, which have a Poisson (simple) flow of requests. Poisson flow of requests has the properties of the recurrent, stationary, ordinary and absence of aftereffects.

If under these conditions the service time of requests for each site submit an exponential law, in this case, the model QN easily described by a system of differential equations.

However, in practice, service time may be more accurately represented with beta-distribution [8].

Substantiation of a choice of the distribution law for the duration of work in the RDC model

QS in which the service process is influenced by random effects [4] it is assumed that the duration of the works is a random variable. It is assumed that the random variables are subject to the duration of the works accepted for this QS distribution law, and the type of distribution adopted the same for all works. As for the distribution parameters, they are set for each work their responsible executives based on either normative data or a priori considerations, or its production experience.

Some QS are given three options: a lower bound *a* of the domain (*optimistic time*), the upper bound *b* (*pessimistic time*) and the mode of the distribution *m* (*most probable time*).

Other systems are given only two parameters – estimates a and b. In almost all QS priori assumed that the density distribution of time estimates for the duration of the work has three properties: a) continuity, b) unimodal, c) has two non-negative point of intersection with the abscissa. Simple distribution with such properties is a beta distribution, which in practice generally postulated. General view of the beta distribution is characterized, in addition to having a large number of random factors, each of which individually have little, insignificant influence, the presence of several, also random factors, the number of which is small, but the effect is material. As a result of the influence of significant factors probability distribution is usually asymmetric. Just such a situation occurs in the implementation of the vast majority of members of the network project work. This implies the possibility to choose a beta distribution as a priori model. The analysis of large amounts of statistical data (duration time of the individual works implementation, normative data, and so on) also confirms the possibility of using the beta distribution as a priori. Formula of beta-distribution density is as follows:

$$B(p,q,x) = \begin{cases} \frac{1}{B(p,q)} x^{p-1} \text{if } 0 \le x \le 1, \\ 0 \text{ if } x < 0, x > 1, \end{cases} \quad (1)$$

Where $B(p,q)$ –beta-function, with

$$B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx = \frac{G(p)G(q)}{G(p+q)}, \quad (2)$$

$$G(z) = \int_0^\infty e^{-t} t^{z-1} dt,$$

and the gamma-function G(z) is defined by the formula

and for integer values z gamma-function G(z) = 1× 2 × ...× (z - 1) = (z - 1)! Initial moment of the r-th order is determined by the formula

$$\frac{1}{B(p,q)} \int_0^1 x^{r+p-1}(1-x)^{q-1}dx = \frac{B(p+r,q)}{B(p,q)}. \quad (3)$$

When r = 1 the mathematical expectation

$$Mx = \frac{p}{p+q}. \quad (4)$$

When r = 2 the dispersion

$$Dx = \frac{pq}{(p+q)^2(p+q+1)}. \quad (5)$$

The probability density of operation time, distributed according to the law of the beta distribution on the interval [a, b] is [4]:

$$f(t) = \begin{cases} \frac{1}{(b-a)^{\alpha+\beta-1}B(\alpha,\beta)}(t-a)^{\alpha-1}(b-t)^{\beta-1} \text{ if } a \le t \le b \\ 0, \quad \text{otherwise}. \end{cases}$$

For example, for the construction of probability theory apparatus of PERT, considered that the duration of any work is a random variable distributed according to the law of the beta distribution on the interval [a, b], and the parameters of the distribution law – mean M and varianceσ^2–defined by the formulas:

$$M = \frac{a+4m+b}{6}, \quad \sigma^2 = \frac{(b-a)^2}{36},$$

Where a, b and m–respectively, optimistic, pessimistic, and most likely (mode) evaluation, establishes by responsible executives of work.

SYSTEM LIFE CYCLE

After entering requests in QN it is possible with a given probability termination service cycle and exit from the RDC. After the exit from the repair subsystem requests are returned to the control subsystem, with a given probability continuing service or leaving the RDC.

Number of storage spaces of each node is not limited to, all subsystems without failure. In the simulated QN used queue discipline – the principle according to which the input to the serving system requirements are selected from the queue for service – defined by the rule FIFO (First In, First Out).

ARPN Journal of Engineering and Applied Sciences

In the current implementation of simulation model time moves discretely, on the principle of shifting a model time to the next event.

