PERFORMANCE BASED LEASE CONTRACT INVOLVING DISCRETE PREVENTIVE MAINTENANCE

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ABSTRACT
Numerous studies have considered penalty as part of lease contract stipulation, which incurs whenever equipment downtime exceed a predetermined level. Under this stipulation, lessor would then choose maintenance effort level just enough to hinder them from penalty. Since it is used for business purpose, lower equipment downtime would always preferable for the lessee but necessarily for the lessor. Lower downtime requires higher maintenance effort, thus higher cost. To make sure that lessor is putting their best effort to achieve the lowest downtime possible a proper incentive needs to be determined. This paper considers not only penalty but also incentive as part of contract stipulation. We adopt the perspective of lessee and build decision making model for determining lease contract price and performance based payment scheme to be offered to the lessor. Principal agent theory is used as modeling approach due to its ability to simultaneously consider the interest of both parties involved.

Keywords: lease contract, performance based scheme, incentive, penalty, principal agent theory.

INTRODUCTION
Businesses need equipment to support their activities. Prior to 1970, most businesses owned the equipment, and maintenance was done in-house. Since 1970, there has been a shift towards maintenance outsourcing. The main reason is primarily due to faster change in technology. The maintenance equipment is expensive, and highly trained maintenance staff was needed, thus it is no longer economical to carry out in-house maintenance.

The trend was shift again in 1990, where businesses choose to lease equipment instead of owning them. According to [1] this trend is unavoidable. Therapid technological advances have resulted in improved equipment appearing in the market and this makes the earlier generation equipment obsolete at an ever-increasing pace. In addition to that, it is often more economical to lease equipment, rather than to buy them, as it involves less initial capital investment and offered attractive tax benefits.

There are numerous studies that consider maintenance for leased equipment. Most of them adopt the perspective of lessor and aimed to determine optimal preventive maintenance (PM) actions needed to hinder lessor from penalty. PM actions are modeled as imperfect, and the effect of such PM actions is described using age reduction method (ARM) or failure rate reduction method (FRRM). Research of [2-6], used ARM to model effect of imperfect PM, while the work of [7-9] applied FRRM.

Research of [2] deals with obtaining the optimal periodical PM while [7], and [9] deal with determining sequential PM policy. These researches concluded that periodical PM is easier to be implemented but yield higher maintenance cost compared to sequential PMs. To get the advantage of the two PM policies, [8] then combines periodical and sequential PM policies in one leasing contract horizon, and the result was satisfying. Opposed to periodical and sequential PMs, [10] introduce failure rate and [3-6] shows age threshold values as a sign to conduct PM.

All papers mentioned above consider only the lessor point of view and how they can achieve maximum profit by choosing the optimal PM policy. This paper aims to determine the optimal leasing price to be offered, which takes into account a lessee perspective, and the performance threshold to be met by a lessor. The most common performance criteria used in lease contracts is downtime. As equipment is used for business purpose, lessee would always prefer the lowest possible equipment downtime and hence they can choose to set downtime limit as low as possible, risking that the lessor would reject the lease contract or they can set a payment scheme which offer lessee a share of the income generated from the lower downtime. This is relevant to the practice of performance based maintenance which has become popular since 2007 [11].

The remainder of this paper is organized as follows. The model formulation is given in section 2, this includes problem description, PM and CM policy and cost, leasing price and solution value. Section 4 provides numerical approach to get the solution value. Section 4 provides numerical example and finally conclusions are drawn in the last section.

MODEL FORMULATION

Problem description
Consider lessee who leased equipment from lessor for \(L_p\) years for a price of \(P\) paid at the beginning of lease period. Lessor as the owner of the equipment is responsible for conducting CM and PM. Lessee then uses the equipment for business purposes that generate operational income \(R\) (dollar) per year. As it is used for business, equipment’s downtime due to failure is highly unfavorable in the eyes of the lessee.

To control lessor performance, a user (lessee) specifies downtime limit \((\bar{D})\), and determine performance based payment associated with this limit. When equipment downtime \((D(t))\) exceeds the limit, lessor is penalized. On the contrary, when
equipment downtime \((D(t))\) is lower than the limit, incentive is given.

