VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

PRESSURE-TRANSIENT ANALYSIS FOR OFF-CENTERED HORIZONTAL WELLS IN HOMOGENEOUS ANISOTROPIC RESERVOIRS WITH CLOSED AND OPEN BOUNDARIES

Freddy Humberto Escobar, Nicolas Manuel Cortes, Oscar David Pabon and Claudia Marcela Hernandez
Universidad Surcolombiana/CENIGAA, Avenida Pastrana, Neiva, Huila, Colombia
E-Mail: fescobar@usco.edu.co

ABSTRACT

Because of the significantly larger drainage area over the vertical wells, it is recognized that horizontal wells produce more than twice the flow rate of a vertical well under the same drawdown pressure which also reduces the occurrence of coning. Then, horizontal well drilling has increased around the well; therefore, it is so important to describe and predict well pressure behavior and develop tools for well test data interpretation. Although, there is already a methodology for pressure transient analysis for the mentioned well configuration, known as *TDS* technique, this does not included neither hemilinear flow, parabolic flow nor open flow boundaries. Besides, most of the equations for gas well have not been introduced in the literature. This paper deals with this situation and presents a detailed synthetic interpretation example of the complemented interpretation technique.

Keywords: horizontal well, anisotropy, *TDS* Technique, hemilinear flow, parabolic flow, open and closed boundaries, pressure transient analysis.

1. INTRODUCTION

In the field of well test analysis for horizontal wells several researches have been conducted and several mathematical models have been presented. The earliest investigations on this topic were presented by Daviau *et al.* (1985), Clonts and Ramey (1986) and Goode and Thambynayagam (1987). They presented mathematical model to study horizontal well pressure behavior and identified several flow regimes encountered during a pressure test run on these wells. They also applied the straight-line conventional analysis as the interpretation technique.

The works conducted by Engler and Tiab (1996a, 1996b) were based on the mathematical models describen well pressure behavior presented by Goode and Thambynayagam (1987) for semi-infinite homogeneous anisotropic and heterogeneous anisotropic oil formations. Engler and Tiab (1996a, 1996b) introduced the *TDS* Technique, Tiab (1993a, 1993b), for the interpretation of pressure tests in horizontal wells in homogeneous and naturally fractured reservoirs. These, of course, use characteristic points found on the pressure and pressure derivative curves. They developed expressions for obtaining information from the following flow regimes: early radial flow, pseudorradial flow, late linear flow. Moreover, they included the intersection point found between different flow regimes, such as, early radial and pseudorradial pressure derivative, early linear and late linear pressure derivative.

Isaaka *et al.* (2000) found between the early linear and pseudorradial flow regimes the presence of the elliptical flow regime in horizontal well. It was recognized by a slope of 0.35 in the pressure derivative. Later on, Chacon, Djebrouni and Tiab (2004) introduced a pressure derivative model for such flow regime. The pressure derivative slope was of 0.36. This model was later used by Escobar *et al.* (2004) and Escobar and Montealegre (2007) to develop pressure test interpretation by *TDS* Technique and conventional analysis, respectively. However, Martinez, Escobar and Bonilla (2012) reformulated a new governing equation for the elliptical flow and provided a corrected version of the *TDS* Technique and conventional analysis for this flow regime.

Later extensions of the *TDS* Technique have been presented by Escobar, Bernal and Olaya-Marin (2014) who developed a practical well test interpretation methodology using characteristic points for fractured horizontal wells in unconventional shale reservoirs using dual-porosity models in the stimulated reservoir volume, and, in the same year Bernal, Escobar and Ghisays (2014) established equations for permeability, half-fracture length, skin factor and reservoir length without considering the model for hydraulically-fractured shale formations using the concept of induced permeability field. Escobar, Zhao and Zhang (2014) presented a TDS methodology for interpretation of pressure tests in horizontal wells included the effect of the threshold pressure gradient.

The purpose of this work is to extend *TDS* Technique for Hemilinear flow, parabolic Flow, open and close boundaries flows for horizontal wells as performed by Escobar, Hernandez and Hernandez (2007) who found the characteristic maximum points and governing equations for vertical wells in long homogeneous reservoirs using *TDS* technique.

