FUZZY FALSE POSITION METHOD FOR SOLVING FUZZY NONLINEAR EQUATIONS

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ABSTRACT

In this paper, we focus on extended numerical methods for solving fuzzy nonlinear equations. An extension of false position method into fuzzy setting is proposed for solving such equations and it will be referred to as fuzzy false position method. An algorithm for this process of solving will be provided. For the purpose of optimization, genetic algorithm will also be incorporated in order to find the best solution for the problem under consideration. Two numerical examples with graphical representations are provided to illustrate the efficiency of the proposed method. The results showed that the proposed method is able to find the best solution for fuzzy nonlinear equations.

Keywords: false position method, nonlinear equation, fuzzy nonlinear equation, numerical methods, fuzzy false position method.

INTRODUCTION

In recent years, system of simultaneous nonlinear equations plays a major function in various fields such as applied mathematics, engineering, statistics and social sciences. Therefore, much attention has been given into developing numerical methods to work out with these systems.

In such cases, to model the real world engineering systems, normally there will be problems of fuzziness in some of the equation's parameters, hence fuzzy numbers are employed rather than crisp number. At first, the concept of fuzzy numbers which include its arithmetic operation were introduced by Zadeh [1]. Then, it was followed by Dubois and Prade [2]. One of the major applications of fuzzy arithmetic is nonlinear systems whose parameters are totally or partially represented by fuzzy numbers [3, 4].

However, in solving fuzzy nonlinear equations, the classical numerical methods cannot be applied directly. Indeed, standard analytical techniques proposed by Buckley and Qu [5], are not suitable for solving the fuzzy equations. Hence, it is necessary to examine and develop new numerical methods to find the roots of fuzzy nonlinear equations. Thus, in [6, 7, 8, 9, 10], Newton's method, Broyden's method and Fixed Point's method were used to solve this type of equations. Another method used is the false position method. The advantage of this method is that it takes account of the observation of the approximate solution. This is done by drawing a secant line from two function values where one is negative and the other one is positive. Then, the root is estimated as the position where it crosses the x-axis or when the function value is equal to zero. Once the false position method comes close to the root, it converges quickly.

Genetic algorithm is a global optimization method used for solving both constrained and unconstrained problem inspired by natural evolution. The algorithm repeatedly modifies a population of individual solutions. The idea is by guessing solutions, then combining the fittest solution to create a new generation of solutions which is better. It is a popular strategy to optimize non-linear systems with a large number of variables. Genetic algorithm might not ensure an optimal solution, but it is able to give a good approximation in a reasonable amount of time.

In this paper, False Position method is proposed to solve these fuzzy nonlinear equations. In the next section, we will recall some basic definitions of fuzzy numbers based on Zadeh's extension principle. Later, we will study the classical False Position method and its algorithm. This include the genetic algorithm for optimization process. Then, in subsequent section, we will extend the False Position method into fuzzy setting for solving fuzzy nonlinear equations. Finally, two numerical examples with graphical representations will be demonstrated to illustrate the proposed algorithm and conclusion will be made.

BASIC CONCEPTS

In this section, we review some important definitions of fuzzy numbers.

Definition 1. [2, 1, 11] A fuzzy number is a fuzzy set like $U : \mathbb{R} \to I = [0,1]$ which satisfies the following properties:

1. U is upper semi continuous,
2. $U(x) = 0$ is outside some interval $[c,d]$,
3. there are real numbers a, b such that $c \leq a \leq b \leq d$,
   a. $U(x)$ is monotonic increasing on $[c,a]$,
   b. $U(x)$ is monotonic decreasing on $[b,d]$,
   c. $U(x) = 1$, $a \leq x \leq b$.

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Definition 2. [12] A fuzzy number U in parametric form is a pair $[\underline{u}^\alpha, \overline{u}^\alpha]$ of functions $\underline{u}^\alpha$, $\overline{u}^\alpha$ for $\alpha \in [0,1]$ which satisfies the following requirements.

1. $\underline{u}^\alpha$ is a bounded monotonic increasing left continuous function,
2. $\overline{u}^\alpha$ is a bounded monotonic decreasing left continuous function,
3. $\underline{u}^\alpha \leq \overline{u}^\alpha$ for $\alpha \in [0,1]$.

A crisp number r is represented by $\underline{u}^\alpha = \overline{u}^\alpha = r$ for $\alpha \in [0,1]$.