**Equipment failure and maintenance cost**

Consider equipment, having \(F(t)\) as its failure distribution function, \(f(t) = \frac{dF(t)}{dt}\) as its failure density function and \(r(t) = \frac{f(t)}{1-F(t)}\) as its failure rate. To reduce the likelihood of failure along lease period, lessor takes periodical PM action. Each PM is conducted by a fixed degree of maintenance effort level \((\tau)\), where \(0 \leq \tau \leq 1\). The value of \(\tau \) indicates proportion of failure rate reduction due to PM. If \(\tau\) indicates the agreed-upon PM interval, failure rate reduction after each PM is given in Equation (1).

\[
\delta_{\text{PM}}(m) = \tau \left( r(t) - r(\hat{t}) \right) \tag{1}
\]

Let \(i = 0, 1, 2, \ldots, n\), and \(\tau_i = i\tau\) denote the time where the \(i\)-th PM is conducted. By default, \(\tau_0 = 0\) and no PM is conducted at \(t = 0\). Equipment failure rate considering PM effect along the lease period \((r_{\text{PM}}(m, t))\) is given in Equation (2).

\[
r_{\text{PM}}(m, t) = \begin{cases} r(t) - i\delta_{\text{PM}}(m), & \text{for } \tau_i \leq t < \tau_{i+1}; i = 0 \\ r(t) - n\delta_{\text{PM}}(m), & \text{for } \tau_n \leq t \leq L_p \end{cases} \tag{2}
\]

If perfect PM cost is \(C_{\text{PPM}}\) and PM cost has quadratic relationship with \(\tau\), then the total PM cost \((C_{\text{PM}}(m, L_p))\) is given in Equation (3).

\[
C_{\text{PM}}(m, L_p) = m^2 C_{\text{PPM}} \tau
\]

CM action is minimal repair; thereby failure process can be modeled as non-homogenous Poisson process with the intensity function stated in (12). The expected number of failures along \(L_p\) is \(\int_0^{L_p} r_{\text{PM}}(m, t) dt\) and hence the expected CM cost and its variance are given in Equation (4) and (5), respectively.

\[
E[C_{\text{CM}}(m, L_p)] = c_{\text{CM}} \int_0^{L_p} r_{\text{PM}}(m, t) dt \tag{4}
\]

\[
\text{Var}[C_{\text{CM}}(m, L_p)] = c_{\text{CM}}^2 \int_0^{L_p} r_{\text{PM}}(m, t) dt \tag{5}
\]

**Equipment downtime**

If the number of failure occurs along lease period is \(N(m, L_p)\), and each failure causes downtime \(X_i\), then the total downtime in the lease period, \((D(m, L_p))\), is given by Equation (6).

\[
D(m, L_p) = \sum_{i=1}^{N(m, L_p)} X_i \tag{6}
\]

If \(X_i\) is exponentially distributed with \(E[X] = \frac{1}{\lambda}\) and \(\text{Var}[X] = \frac{1}{\lambda^2}\). The expected total downtime along \(L_p\), is given by Equation (7), while its variance is in Equation (8).

\[
E[D(m, L_p)] = \frac{1}{\lambda} \left( \int_0^{L_p} r_{\text{PM}}(m, t) dt \right) \tag{7}
\]

\[
\text{Var}[D(m, L_p)] = \frac{2}{\lambda^2} \left( \int_0^{L_p} r_{\text{PM}}(m, t) dt \right) \tag{8}
\]

**Equipment operational revenue**

As lessee uses equipment for business purpose which generates revenue \(R\) (dollar/years), then the expected value and variance of equipment operational revenue are given by Equation (9) and (10), respectively.

\[
E[O_R(m)] = R \left( L_p - E[D(m, L_p)] \right) \tag{9}
\]

\[
\text{Var}[O_R(m)] = R^2 \text{Var}[D(m, L_p)] \tag{10}
\]

**Lease contract scheme**

The amount of payment specified in a lease contract is given in Equation (11). There are fixed payment \((P)\) given to the lessor at the beginning of lease period and \(\Psi(m, L_p)\) as performance related payment given at the end of lease period.

\[
P + \Psi(m, L_p) \tag{11}
\]

The value of performance related payment can be either positive, or negative. Positive performance payment means incentive and negative performance payment means penalty. Incentive occurs when the realization of equipment downtime \(D(m, L_p)\) is lower than a predetermined limit \(D\), while penalty occurs when the opposite happens. The expected value of performance related payment and its variance are given in Equation (12) and (13), respectively.

\[
E[\Psi(m, L_p)] = \phi R \left( E[D(m, L_p)] \right) \tag{12}
\]

\[
\text{Var}[\Psi(m, L_p)] = \phi^2 R^2 \text{Var}[D(m, L_p)] \tag{13}
\]

Notice that performance related payment is a percentage (denoted by \(\phi\)) of operational revenue gained [loss] due to downtime realization that is higher [lower] than the predetermined limit.