2. MATHEMATICAL FORMULATION

2.1. Mathematical model

Goode and Thambynayagam (1987) presented the mathematical solution for horizontal well pressure reservoir pressure behavior in both homogeneous and heterogeneous reservoirs. Its homogeneous model was

ARPN Journal of Engineering and Applied Sciences

used by Engler and Tiab (1996a, 1996b). This work also adopts this mathematical solution.

Engler and Tiab (1996b) proposed the following definitions of the dimensionless quantities:

$$P_D = \frac{k_y L_w \Delta P}{141.2 q \mu B} \tag{1}$$

$$t_D * P_D' = \frac{k_y L_w}{141.2 q \mu B}\left[t * \Delta P'\right] \tag{2}$$

$$t_D = \frac{0.0002637 k_y t}{\phi \mu c_t r_w^2} \tag{3}$$

$$t_{DA} = t_D \frac{r_w^2}{A} = \frac{0.0002637 k_y t}{\phi \mu c_t A} \tag{4}$$

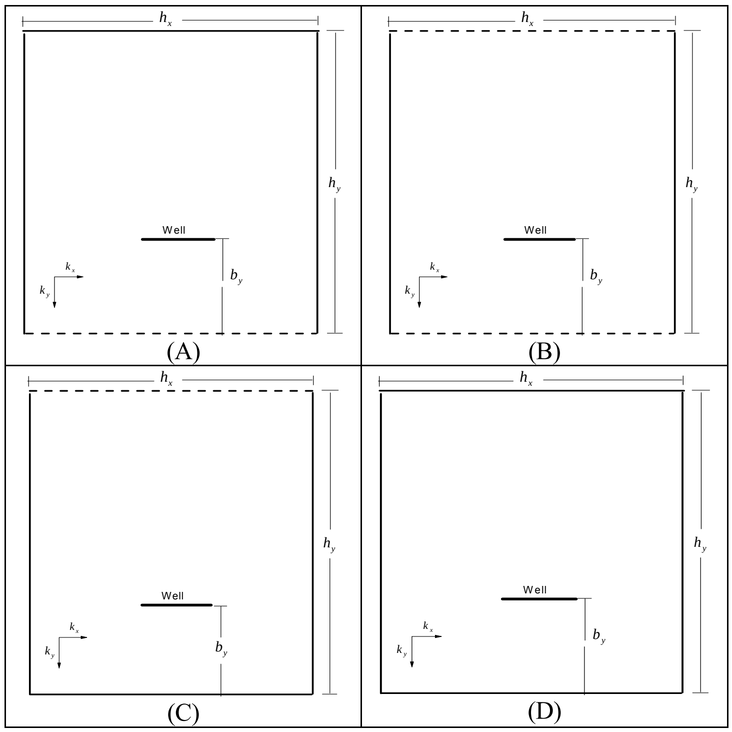

Figure-1. Reservoir geometry cases for off-centered horizontal wells in the reservoir. (A) Well located near the open boundary and the other boundary is closed. (B) Well is located within two open boundaries. (C) Well located near the closed boundary and the other boundary is open. (D) All closed boundaries. After Cortes and Pabon (2016).

VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

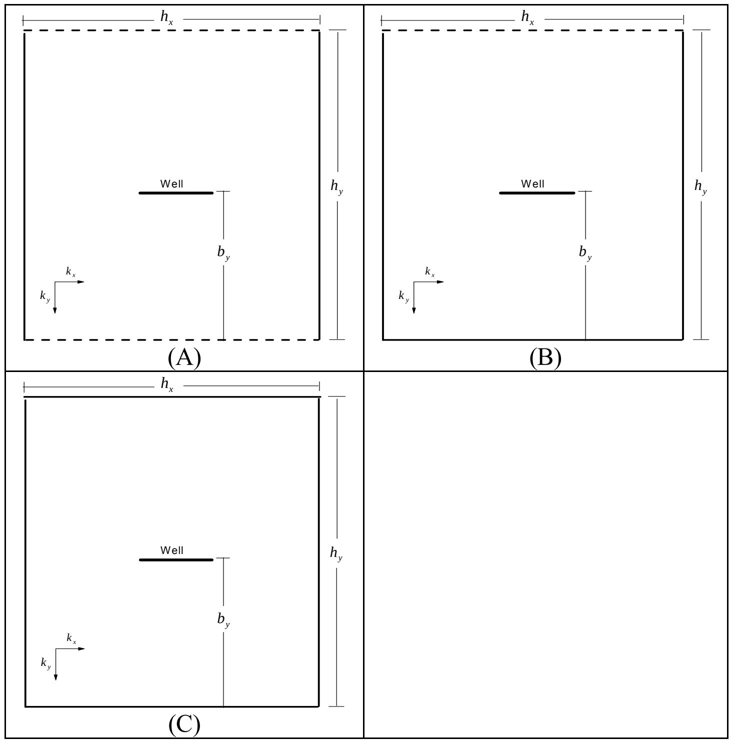

Figure-2. Reservoir geometry cases for centered horizontal wells in the reservoir (A) The reservoir has both open boundaries. (B) The reservoir has one open boundary. (C) The reservoir has all closed boundaries. After Cortes and Pabon (2016).

$$Y_D = \frac{2b_y}{h_y} \tag{5}$$

2.2. *TDS* Technique

Engler and Tiab (1996) formulated the following equations to find various characteristic parameters and skin for each flow regime:

For Early radial flow:

$$\sqrt{k_y k_z} = \left(\frac{70.6 q \mu B}{L_w (t*\Delta P')_{er1}} \right) \tag{6}$$

$$s_m = \frac{1}{2}\left[\frac{\Delta P_{er}}{(t*\Delta P')_{er}} - \ln\left(\frac{\sqrt{k_y k_z}\, t_{er}}{\phi \mu c_t r_w^{\ 2}} \right) + 7.43 \right] \tag{7}$$

For early linear flow:

$$L_w = \frac{4.064 qB}{h_z (t*\Delta P')_{el}} \sqrt{\frac{\mu t_{el}}{k_y \phi c_t}} \tag{8}$$

$$k_y = \left(\frac{4.064 qB}{h_z (t*\Delta P')_{el}} \right)^2 \frac{\mu t_{el}}{L_w^{\ 2} \phi c_t} \tag{9}$$

$$s_m + s_z = \frac{0.029}{h_z} \sqrt{\frac{k_z t_{el}}{\phi \mu c_t}} \left[\frac{\Delta P_{el}}{(t*\Delta P')_{el}} - 2.0 \right] \tag{10}$$

For pseudorradial flow:

$$\sqrt{k_x k_y} = \frac{70.6 q \mu B}{h_z (t*\Delta P')_{pr1}} \tag{11}$$

$$s_m + s_z = \frac{L_w}{2h_z} \sqrt{\frac{k_z}{k_y}} \left[\frac{\Delta P_{pr}}{(t*\Delta P')_{pr}} - \ln\left(\frac{k_x t_{pr}}{\phi \mu c_t L_w^{\ 2}} \right) + 4.659 \right] \tag{12}$$

For late linear flow:

$$h_x = \frac{4.064 qB}{h_z (t*\Delta P')_{ll}} \sqrt{\frac{\mu t_{ll}}{k_y \phi c_t}} \tag{13}$$

$$k_y = \left(\frac{4.064 qB}{h_z (t*\Delta P')_{ll}} \right)^2 \left(\frac{\mu t_{ll}}{h_x^2 \phi c_t} \right) \tag{14}$$

$$(s_m + s_z + s_x) = \frac{0.029 L_w}{h_x h_z} \sqrt{\frac{k_z t_{ll}}{\phi \mu c_t}} \left[\frac{\Delta P_{ll}}{(t*\Delta P')_{ll}} - 2 \right] \tag{15}$$

VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

For the elliptical flow regime, Martinez *et al.* (2012) presented the following relationships:

$$L_w = \left[5.5962 \frac{q\mu^{0.64}B}{k_y^{0.5}k_x^{0.14}h_z\left[t*\Delta P'\right]_{Ell}} \left(\frac{t_{Ell}}{\phi c_t}\right)^{0.36} \right]^{1/0.72} \quad (16)$$

$$k_y = \left[5.5962 \frac{q\mu^{0.64}B}{L_w^{0.72}k_x^{0.14}h_z\left[t*\Delta P'\right]_{Ell}} \left(\frac{t_{Ell}}{\phi c_t}\right)^{0.36} \right]^{1/0.5} \quad (17)$$

$$s_{Ell} = \frac{1}{25.231296}\left[\frac{\Delta P_{Ell}}{\left[t*\Delta P'\right]_{Ell}} - 2.77778\right] \left[\frac{L_w^{0.28}k_y^{0.5}}{h_z k_x^{0.14}}\left(\frac{t_{Ell}}{\phi\mu c_t}\right)^{0.36}\right] \quad (18)$$