Definition 3. A triangular fuzzy number is a fuzzy number represented with three points, $u = (a,b,c)$, and its membership function can be represented as

$$u(x) = \begin{cases} \dfrac{x-a}{c-a}, & a \leqslant x \leqslant c, \\ \dfrac{x-b}{c-b}, & c \leqslant x \leqslant b, \end{cases}$$

where $c \neq a$, $c \neq b$ and hence its parametric form is

$$[\underline{u}^\alpha, \overline{u}^\alpha] = [a + (c-a)\alpha,\, b + (c-b)\alpha].$$

Let $\mathcal{F}(\mathcal{R})$ be the set of all triangular fuzzy numbers.

Definition 4. [12] The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows.

For arbitrary $U^\alpha = [\underline{u}_\alpha, \overline{u}_\alpha]$, $V^\alpha = [\underline{v}_\alpha, \overline{v}_\alpha]$, and $k > 0$, the addition $U^\alpha + V^\alpha$and multiplication by real number $k > 0$ are defined as

1. addition, $U^\alpha \oplus V^\alpha = [\underline{u}^\alpha + \underline{v}^\alpha, \overline{u}^\alpha + \overline{v}^\alpha]$,
2. subtraction, $U^\alpha \ominus V^\alpha = [\underline{u}^\alpha - \overline{v}^\alpha, \overline{u}^\alpha - \underline{v}^\alpha]$,
3. scalar multiplication,

$$k \odot U^\alpha = \begin{cases} [k\underline{u}^\alpha, k\overline{u}^\alpha], & k \geqslant 0, \\ [k\overline{u}^\alpha, k\underline{u}^\alpha], & k < 0. \end{cases}$$

If $k = -1$, then $k \odot U^\alpha = -U^\alpha$.

THE CLASSICAL FALSE POSITION METHOD FOR SOLVING NONLINEAR EQUATION

The method of false position is a root finding algorithm that hybrid the features from bisection and secant method. It involves the bracketing of bisection method and the secant line in secant method. It was introduced to improve the bisection method which converges at a fairly slow speed. Like bisection method, it starts with two proper

values of x_L (lower bound value) and x_U (upper bound value) for the current bracket, such that

$$f(x_L)f(x_U) < 0. \qquad (1)$$

This is based on the following intermediate value theorem.

Theorem 1. If f is continuous on $[a,b]$ and k is a value between $f(a)$ and $f(b)$, then there exists a value c in $[a,b]$ such that $f(c) = k$.

In bisection method, sometimes it may not be efficient because it does not take into consideration that $f(x_L)$ is much closer to the zero of the function $f(x)$ as compared to $f(x_U)$. The idea for False Position method, on advantage of this observation, is to connect function value at x_L to the function value at x_U by drawing a secant line, and then estimates the root, x_r, as where it crosses the x-axis.

Firstly, we write down two versions of the slope m of the secant line from $(x_L, f(x_L))$ to $(x_r, 0)$ and from $(x_L, f(x_L))$ to $(x_U, f(x_U))$.

$$m = \frac{0 - f(x_L)}{x_r - x_L},$$

and

$$m = \frac{f(x_L) - f(x_U)}{x_L - x_U}.$$

So,

$$\frac{0 - f(x_L)}{x_r - x_L} = \frac{f(x_L) - f(x_U)}{x_L - x_U}.$$

The above equation can be solved to obtain the next predicted root x_r as

$$f(x_L)[x_L - x_U] = x_r - x_L[f(x_L) - f(x_U)].$$

Through simple algebraic manipulations, it can be expressed as

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$$x_r = x_L - \frac{f(x_L)[x_L - x_U]}{f(x_L) - f(x_U)}.$$

False position algorithm

The steps to apply the false-position method to find the root of the equation $f(x) = 0$ are as follows.

1. Choose x_L and x_U as two guesses for the root such that $f(x_L)f(x_U) < 0$, or in other words, $f(x)$ changes sign between x_L and x_U.

2. Estimate the root, x_r of the equation $f(x) = 0$ as

$$x_r = x_L - \frac{f(x_L)[x_L - x_U]}{f(x_L) - f(x_U)}.$$

3. Now check the following.

   a. If $f(x_L)f(x_U) < 0$, this implies that the root lies between x_L and x_R, then replace $x_U = x_r$.

   b. If $f(x_U)f(x_) < 0$, this implies that the root lies between x_U and x_R, then replace $x_L = x_r$.

   c. If $f(x_L)f(x_r) = 0$ or $f(x_U)f(x_r) = 0$, implies that the root is x_R. Stop the algorithm .

4. Find the new estimate root

$$x_r = x_L - \frac{f(x_L)[x_L - x_U]}{f(x_L) - f(x_U)}.$$

5. Find the new absolute relative approximate error,

$$|\epsilon_a| = \left|x_r^{\,old} - x_r^{\,new}\right|,$$

   where x_r^{new} =estimated root from current iteration and x_r^{old} =estimated root from previous iteration.