**Utility function of lessee and lessor**

We consider the form of mean variance utility function \(U(X) = E[X] - \frac{1}{2} \text{Var}[X]\) to describe utility of the lessee and lessor as it offer tractability [3]. The value of \(r\) represents the parties’ attitude towards risk, 0 if they are risk neutral and 1 if they are risk averse.
Lessee’s utility is a function of its profit ($\phi_1$) which is influenced by equipment operational revenue ($E[U_1(m)]$), leasing contract fixed payment ($F(m, L_p)$), and equipment operational cost ($O_{opr}$), given by Equation (14).

$$E[U_1(\phi_1, P, m)] = E[O_{opr}(m)] - \{P + E[\Psi(m, L_p)] \}$$

$$+ \frac{1}{2} \sqrt{\text{Var}[O_{opr}(m)]}$$

$$+ \sqrt{\text{Var}[\Psi(m, L_p)]}$$

(14)

On the other side, lessor’s utility is a function of its profit ($\phi_2$) which is influenced by their fixed income ($P$), performance related payment ($\Psi(m, L_p)$), corrective maintenance cost ($C_{CM}(m, L_p)$) and preventive maintenance cost ($C_{PM}(m, L_p)$) as in Equation (16).

$$E[U_2(\phi_2, P, m)] = P + E[\Psi(m, L_p)]$$

$$- \{E[C_{CM}(m, L_p)] + C_{PM}(m, L_p) \}$$

$$+ \frac{1}{2} \sqrt{\text{Var}[\Psi(m, L_p)]}$$

$$+ \sqrt{\text{Var}[C_{CM}(m, L_p)]}$$

(16)

or

$$E[U_2(\phi_2, P, m)] = P$$

$$+ \phi R \left( \bar{D} - \frac{1}{\lambda} \int_0^{\infty} \tau_m(m, t) dt \right)$$

$$- \left( c_{cm} \int_0^{L_p} \tau_m(m, t) dt \right)$$

$$+ m^2 c_{pm} n$$

$$- \frac{1}{2} \phi^2 R^2 \left( \frac{1}{\lambda} \int_0^{L_p} \tau_m(m, t) dt \right)$$

$$+ c_{cm}^2 \int_0^{L_p} \tau_m(m, t) dt$$

(17)

Both parties are interested to maximize their utility function subject to the decision taken by the other.

**Reserved utility**

To assure that lessor would accept the contract offered, the amount of a lease contract payment should meet their minimal utility level (reserved utility). Research of [11] has considered fixed utility level as common practice on the study of principal agent theory, but here we consider lessor’s reserved utility as function of cost they incurred. It is reasonable because we would demand higher return as we are asked to give higher effort. The amount of reserved utility is given on Equation (18).

$$R_u(m, L_p) = y \left( E[C_{CM}(m, L_p)] + C_{PM}(m, L_p) \right)$$

(18)

Lessee’s decision making model

Here, we build a decision making model from the perspective of lessee as follows:

$$\max_{\phi_2} E \left[ U_2(\phi_2, P, m) \right]$$

Subject to

$$\max_{\phi_2} E \left[ U_2(\phi_2, P, m) \right] \geq R_u(m, L_p)$$

(16)

Lessee would maximize their utility level by determining the optimal leasing price ($P$) and performance related payment ($\Psi(m, L_p)$). This is done by considering two constraints. The first is that the combined payment (fixed and performance related) should meet lessor’s minimum utility requirement ($R_u(m, L_p)$) to make sure that lessor accept the contract (c.1). The second one is that the PM effort level preferred by lessee should also generate maximum utility for lessor (c.2), thus making sure that lessor would always choose to implement it.