Model time shifted forward on a not fixed value, but exactly to the time of occurrence the earliest of the next events. It has several advantages: the events are considered and modeled in the moments of their accomplishments, and both, if they have the same time of occurrence. Periods of time when in the model nothing happens, passed without much computer time.

The possible variant shifts the time upon the occurrence of major – leading to changes in the system state – event has the form shown in Figure-3.

Timing diagrams for this QN, representing the model time shift in accordance with the occurrence of events on this basis will be as follows:

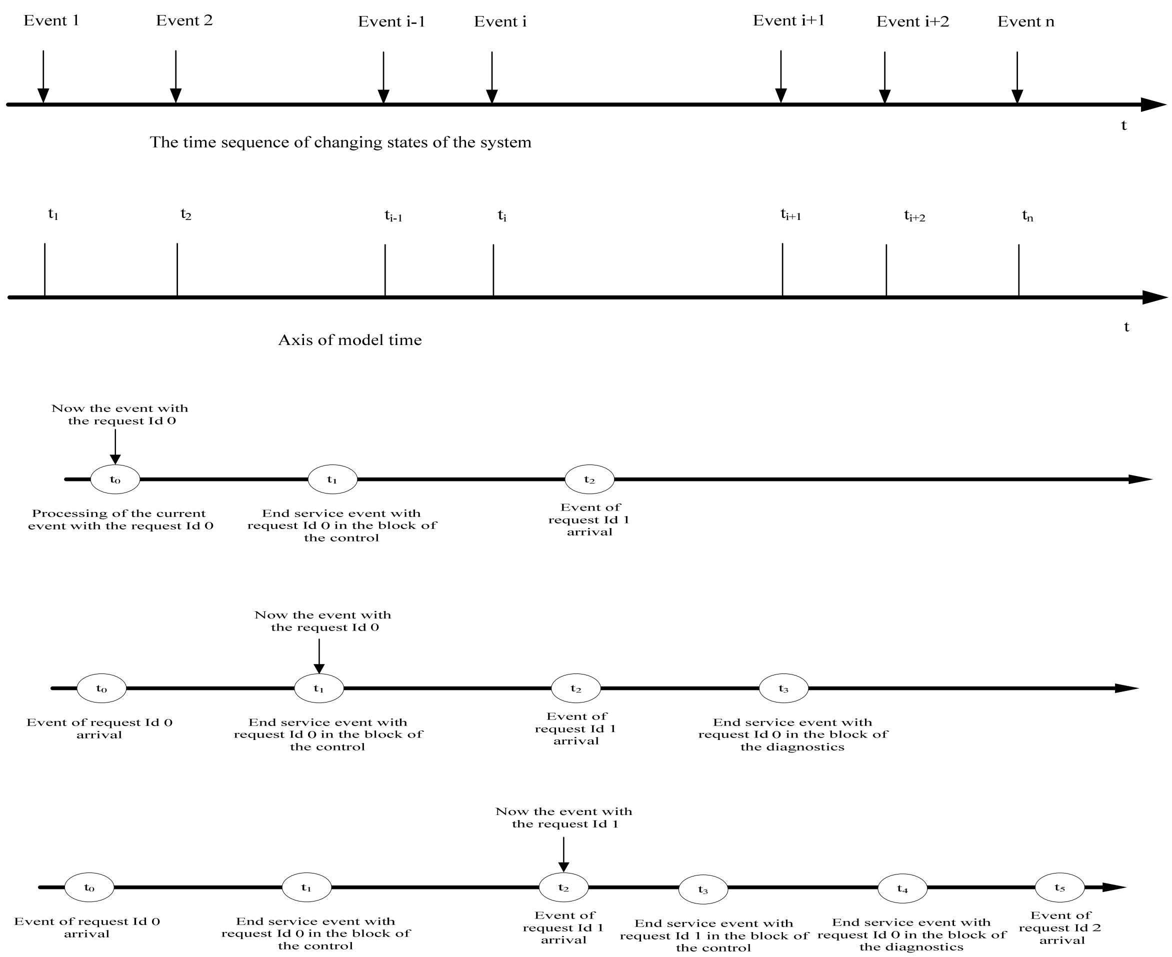

Figure-3. Model time shift.