6. Compare the absolute relative approximate error, $|\epsilon_a|$ with the pre-specified relative error tolerance, ϵ. If $|\epsilon_a| > \epsilon$, go back to step 3, else, stop the algorithm.

FUZZY FALSE POSITION METHOD FOR SOLVING FUZZY NONLINEAR EQUATIONS

The method of False Position to solve fuzzy nonlinear equations, can be described as follows.

1. Choose two initial values of fuzzy numbers for which their function value have opposite sign. Let us assume two initial values are

$$A(a_1, a_2, a_3) \text{ and } B(b_1, b_2, b_3).$$

To fulfill that condition, check $f(a_2)f(b_2) < 0$.

2. Transform the initial fuzzy values $A(a_1, a_2, a_3)$ and $B(b_1, b_2, b_3)$ into their parametric form

$$[A]^\alpha = [\underline{a}^\alpha, \overline{a}^\alpha],$$

where

$$\underline{a}^\alpha = a_1 + \alpha(a_2 - a_1), \; \overline{a}^\alpha = a_3 - \alpha(a_3 - a_2),$$

and

$$[B]^\alpha = [\underline{b}^\alpha, \overline{b}^\alpha],$$

where

$$\underline{b}^\alpha = b_1 + \alpha(b_2 - a_1), \; \overline{b}^\alpha = b_3 - \alpha(b_3 - b_2).$$

For iteration, we discretise α in the form $\alpha_0 < \alpha_1 < \ldots < \alpha_{n-1} < \alpha_n$, where $\alpha_0 = 0$ and $\alpha_n = 1$. The discretised α are equally spaced, that is $\alpha_1 = \alpha_0 + i\Delta h$, for $i = 0, 1, 2, \ldots, n$, and $\Delta h = \frac{1}{n} > 0$. In this study, Δh is called the discretization spacing. After discretisation, we have a set of α with $(n+1)$ elements.

3. Then, find $[X]^\alpha = [\underline{x}^\alpha, \overline{x}^\alpha]$ for $\alpha \in [0, 1]$ with $h = 0.1$,

$$[\underline{x}^\alpha] = \min\{G(a,b) | a \in [\underline{a}^\alpha, \overline{a}^\alpha], b \in [\underline{b}^\alpha, \overline{b}^\alpha]\},$$

$$[\overline{x}^\alpha] = \max\{G(a,b) | a \in [\underline{a}^\alpha, \overline{a}^\alpha], b \in [\underline{b}^\alpha, \overline{b}^\alpha]\},$$

where

$$G(a,b) = b - \frac{f(b)(b-a)}{f(b) - f(a)}.$$

In order to find minimum and maximum of $G(a, b)$, we use the genetic algorithm approach. A genetic algorithm is a method for solving both constrained and

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unconstrained optimization problems. Unlike classical algorithm which only generates a single point, this genetic algorithm generates a population of points from given interval. In addition, the best point in that population approaches an optimal solution. This method generates optimum solution at each iteration for every $\alpha \in [0,1]$ with $h = 0.1$.

4. When $\alpha = 1$, then $[\underline{x}]^1 = [\overline{x}]^1$, check $f(x)$.

a. If $f(x)f(a_2) < 0$, this implies that the fuzzy root lies between X and A, then $B = X$.

b. If $f(x)f(b_2) < 0$, this implies that the fuzzy root lies between X and B, then $A = X$.

c. If $f(x)f(a_2) = 0$ or $f(x)f(b_2) = 0$, this implies that the fuzzy root is X.

5. Continue the iteration for every α until the stopping criteria as follows is fulfilled.

$$|\epsilon_a| = |x^{old} - x^{new}|,$$

where x^{new} = estimated root from current iteration and x^{old} = estimated root from previous iteration when $\alpha = 1$.

6. Compare the absolute relative approximate error $|\epsilon_a|$ with the pre-specified relative error tolerance ϵ. If $|\epsilon_a| > \epsilon$, then go to step 3, else, stop the algorithm.

NUMERICAL APPLICATIONS

Example 1. We try to solve the following fuzzy nonlinear equation with fuzzy false position method.

$$f(u_x) = x^2 - 3,$$
$$f_0 = [\underline{f}(u_0), \overline{f}(u_0)],$$

where the tolerance, $\epsilon = 0.0001$. Firstly, choose two initial fuzzy values for which their function values have opposite sign: $A(0,1,2)$ and $B(1,2,3)$ where $f(1) < 0$ and $f(2) > 0$. Hence, $f(1)f(2) < 0$.