**SOLUTION METHOD**

We present here a numerical approach to find the optimal solutions.

**Step 1**

Use constraint c.2, determine $m$ as function of $\phi_2$ by taking the derivative of lessor’s utility function towards $m$, then have $m(\phi_2)$.

**Step 2**

Take constraint c.1 which is binding at $R_u(m, L_p)$ and $m(\phi_2)$, substitute them into utility function (Eq. 15). This creates lessee’s utility as function of $\phi_2$.

**Step 3**

Determine $\phi_2$ that maximizes lessee’s utility function, check for feasibility of ($0 \leq \phi_2 \leq 1$). If $\phi$ is not feasible, STOP.

If $\phi$ is feasible, continue to step 4.

**Step 4**

Find the value of $\phi_2$ given $m$ from step 3. Check for feasibility of $m$ ($0 \leq m \leq 1$). If $m$ is not feasible, STOP.

If $m$ is feasible, continue to step 5.

**Step 5**

Find $P$ by substituting $m$ (from step 4) and $\phi_2$ (from step 3) into (c.1) which is binding at $R_u(m, L_p)$. Check feasibility of $P$ ($\phi_2 > 0$). If $P$ is not feasible, STOP.

If $P$ is feasible, continue to step 6.

**Step 6**

Denote $m$, $P$, and $\phi_2$ as solutions of the model.
NUMERICAL EXAMPLE

We consider equipment with failure modeled by two parameter Weibull distribution, \( F(t, \alpha) = 1 - e^{-\alpha t^\beta} \). The value of \( \alpha = 1 \) years and \( \beta = 2 \). Leasing contract period is planned for \( L_p = 5 \) years. Each failure causes downtime with parameter \( \lambda = 50 \). Other parameters are given in Table-1.

<table>
<thead>
<tr>
<th>Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
</tr>
<tr>
<td>( g10^3 )</td>
</tr>
</tbody>
</table>

During the contract, lessor is responsible to conduct PM and CM. Penalty is given if the equipment’s downtime along leasing period falls below \( D = 5\%L_p \), and incentive is given when the opposites happen.

Table-2. Payment scheme solution for various risk attitude setting.

<table>
<thead>
<tr>
<th>Risk Attitude Setting</th>
<th>( P )</th>
<th>( \varphi )</th>
<th>( m )</th>
<th>( U_{lesser} )</th>
<th>( U_{lesser} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both Risk Neutral</td>
<td>54.127</td>
<td>0.769</td>
<td>0.183</td>
<td>2305.629</td>
<td>16.137</td>
</tr>
<tr>
<td>Risk Averse Lessor</td>
<td>109.327</td>
<td>0.016</td>
<td>0.128</td>
<td>2259.843</td>
<td>15.037</td>
</tr>
<tr>
<td>Risk Averse User</td>
<td>50.085</td>
<td>0.995</td>
<td>0.218</td>
<td>2305.025</td>
<td>17.189</td>
</tr>
</tbody>
</table>

This particular scenario is selected to avoid the case when zero PM effort level to be sufficient for maintaining downtime below target. Table-2 show results for various set of risk attitudes setting.

By simple calculation we could notice that for \( \alpha = 1 \), putting maintenance effort level \( m = 0 \) would be sufficient to hinder lessor from penalty, but Table-2 shows that the optimal \( m \) is always greater than zero. This shows that under payment scheme offered, lessor would be induced to perform better than target.

The result pattern shown for various risk attitude setting in Table-2 is described as follows. When both parties are risk averse, there is no lost due to risk premium, combination of fixed price and performance related payment are optimal to induce \( m \). When lessor is risk averse, they would avoid risk due to uncertainty of income, the value of \( \varphi \) on this second setting is much smaller than on the first. The opposite of this shows on the third setting, where lessee is risk averse. They would then push risk to the lessor showed by the value of \( \varphi \) which is much larger than on the first.

CONCLUSIONS

We have determined the optimal lease payment scheme (fixed price and performance related payment) to be offered by the lessor at the beginning of a lease contract. This lease payment meets lessor’s minimum utility requirement thus assured contract acceptance and induced lessee’s preferred maintenance effort level to the lessor. We also consider more relevant scenario where lessee’s reserved utility is not constant, but dependent on the effort they take.

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REFERENCES