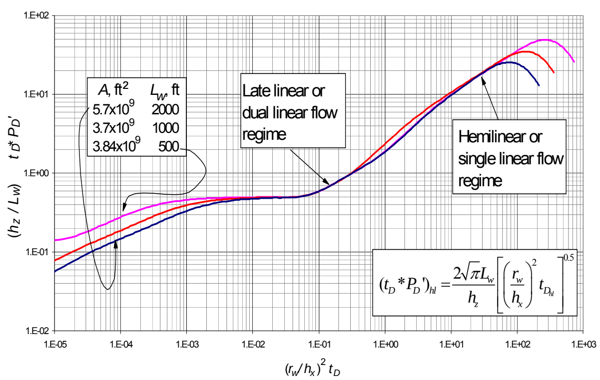

Figure-3. Dimensionless $(r_w/h_x)^2\ t_D$ versus $(h_z/L_w)\ t_D*P_D'$ log-log plot with unified late-linear and hemilinear flow regimes. After Cortes and Pabon (2016).

For vertical wells, Escobar *et al.* (2007) differentiated between linear flow (dual linear flow) when there exist two linear flows at both sides of the well and single linear (hemilinear flow) in which a linear flow takes place at only one lateral side of the well. This definition is used by Cortes and Pabon (2016). Hemilinear flow occurs within the reservoir when the horizontal well is located near the closed boundary; regardless the far boundary is closed (late pseudosteady- state period) or open (steady state). Reservoir geometries of this flow regime are shown in Figure-1 (C) and (D). The governing equation of the pressure derivative for hemilinear flow regime was empirically obtained from Figure-3 by Cortes and Pabon (2016). Then, the governing pressure equation was obtained by integration:

$$P_{D_{hl}} = \frac{4L_w r_w}{h_z h_x}\sqrt{\pi t_{D_{hl}}} + \sqrt{\frac{k_y}{k_z}}(s_x + s_z + s_m + s_{hl}) \quad (19)$$

The dimensionless governing pressure derivative is:

$$(t_D * P_D')_{hl} = \frac{2\sqrt{\pi}L_w}{h_z}\left[\left(\frac{r_w}{h_x}\right)^2 t_{D_{hl}}\right]^{0.5} \quad (20)$$

Replacing the dimensionless quantities given by Equations (2) and (3) into Equation (20), it yields:

$$k_y = 66.067\left(\frac{\mu t_{hl}}{\phi c_t}\right)\left[\frac{qB}{h_z h_x [t*\Delta P']_{hl}}\right]^2 \quad (21)$$

$$h_x = 8.1282\frac{qB}{h_z[t*\Delta P']_{hl}}\left(\frac{\mu t_{hl}}{k_y\phi c_t}\right)^{0.5} \quad (22)$$

Dividing Equation (19) by Equation (20) and, then, substituting Equations (1), (2) and (3) into the resulting expression, it yields:

$$(s_x + s_z + s_m + s_{hl}) = \frac{1}{17.3716}\left[\frac{L_w}{h_z h_x}\sqrt{\frac{k_z t_{hl}}{\phi\mu c_t}}\right]\left[\frac{\Delta P_{hl}}{[t*\Delta P']_{hl}} - 2\right] \quad (23)$$

Parabolic flow appears within the reservoir when the horizontal well is located near the open boundary regardless the far boundary is either close or open to flow. Reservoir geometries of this flow regime are shown in Figure-1 (A) and (B). It determined dimensionless governing equations of Parabolic flow.