Then, the parametric form of A and B are as follows.

$$[A]^\alpha = [\alpha, 2-\alpha],$$

where

$$\underline{a}^\alpha = 0 + \alpha(1-0),\ \overline{a}^\alpha = 2 - \alpha(2-1),$$

and

$$[B]^\alpha = [1+\alpha, 3-\alpha],$$

where

$$\underline{b}^\alpha = 1 + \alpha(2-1),\ \overline{b}^\alpha = 3 - \alpha(3-2).$$

For the first iteration, find

$$[\underline{x}]^\alpha = \min\{G(a,b) | a \in [\underline{a}^\alpha, \overline{a}^\alpha], b \in [\underline{b}^\alpha, \overline{b}^\alpha]\},$$

and

$$[\overline{x}]^\alpha = \max\{G(a,b) | a \in [\underline{a}^\alpha, \overline{a}^\alpha], b \in [\underline{b}^\alpha, \overline{b}^\alpha]\},$$

where

$$G(a,b) = b - \frac{f(b)(b-a)}{f(b)-f(a)}.$$

By using MATLAB, for $\alpha \in [0,1]$ with $h = 0.1$, the first iteration is generated as shown in Table-1.

Table-1. First iteration for example-1.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[0.0,2.0]	[1.0,3.0]	[1.0007,2.9995]
0.1	[0.1,1.9]	[1.1,2.9]	[1.0969,2.5917]
0.2	[0.2,1.8]	[1.2,2.8]	[1.1867,2.3141]
0.3	[0.3,1.7]	[1.3,2.7]	[1.2703,2.1187]
0.4	[0.4,1.6]	[1.4,2.6]	[1.3468,1.9778]
0.5	[0.5,1.5]	[1.5,2.5]	[1.4167,1.8750]
0.6	[0.6,1.4]	[1.6,2.4]	[1.4801,1.8000]
0.7	[0.7,1.3]	[1.7,2.3]	[1.5367,1.7458]
0.8	[0.8,1.2]	[1.8,2.2]	[1.5867,1.7149]
0.9	[0.9,1.1]	[1.9,2.1]	[1.6300,1.6966]
1.0	[1.0,1.0]	[2.0,2.0]	[1.6667,1.6667]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(2)f(1.6667) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$.

Then, we continue the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The second iteration is shown in Table-2.

Table-2. Second iteration for example-1.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.0007,2.9995]	[1.0,3.0]	[1.5005,1.9996]
0.1	[1.0969,2.5917]	[1.1,2.9]	[1.5465,1.9148]
0.2	[1.1867,2.3141]	[1.2,2.8]	[1.5860,1.8536]
0.3	[1.2703,2.1187]	[1.3,2.7]	[1.6203,1.8097]
0.4	[1.3468,1.9778]	[1.4,2.6]	[1.6474,1.7786]
0.5	[1.4167,1.8750]	[1.5,2.5]	[1.6725,1.7571]
0.6	[1.4801,1.8000]	[1.6,2.4]	[1.6888,1.7429]
0.7	[1.5367,1.7458]	[1.7,2.3]	[1.7034,1.7340]
0.8	[1.5867,1.7149]	[1.8,2.2]	[1.7141,1.7317]
0.9	[1.6300,1.6966]	[1.9,2.1]	[1.7220,1.7304]
1.0	[1.6667,1.6667]	[2.0,2.0]	[1.7273,1.7273]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(2)f(1.7273) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |1.7273 - 1.6667|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The third iteration is shown in Table-3.

Table-3. Third iteration for example-1.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.5005,1.9996]	[1.0,3.0]	[1.6671,1.7998]
0.1	[1.5465,1.9148]	[1.1,2.9]	[1.6833,1.7764]
0.2	[1.5860,1.8536]	[1.2,2.8]	[1.7026,1.7599]
0.3	[1.6203,1.8097]	[1.3,2.7]	[1.7070,1.7486]
0.4	[1.6474,1.7786]	[1.4,2.6]	[1.7147,1.7413]
0.5	[1.6725,1.7571]	[1.5,2.5]	[1.7211,1.7366]
0.6	[1.6888,1.7429]	[1.6,2.4]	[1.7252,1.7338]
0.7	[1.7034,1.7340]	[1.7,2.3]	[1.7280,1.7323]
0.8	[1.7141,1.7317]	[1.8,2.2]	[1.7299,1.7320]
0.9	[1.7220,1.7304]	[1.9,2.1]	[1.7311,1.7320]
1.0	[1.7273,1.7273]	[2.0,2.0]	[1.7317,1.7317]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(2)f(1.7317) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |1.7317 - 1.7273|$.