In implementing the program models QN is advisable to use an object-oriented approach. The structure of generalized into one QN elements as follows:

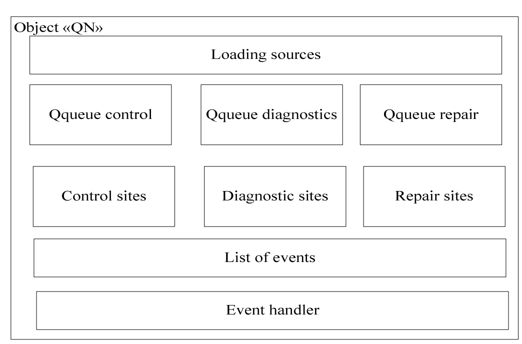

Figure-4.QN structure.

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Upon the occurrence of each major events that simulates by sampling the nearest event from the list of current events, there is a shift of model time and delete the event from the list as processed. The main steps for modeling of object with structural scheme, which provides more than one source of load and joint control and diagnostic services, have the following sequence shown in Figure-5:

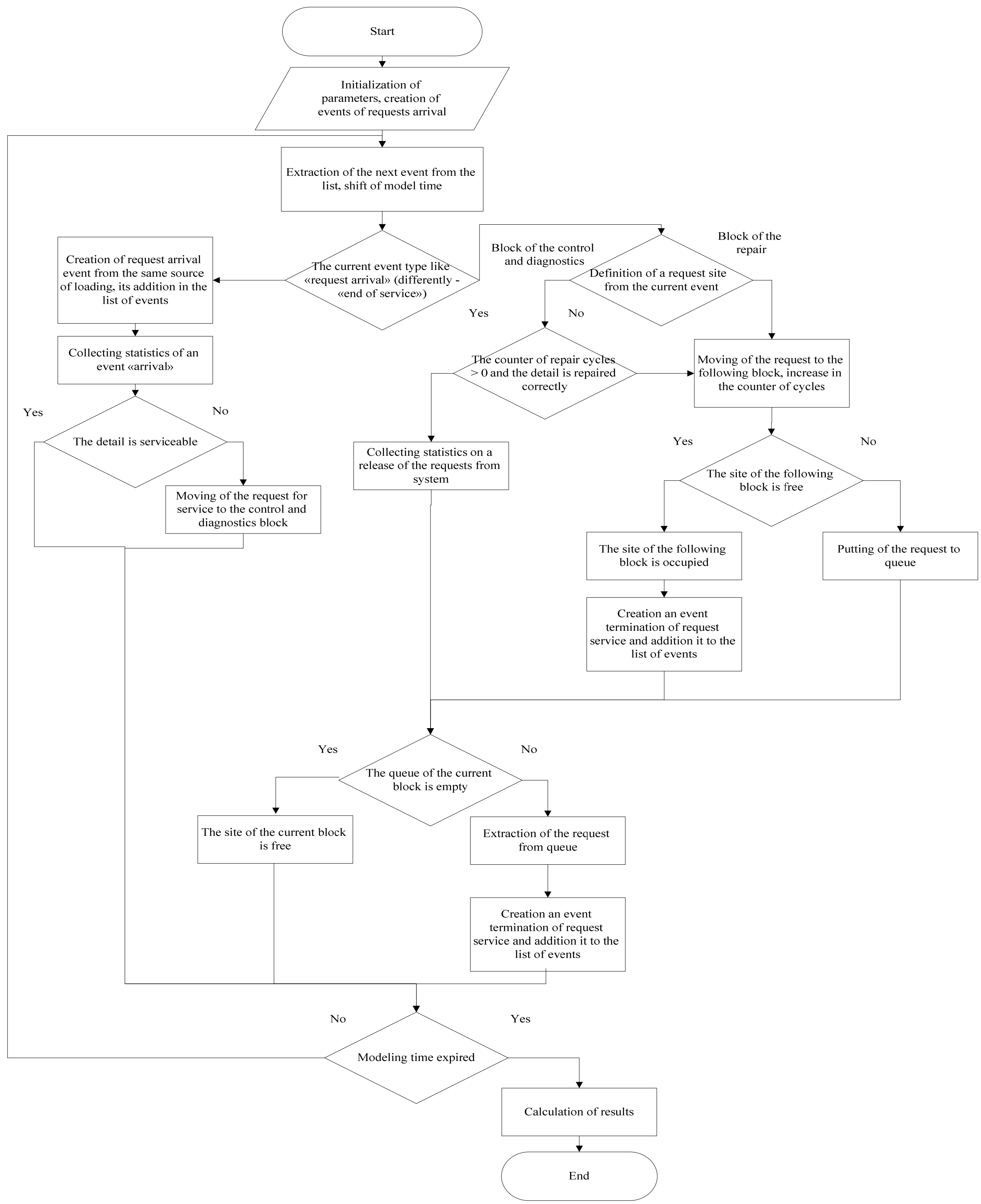

Figure-5. Graph algorithm simulation functioning of RDC.