The dimensionless governing pressure is:

$$P_D = -11.1326\frac{L_w^{0.62}b_y^{2.3}}{h_z^{0.62}h_y^{1.05}r_w^{1.25}}(t_{D_{PB}})^{-0.5} + \sqrt{\frac{k_y}{k_z}}(s_x + s_z + s_m + s_{PB}) \quad (24)$$

The dimensionless governing pressure derivative is:

$$(t_D * P_D')_{PB} = 5.5663\frac{L_w^{0.62}b_y^{2.3}}{h_z^{0.62}h_y^{1.05}r_w^{1.25}}(t_{D_{PB}})^{-0.5} \quad (25)$$

Replacing the dimensionless parameters given by Equations (2) and (3) into Equation (25), it yields:

$$b_y = \left[\frac{\frac{1}{48403.5189}\frac{k_y^{1.5}L_w^{0.38}h_z^{0.62}h_y^{1.05}r_w^{0.25}}{q\mu^{1.5}B}}{(t*\Delta P')_{PB}\left(\frac{t_{PB}}{\phi c_t}\right)^{0.5}}\right]^{1/2.3} \quad (26)$$

VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

Dividing Equation (24) by Equation (25) and, then, substituting Equation (1), (2) and (3) into the result, we obtain:

$$(s_x + s_z + s_m + s_{PB}) = \left[\frac{342.777 \dfrac{L_w^{0.62} b_y^{2.3}}{k_y h_z^{0.62} h_y^{1.05} r_w^{0.25}}}{\left(\dfrac{k_z \phi \mu c_t}{t_{PB}}\right)^{0.5}} \right] \left[\frac{\Delta P_{PB}}{[t*\Delta P']_{PB}} + 2 \right] \quad (27)$$

For the first case of steady state, the reservoir geometry is shown in Figure-1 (B). The dimensionless governing pressure derivative equation is:

$$(t_D * P_D')_{ss1} = 361699.2 \frac{L_w^{0.17}}{h_z^{0.75}} \left(\frac{b_y}{h_y}\right)^3 t_{DA_{ss1}}^{-1} \quad (28)$$

Replacing the dimensionless quantities given by Equations (2) and (3) into Equation (28), it yields:

$$A = \frac{1}{1.93674\times10^{11}} \frac{k_y^2 L_w^{0.83}}{\mu^2} \frac{h_z^{0.75}}{L_w^{0.17}} [t*\Delta P']_{ss1} \left(\frac{t_{ss1}}{qB\phi c_t}\right)\left(\frac{h_y}{b_y}\right)^3 \quad (29)$$

The second case of steady state, the reservoir geometry is shown in Figure-1 (A). The dimensionless governing pressure derivative expression is:

$$(t_D * P_D')_{ss2} = 322427.2182 \frac{L_w^{0.17} b_y^{3.07}}{h_z^{0.72} h_y^{2.95}} t_{DA_{ss2}}^{-1} \quad (30)$$

Replacing the dimensionless parameters given by Equations (2) and (3) into Equation (30), it yields:

$$A = \frac{1}{1.726458976\times10^{11}} \frac{k_y^2 L_w^{0.83}}{\mu^2} \frac{h_z^{0.72} h_y^{2.95}}{b_y^{3.07}} [t*\Delta P']_{ss2} \left(\frac{t_{ss2}}{qB\phi c_t}\right) \quad (31)$$

The third case of steady state is sketched by the reservoir geometry of Figure-1 (C). The dimensionless governing pressure derivative equation is:

$$(t_D * P_D')_{ss3} = \frac{1}{242.94} \left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}}\right) t_{DA_{ss3}}^{-1} \quad (32)$$

Replacing Equations (2) and (3) into Equation (32), it yields:

$$h_y = \left[\frac{1}{2204.07} \frac{k_y^2 L_w^{0.15} h_z^{0.75} [t*\Delta P']_{ss3}}{\mu^2 h_x^{0.2}} \left(\frac{t_{ss3}}{qB\phi c_t}\right) \right]^{0.4} \quad (33)$$

The fourth case of steady state behavior is represented by the reservoir geometry shown in Figure-2 (B). The dimensionless governing pressure derivative equation is:

$$(t_D * P_D')_{ss4} = \frac{1}{468.124} \left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}}\right) (t_{DA})_{ss4}^{-1} \quad (34)$$

Replacing the dimensionless parameters given by Equations (2) and (3) into Equation (34), it yields:

$$h_y = \left[\frac{1}{1143.836} \frac{k_y^2 L_w^{0.15} h_z^{0.75} [t*\Delta P']_{ss4}}{h_x^{0.2} \mu^2} \left(\frac{t_{ss4}}{qB\phi c_t}\right) \right]^{0.4} \quad (35)$$