Since $|\epsilon_n| > \epsilon$, proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The fourth iteration is shown in Table-4.

Table-4. Fourth iteration for example-1.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.6671,1.7998]	[1.0,3.0]	[1.7144,1.7499]
0.1	[1.6833,1.7764]	[1.1,2.9]	[1.7201,1.7431]
0.2	[1.7026,1.7599]	[1.2,2.8]	[1.7251,1.7375]
0.3	[1.7070,1.7486]	[1.3,2.7]	[1.7266,1.7356]
0.4	[1.7147,1.7413]	[1.4,2.6]	[1.7286,1.7339]
0.5	[1.7211,1.7366]	[1.5,2.5]	[1.7301,1.7328]
0.6	[1.7252,1.7338]	[1.6,2.4]	[1.7310,1.7323]
0.7	[1.7280,1.7323]	[1.7,2.3]	[1.7315,1.7321]
0.8	[1.7299,1.7320]	[1.8,2.2]	[1.7318,1.7321]
0.9	[1.7311,1.7320]	[1.9,2.1]	[1.7320,1.7320]
1.0	[1.7317,1.7317]	[2.0,2.0]	[1.7320,1.7320]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(2)f(1.7320) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |1.7320 - 1.7317|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The fifth iteration is shown in Table-5.

Table-5. Fifth iteration for Example-1.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.7144,1.7499]	[1.0,3.0]	[1.7273,1.7368]
0.1	[1.7201,1.7431]	[1.1,2.9]	[1.7291,1.7347]
0.2	[1.7251,1.7375]	[1.2,2.8]	[1.7304,1.7333]
0.3	[1.7266,1.7356]	[1.3,2.7]	[1.7309,1.7328]
0.4	[1.7286,1.7339]	[1.4,2.6]	[1.7314,1.7324]
0.5	[1.7301,1.7328]	[1.5,2.5]	[1.7317,1.7322]
0.6	[1.7310,1.7323]	[1.6,2.4]	[1.7319,1.7321]
0.7	[1.7315,1.7321]	[1.7,2.3]	[1.7320,1.7321]
0.8	[1.7318,1.7321]	[1.8,2.2]	[1.7320,1.7321]
0.9	[1.7320,1.7320]	[1.9,2.1]	[1.7320,1.7321]
1.0	[1.7320,1.7320]	[2.0,2.0]	[1.7320,1.7320]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(2)f(1.7320) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |1.7320 - 1.7320|$.

As $|\epsilon_n| < \epsilon$, we stop the iteration. Hence, the fuzzy root for $f(u_x) = x^2 - 3$ is $(1.7273, 1.7320, 1.7368)$. The graphical representation of iterations done is illustrated in Figure-1.

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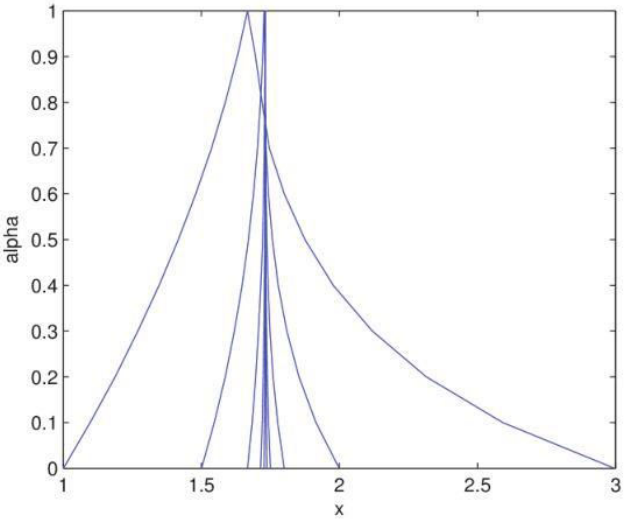

Figure-1. Graphical representation of optimum solution of a fuzzy non-linear equation, $f(u_x) = x^2 - 3$.

Example 2. Consider the following fuzzy nonlinear equation.

$$f(u_x) = x^3 - 5x + 1,$$
$$f_0 = [\underline{f}(u_0), \overline{f}(u_0)],$$

where the tolerance $\epsilon = 0.0001$.

To find the solution using fuzzy false position method, first, we assume two initial fuzzy values for which their function values have opposite sign: $A(1.5, 2.0, 2.5)$

and $B(2.5, 3.0, 3.5)$ where $f(2) > 0$ and $f(3) > 0$. Hence $f(2)f(3) < 0$.