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QN IMPLEMENTATION AND THE PROCESS OF ITS FUNCTIONING

All the elements that make up the QN, as well as objects and concepts necessary for the implementation of functional and structural models specified QN represent the corresponding instances of classes. Some of the classes are combined to the library as belonging to one category of objects.

The classes are used to represent the structural organization of the model:

- component library of QS «Components Library», that includes:
  - class of requests «Request»;
  - class of service site «Device»;
- various enumeration describing such entities as the type site «Type» and its condition «State», the current location of the requests «Location» in the system, etc. (implemented appropriate fields propertiese numeration types – enum);
- class of QS. This class is formed from the corresponding real object abstract elements that make up the QN: service sites, queues, etc.

To implement the functional components of the models used:

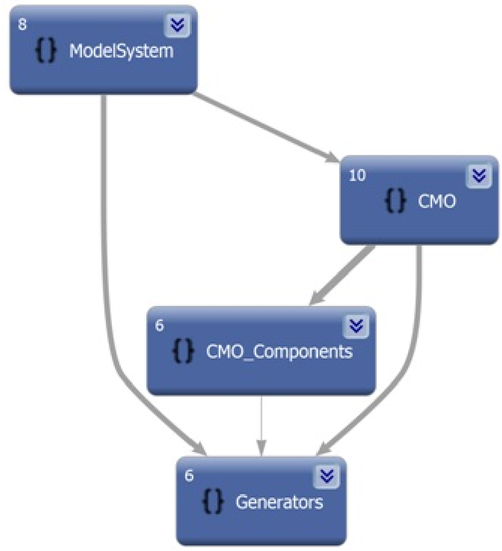
- class of event «Basic Event», occurring in the system, with the appropriate type «Type Event»;
- class of even than dler«HandlerEvent», occurring during the QS functioning;
- library of incoming requests generators operating in accordance with a given distribution law «Generators RV» (all classes are inherited from a parent abstract class «Distribution»):
  - Exponential Distribution;
  - Gamma Distribution;
  - Beta Distribution;
  - Uniform Distribution;
  - Probability.

To collect, intermediate, final processing and presentation of results are created:

- class of statistics garbage «Garbage Statistic»;
- class of logging «Log Modeling».

As a development environment software tools for modeling QN selected IDE Microsoft Visual Studio 2010. Choice is due to a wide range of tools for writing and debugging code, the presence of built-in libraries with classes of random number generators to create a sufficiently uniform pseudo-random sequence of values. Application is implemented on the basis of object-oriented programming language C# [6].

One of the tools of Visual Studio 2010 allowing a more graphically demonstrated the application architecture is the Dependency Graph using DGML (Directed Graph Markup Language). This tool is used for source code analysis and visualization of relationships between the various artifacts of the application code (classes, interfaces, namespaces, etc.) For presentation infrastructure of created application and documenting code are constructed corresponding diagrams dependencies. Diagram dependencies of namespaces:



Figur-6. Dependency diagram of namespaces.

To achieve the acceptable performance of program carried out the separation process simulation and interact with the user through the organization of multi-threaded execution. Moreover, each network simulation functioning occurs independently from each other, in parallel.

DESCRIPTION OF THE DEVELOPED SOFTWARE APPLICATIONS FOR MODELING RDC

The program has some settings, providing the user a more comfortable work (choice of language, the ability to conduct logging modeling steps for all major events), as well as the corresponding menu commands start and stop the process.

The interface of user workspace is implemented in accordance with the structural schema of the specifies QN that provides visibility and ease of its initialization parameters and allows you to immediately get an idea about the basic principles of operation [7].

Customizable settings are divided into groups according to the QN logic structure: allocated control subsystem, diagnostics and repair subsystems, separately presented unit characteristics to form the input streams of requests. On the system settings panel is also carried out the task of simulation time.