Then, the parametric form of A and B are as follows.

$$[A]^\alpha = [1.5 + \alpha(0.5), 2.5 - \alpha(0.5)],$$

where

$$\underline{a}^\alpha = 1.5 + \alpha(2.0 - 1.5),\ \overline{a}^\alpha = 2.5 - \alpha(2.5 - 2.0),$$

and

$$[B]^\alpha = [2.5 + \alpha(0.5), 3.5 - \alpha(0.5)],$$

where

$$\underline{b}^\alpha = 2.5 + \alpha(3.0 - 2.5),\ \overline{b}^\alpha = 3.5 - \alpha(3.5 - 3.0).$$

For the first iteration, find

$$[\underline{x}]^\alpha = \min\{G(a,b) | a \in [\underline{a}^\alpha, \overline{a}^\alpha], b \in [\underline{b}^\alpha, \overline{b}^\alpha]\},$$

and

$$[\overline{x}]^\alpha = \max\{G(a,b) | a \in [\underline{a}^\alpha, \overline{a}^\alpha], b \in [\underline{b}^\alpha, \overline{b}^\alpha]\},$$

where

$$G(a,b) = b - \frac{f(b)(b-a)}{f(b) - f(a)}.$$

By using MATLAB, for $\alpha \in [0, 1]$ with $h = 0.1$, the first iteration is generated as shown in Table-6.

Table-6. First iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.50,2.50]	[2.50,3.50]	[1.7126,2.3146]
0.1	[1.55,2.45]	[2.55,3.45]	[1.7670,2.2882]
0.2	[1.60,2.40]	[2.60,3.40]	[1.7995,2.2621]
0.3	[1.65,2.35]	[2.65,3.35]	[1.8406,2.2364]
0.4	[1.70,2.30]	[2.70,3.30]	[1.8831,2.2112]
0.5	[1.75,2.25]	[2.75,3.25]	[1.9170,2.1864]
0.6	[1.80,2.20]	[2.80,3.20]	[1.9539,2.1621]
0.7	[1.85,2.15]	[2.85,3.15]	[1.9854,2.1385]
0.8	[1.90,2.10]	[2.90,3.10]	[2.0163,2.1172]
0.9	[1.95,2.05]	[2.95,3.05]	[2.0450,2.0955]
1.0	[2.00,2.00]	[3.00,3.00]	[2.0714,2.0714]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.0714) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$.

Then, we continue the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The second iteration is shown in Table-7.

Table-7. Second iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.7126,2.3146]	[2.50,3.50]	[1.8700,2.2263]
0.1	[1.7670,2.2882]	[2.55,3.45]	[1.9108,2.2113]
0.2	[1.7995,2.2621]	[2.60,3.40]	[1.9373,2.1968]
0.3	[1.8406,2.2364]	[2.65,3.35]	[1.9748,2.1828]
0.4	[1.8831,2.2112]	[2.70,3.30]	[1.9942,2.1694]
0.5	[1.4167,1.8750]	[2.75,3.25]	[2.0167,2.1566]
0.6	[1.9539,2.1621]	[2.80,3.20]	[2.0397,2.1445]
0.7	[1.9854,2.1385]	[2.85,3.15]	[2.0583,2.1331]
0.8	[2.0163,2.1172]	[2.90,3.10]	[2.0756,2.1240]
0.9	[2.0450,2.0955]	[2.95,3.05]	[2.0907,2.1148]
1.0	[2.0714,2.0714]	[3.00,3.00]	[2.1038,2.1038]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1038) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

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$|\epsilon_n| = |2.1038 - 2.0174|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The third iteration is shown in Table-8.

Table-8. Third iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.8700,2.2263]	[2.50,3.50]	[1.9748,2.1812]
0.1	[1.9108,2.2113]	[2.55,3.45]	[2.0029,2.1724]
0.2	[1.9373,2.1968]	[2.60,3.40]	[2.0211,2.1640]
0.3	[1.9748,2.1828]	[2.65,3.35]	[2.0449,2.1562]
0.4	[1.9942,2.1694]	[2.70,3.30]	[2.0575,2.1490]
0.5	[2.0167,2.1566]	[2.75,3.25]	[2.0713,2.1422]
0.6	[2.0397,2.1445]	[2.80,3.20]	[2.0848,2.1361]
0.7	[2.0583,2.1331]	[2.85,3.15]	[2.0948,2.1306]
0.8	[2.0756,2.1240]	[2.90,3.10]	[2.1043,2.1267]
0.9	[2.0907,2.1148]	[2.95,3.05]	[2.1117,2.1228]
1.0	[2.1038,2.1038]	[3.00,3.00]	[2.1179,2.1179]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1179) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |2.1179 - 2.1038|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The fourth iteration is shown in Table-9.

Table-9. Fourth iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[1.9748,2.1812]	[2.50,3.50]	[2.0446,2.1572]
0.1	[2.0029,2.1724]	[2.55,3.45]	[2.0581,2.1519]
0.2	[2.0211,2.1640]	[2.60,3.40]	[2.0697,2.1471]
0.3	[2.0449,2.1562]	[2.65,3.35]	[2.0841,2.1427]
0.4	[2.0575,2.1490]	[2.70,3.30]	[2.0917,2.1388]
0.5	[2.0713,2.1422]	[2.75,3.25]	[2.0997,2.1352]
0.6	[2.0848,2.1361]	[2.80,3.20]	[2.1076,2.1321]
0.7	[2.0948,2.1306]	[2.85,3.15]	[2.1125,2.1294]
0.8	[2.1043,2.1267]	[2.90,3.10]	[2.1174,2.1277]
0.9	[2.1117,2.1228]	[2.95,3.05]	[2.1211,2.1261]
1.0	[2.1179,2.1179]	[3.00,3.00]	[2.1239,2.1239]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1239) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |2.1239 - 2.1179|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The fifth iteration is shown in Table-10.

Table-10. Fifth iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[2.0446,2.1572]	[2.50,3.50]	[2.0810,2.1442]
0.1	[2.0029,2.1724]	[2.55,3.45]	[2.0899,2.1411]
0.2	[2.0581,2.1519]	[2.60,3.40]	[2.0968,2.1383]
0.3	[2.0841,2.1427]	[2.65,3.35]	[2.1065,2.1358]
0.4	[2.0917,2.1388]	[2.70,3.30]	[2.1099,2.1337]
0.5	[2.0997,2.1352]	[2.75,3.25]	[2.1141,2.1318]
0.6	[2.1076,2.1321]	[2.80,3.20]	[2.1184,2.1302]
0.7	[2.1125,2.1294]	[2.85,3.15]	[2.1210,2.1289]
0.8	[2.1174,2.1277]	[2.90,3.10]	[2.1234,2.1282]
0.9	[2.1211,2.1261]	[2.95,3.05]	[2.1252,2.1275]
1.0	[2.1239,2.1239]	[3.00,3.00]	[2.1265,2.1265]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1265) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |2.1265 - 2.1239|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The sixth iteration is shown in Table-11.

Table-11. Sixth iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[2.0810,2.1442]	[2.50,3.50]	[2.1028,2.1371]
0.1	[2.0899,2.1411]	[2.55,3.45]	[2.1082,2.1353]
0.2	[2.0968,2.1383]	[2.60,3.40]	[2.1116,2.1336]
0.3	[2.1065,2.1358]	[2.65,3.35]	[2.1176,2.1323]
0.4	[2.1099,2.1337]	[2.70,3.30]	[2.1190,2.1311]
0.5	[2.1141,2.1318]	[2.75,3.25]	[2.1213,2.1301]
0.6	[2.1184,2.1302]	[2.80,3.20]	[2.1236,2.1293]
0.7	[2.1210,2.1289]	[2.85,3.15]	[2.1249,2.1286]
0.8	[2.1234,2.1282]	[2.90,3.10]	[2.1262,2.1283]
0.9	[2.1252,2.1275]	[2.95,3.05]	[2.1270,2.1280]
1.0	[2.1265,2.1265]	[3.00,3.00]	[2.1276,2.1276]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1276) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

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$|\epsilon_n| = |2.1276 - 2.1265|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The seventh iteration is shown in Table-12.

Table-12. Seventh iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[2.1028,2.1371]	[2.50,3.50]	[2.1142,2.1332]
0.1	[2.1082,2.1353]	[2.55,3.45]	[2.1176,2.1321]
0.2	[2.1116,2.1336]	[2.60,3.40]	[2.1197,2.1312]
0.3	[2.1176,2.1323]	[2.65,3.35]	[2.1228,2.1304]
0.4	[2.1190,2.1311]	[2.70,3.30]	[2.1236,2.1298]
0.5	[2.1213,2.1301]	[2.75,3.25]	[2.1250,2.1292]
0.6	[2.1236,2.1293]	[2.80,3.20]	[2.1261,2.1288]
0.7	[2.1249,2.1286]	[2.85,3.15]	[2.1268,2.1285]
0.8	[2.1262,2.1283]	[2.90,3.10]	[2.1274,2.1284]
0.9	[2.1270,2.1280]	[2.95,3.05]	[2.1278,2.1283]
1.0	[2.1276,2.1276]	[3.00,3.00]	[2.1281,2.1281]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1281) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$$|\epsilon_n| = |2.1281 - 2.1276|.$$

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The eighth iteration is shown in Table-13.