The required settings queuing network include:

- set the number of sites in the nodes of control, diagnostics and repair;
- choice of distribution laws for time intervals of requests receipt in the system;
- choice of distribution laws of service intervals in all service nodes;
- parameterization all distribution laws;
- set the probability of getting a request in the system that does not require full service (serviceable);

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- set the probability of faulty repair, that is the probability of repeating the cycle of full service.
- The following distribution laws of random variables require input parameters:
- for exponential– λ;
- for gamma – $\theta$and k;
- for beta – αandβ;

for uniform – a and b.

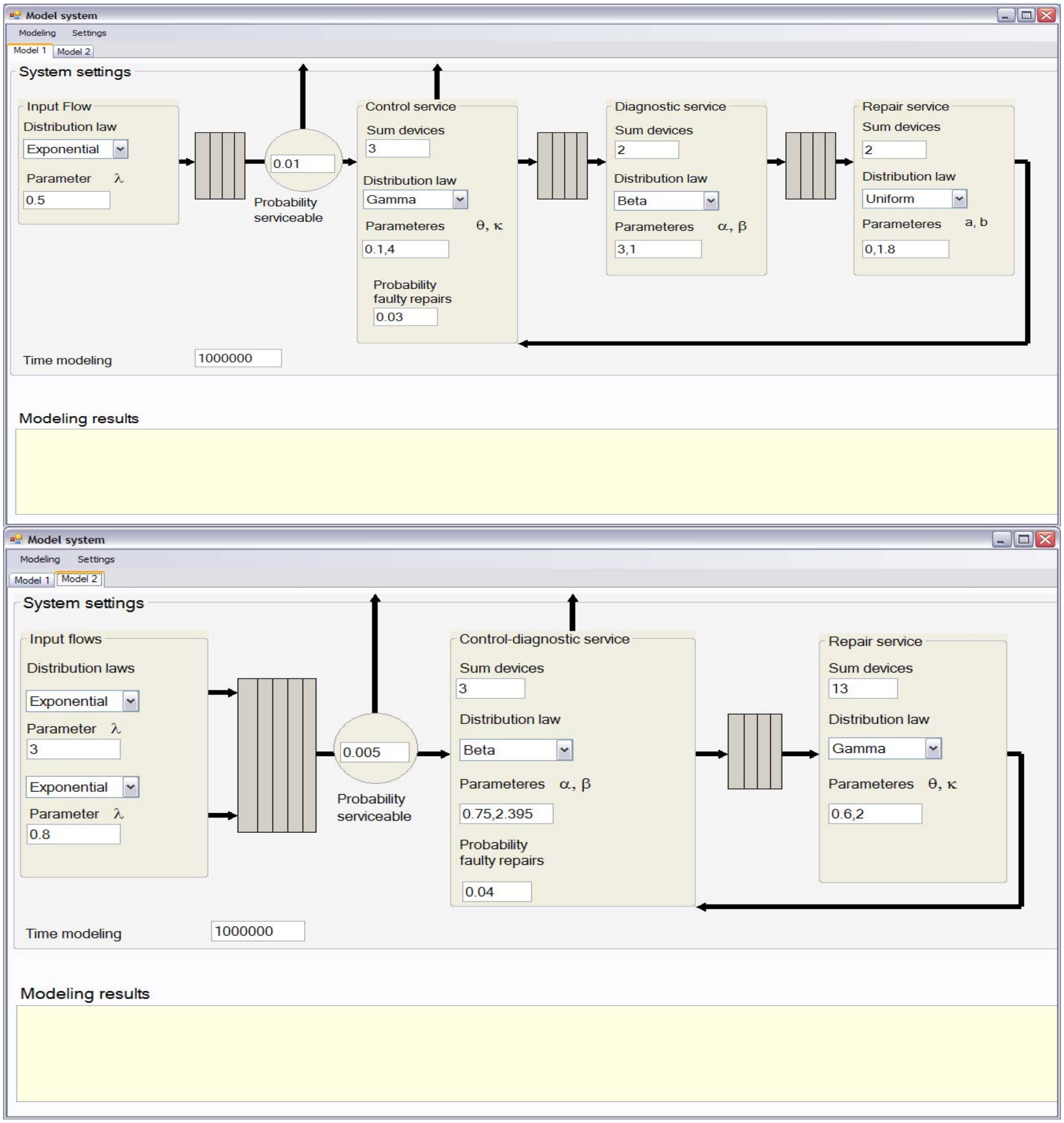

Figure-7. Interface developed computer program.

In the text box located below the input interface displays information about the current state of the application: warning of the need to wait for completion of the modeling process and at the end of it, the results of the model.

The most important desired characteristic, first of all, the average service time of request in the system. That this is a key parameter for evaluating the efficiency of the system as a whole.

Producing runs of software model wish using different parameters, it is possible to determine which subsystem is making the greatest delays in the complex. This further allows to adjust its settings and to select the optimal configuration of the node, including determines

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the number of service sites capable of maintaining the load factor in an acceptable range. Thus, the software RDC model, perform simulations of its functioning, using the basic concepts of the queuing systems theory and simulation methods, allows us to predict the behavior of the system and makes it possible to theoretically (not conduct experiments with real equipment) calculate data required for improvement specific RDC.

Window to display the simulation results contain the following information:

Time modeling: 1000000,17214081

Number of requests not served during the simulation: 1

Number of requests served during the simulation: 500252

Average time input intervals: 1,99898885592053

Average time service: 2,5555980944717

DEVELOPMENT OF QN MODELS IN GPSS

To assess the quality of the developed program was produced QN simulation by means of a general-purpose modeling system GPSS [3].

Taking into account the principles of building models in the GPSS environment algorithm for simulating the process of specified QN functioning in general is reduced to the following items:

1. To provide the generation of incoming transacts with an exponential distribution of an arrival time intervals;
2. To describe a route of the transact:

a. to place the transact in object of the equipment «queue of control»;
b. if further service isn't required, then to remove the transact from system;
c. to move the transact to object of the equipment «multichannel QS of control»;
d. to release a place in «queue of control»;
e. to serve the transact in «multichannel QS of control»;
f. if the transact previously successfully passed the full service cycle, then to remove the transact from system;
g. to place the transact in object of the equipment «queue of diagnostics»;
h. to move the transact to object of the equipment «multichannel QS of diagnostics»;
i. to release a place in «queue of diagnostics»;
j. to serve the transact in «multichannel QS of diagnostics»;
k. to place the transact in object of the equipment «queue of repair»;
l. to move the transact to object of the equipment «multichannel QS of repair»;
m. to release a place in «queue of repair»;
n. to serve the transact in «multichannel QS of repair»;
o. to move the transact to object of the equipment «queue of control » and go to step c;

3. To provide transacts service in all multichannel QS in accordance with the selected distributions.

The route for the second QS model looks similarly except association stages of passage control and diagnostics. However, the algorithm has one important addition: in this case the receipt requests is provided from two loading sources, so necessary to provide the presence of two generators input transacts streams. Requests from both streams come into one queue that preceding multichannel QS control diagnostics.

When creating QN model is necessary to consider the fact that the probability of completion of transact service in the system and out of her at the time of initial entry into the system and at the time of completion of the cycle (i.e. after the passage of the service in all sites – control, diagnostics, repair, secondary control) are different. The first probability corresponds to receipt in system of initially serviceable detail that doesn't require further care, and the second – an unsuccessful service in the system, during which functionality of a detail wasn't restored, i.e. faulty repair.

For technical realization of this feature it is necessary to establish in the beginning, at what stage of service there is an active transact, and only then to play random variable and send it to further route in accordance with a given probability.

The source code of the program for the QN model with a single loading source and differentiated stages control and diagnostics:

```
Simulate
      INITIAL X$c,0
      INITIAL X$t,0
      Control STORAGE 3
      Diagnostic STORAGE 2
      Repair STORAGE 2
      GENERATE (EXPONENTIAL(1,0,2))
      ASSIGN 1,0
      QUEUE WaitServiceInControl
      TRANSFER 0.01,Service,NotService
NotService DEPART WaitServiceInControl
      TRANSFER ,LeaveCeMO
Service     ENTER Control
      DEPART WaitServiceInControl
      ADVANCE (GAMMA(2,0,4,0.1))
      LEAVE Control
      TEST G P1,0,ContinueService
      TRANSFER
0.03,LeaveCeMO,ContinueService
ContinueService   QUEUE
WaitServiceInDiagnostic
      ENTER Diagnostic
      DEPART WaitServiceInDiagnostic
      ADVANCE   (BETA(2,0.5,3,0.1,0.9))

      LEAVE Diagnostic
      QUEUE WaitServiceInRepair
      ENTER Repair
      DEPART WaitServiceInRepair
      ADVANCE (UNIFORM(2,0,1.8))
      LEAVE Repair
      ASSIGN 1+,1
```