Table-13. Eighth iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[2.1142,2.1332]	[2.50,3.50]	[2.1207,2.1311]
0.1	[2.1176,2.1321]	[2.55,3.45]	[2.1229,2.1304]
0.2	[2.1197,2.1312]	[2.60,3.40]	[2.1238,2.1299]
0.3	[2.1228,2.1304]	[2.65,3.35]	[2.1255,2.1295]
0.4	[2.1236,2.1298]	[2.70,3.30]	[2.1260,2.1291]
0.5	[2.1250,2.1292]	[2.75,3.25]	[2.1267,2.1287]
0.6	[2.1261,2.1288]	[2.80,3.20]	[2.1273,2.1286]
0.7	[2.1268,2.1285]	[2.85,3.15]	[2.1277,2.1285]
0.8	[2.1274,2.1284]	[2.90,3.10]	[2.1280,2.1284]
0.9	[2.1278,2.1283]	[2.95,3.05]	[2.1281,2.1284]
1.0	[2.1281,2.1281]	[3.00,3.00]	[2.1283,2.1283]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1283) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |2.1283 - 2.1281|$.

Since $|\epsilon_n| > \epsilon$, we proceed the iteration with the new $[A]^\alpha$ and existing $[B]^\alpha$. The ninth iteration is shown in Table-14.

Table-14. Ninth iteration for example-2.

α	$[A]^\alpha$	$[B]^\alpha$	$[X]^\alpha$
0.0	[2.1142,2.1332]	[2.50,3.50]	[2.1207,2.1311]
0.1	[2.1176,2.1321]	[2.55,3.45]	[2.1229,2.1304]
0.2	[2.1197,2.1312]	[2.60,3.40]	[2.1238,2.1299]
0.3	[2.1228,2.1304]	[2.65,3.35]	[2.1255,2.1295]
0.4	[2.1236,2.1298]	[2.70,3.30]	[2.1260,2.1291]
0.5	[2.1250,2.1292]	[2.75,3.25]	[2.1267,2.1287]
0.6	[2.1261,2.1288]	[2.80,3.20]	[2.1273,2.1286]
0.7	[2.1268,2.1285]	[2.85,3.15]	[2.1277,2.1285]
0.8	[2.1274,2.1284]	[2.90,3.10]	[2.1280,2.1284]
0.9	[2.1278,2.1283]	[2.95,3.05]	[2.1281,2.1284]
1.0	[2.1281,2.1281]	[3.00,3.00]	[2.1283,2.1283]

For $\alpha = 1$, substitute $[X]^1$ into $f(u_x)$. By checking $f(b_2)f(x) < 0$ where $f(3)f(2.1284) < 0$, this implies that the fuzzy root lies between B and X. Then, replace $A = X$. Check

$|\epsilon_n| = |2.1284 - 2.1283|$.

As $|\epsilon_n| < \epsilon$, we stop the iteration. Hence, the fuzzy root for $f(u_x) = x^3 - 5x + 1$ is $(2.1241, 2.1284, 2.1299)$. The graphical representation of iterations done is illustrated in Figure-2.

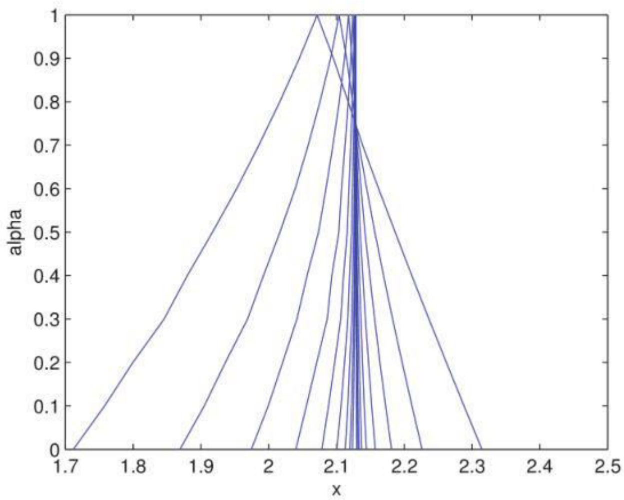

Figure-2. Graphical representation of optimum solution of a fuzzy non-linear equation, $f(u_x) = x^3 - 5x + 1$.

CONCLUSIONS

We have studied False Position's method for solving fuzzy nonlinear equations instead of standard analytical techniques which are not suitable. Several modifications have been made to guarantee the convexity of fuzzy solutions. First, the fuzzy nonlinear equation is written in parametric form and then is solved step by step using fuzzy false position's algorithm. Genetic algorithm has been proposed in order to determine the optimum solution at each iteration.

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