VOL. 11, NO. 16, AUGUST 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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```
        QUEUE WaitServiceInControl
        TRANSFER ,Service
LeaveCeMO   SAVEVALUE   t+,M1
        SAVEVALUE   c+,1
        SAVEVALUE   aveT,(X$t/X$c)
        TERMINATE
        GENERATE 1000000
        TERMINATE 1
```

The source code of the program for the QN model with two loading sources and joint control and diagnostics:

```
Simulate
        INITIAL X$c,0
        INITIAL X$t,0
        ControlDiagnostic STORAGE 3
        Repair STORAGE 13
        GENERATE (EXPONENTIAL(1,0,1/3))
        ASSIGN 1,0
        Transfer ,InQueue
        GENERATE (EXPONENTIAL(2,0,1.25))
        ASSIGN 1,0
InQueue     QUEUE WaitServiceInControl
        TRANSFER 0.005,Service,NotService
NotService DEPART WaitServiceInControl
        TRANSFER ,LeaveCeMO
Service     ENTER ControlDiagnostic
        DEPART WaitServiceInControl
        ADVANCE (BETA(2,0.1,1,1,5.5)) ;
M[x]=0.23846
        LEAVE ControlDiagnostic
        TEST G P1,0,ContinueService
        TRANSFER
0.04,LeaveCeMO,ContinueService
ContinueService   QUEUE
WaitServiceInRepair
        ENTER Repair
        DEPART WaitServiceInRepair
        ADVANCE (GAMMA(2,0,2,0.6))
        LEAVE Repair
        ASSIGN 1+,1
        QUEUE WaitServiceInControl
        TRANSFER ,Service
LeaveCeMO   SAVEVALUE   t+,M1
        SAVEVALUE   c+,1
        SAVEVALUE   aveT,(X$t/X$c)
        TERMINATE
        GENERATE 1000000
        TERMINATE 1
```

COMPARATIVE ANALYSIS OF THE QN MODELS IMPLEMENTATIONS

The simulation process with the given parameters (used parameters presented in Figure-5) showed the following results:

Characteristic	Software tool	GPSS World
Average service time in the first QN model	2.545	2.555
Average service time in the second QN model	1.873	1.840

Parameters differ hundredths, which proves the correctness and efficiency of the developed software applications. The difference is due to using different random number generators for random variables distributed uniformly on the interval {0, 1}, and using a variety of algorithms for generating random variables with given distribution.

For the duration of the simulation, i.e. computer time simulation, which reflects the cost of the resource computer time, measurements have shown:

Characteristic	Software tool	GPSS World
Machine simulation time of the first QN	~12.3 s	~12.4 s
Machine simulation time of the second QN	~77 s	~54 s

Such result proves the similar productivity of implementations for the model in both environments.

FINDINGS

As a result of consideration of the construction features RDC for GDTS revealed the general structure of such systems, thereby creating the appropriate structural and functional scheme of these objects. The received data formed the basis of the RDC models that are created using the tools of the queuing systems theory. Through the use of computer simulation tools, using appropriate methods of statistical tests, the probability theory and probability distributions, it was possible to develop software models, adequate to real RDC objects. Achieving similar work allowed to make virtual tests of these complexes according to the developed scheme of simulation their functioning which is essential for the analysis of the RDC. Analysis will improve the operation of the systems by finding the most optimal settings, and as a result, maximize the efficiency of the entire GDTS.

Comparing the use of different implementations of the models can be said about such advantages of the created software tools such as:

- clear presentation of the simulated QN;
- clear and accessibility of the interface;
- simplicity of use;
- no need to perform preliminary calculations at initialization of parameters;
- output the results in an accessible and understandable form;
- protection from entering invalid parameters.

The disadvantage of the created program is less informative report (the smaller number of QN functioning characteristics and statistical parameters are defined), due to the specifics and requirements technical specification.

At the same time designed program doesn't concede the model written in GPSS World system in terms of accuracy and speed.

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