For the fifth case of steady state behavior, the reservoir geometry is shown in Figure-2 (A) and the dimensionless equation of the governing pressure derivative is given by:

$$(t_D * P_D')_{ss5} = \frac{1}{4085.7986} \left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}}\right) (t_{DA})_{ss5}^{-1} \quad (36)$$

Replacing the dimensionless quantities given by Equations (2) and (3) into Equation (36), it yields:

$$h_y = \left[\frac{1}{131.053} \frac{k_y^2 L_w^{0.15} h_z^{0.75} [t*\Delta P']_{ss5}}{\mu^2 h_x^{0.2}} \left(\frac{t_{ss5}}{qB\phi c_t}\right) \right]^{0.4} \quad (37)$$

For the pseudosteady-state period, the reservoir geometry is shown in Figure-1 (D) and Figure-2 (C). The dimensionless governing pressure derivative equation is:

$$(t_D * P_D')_{pss} = 2\pi \left(\frac{L_w}{h_z}\right) (t_{DA})_{pss} \quad (38)$$

Replacing Equations (2) and (3) into Equation (38), it yields:

$$A = \frac{1}{4.2744} \frac{qBt_{pss}}{h_z \phi c_t [t*\Delta P']_{pss}} \quad (39)$$

ARPN Journal of Engineering and Applied Sciences

2.2.1. Intersection points between flow regimes

Engler and Tiab (1996b) presented the following two equations, for the intersections (early radial-early linear) and (early radial-late linear),

$$k_z = 301.7727 \phi \mu c_t \frac{h_z^2}{t_{er-eli}} \tag{40}$$

$$k_z = 301.7727 \left(\frac{h_x h_z}{L_w} \right)^2 \frac{\phi \mu c_t}{t_{er-lli}} \tag{41}$$

Cortes and Pabon (2016) intercepted several pressure derivative equations to obtain useful intersection point expression. For instance, the time for intersection between early radial flow and elliptical flow regimes, formerly given by Martinez *et al.* (2012) is given by:

$$(t_{D_{er-Elli}})^{0.36} = \frac{1}{1.53989388} \sqrt{\frac{k_y}{k_z}} \frac{h_z}{r_w^{0.72} L_w^{0.28}} \left(\frac{k_x}{k_y} \right)^{0.14} \tag{42}$$

This becomes in real units;

$$k_z = \left[12.6156 \frac{h_z k_x^{0.14}}{L_w^{0.28}} \left(\frac{\phi \mu c_t}{t_{er-Elli}} \right)^{0.36} \right]^2 \tag{43}$$

The intersection time between early radial and hemilinear flow regime is given by:

$$(t_{D_{er-hli}})^{0.5} = \frac{1}{2} \sqrt{\frac{k_y}{k_z}} \frac{h_z}{(2\sqrt{\pi}) L_w} \left(\frac{h_x}{r_w} \right) \tag{44}$$

After replacing the dimensionless parameters in the above expression, it yields:

$$k_z = 75.4432 \left(\frac{h_z h_x}{L_w} \right)^2 \left(\frac{\phi \mu c_t}{t_{er-hli}} \right) \tag{45}$$

The obtained expression for the intersection time between early radial and parabolic flow regime is:

$$(t_{D_{er-PBi}})^{0.5} = 11.1326 \frac{L_w^{0.62} b_y^{2.3}}{h_z^{0.62} h_y^{1.05} r_w^{1.25}} \sqrt{\frac{k_z}{k_y}} \tag{46}$$

in real units, this becomes:

$$b_y = \left[\frac{1}{685.5538} \frac{h_z^{0.62} h_y^{1.05} r_w^{0.25}}{L_w^{0.62}} \sqrt{\frac{k_y}{k_z}} \left(\frac{k_y t_{er-PBi}}{\phi \mu c_t} \right)^{0.5} \right]^{1/2.3} \tag{47}$$

The time of intersection between early radial and first case of steady-state period is governed by:

$$t_{DA_{er-ss1i}} = 723398.4 \frac{L_w^{0.17}}{h_z^{0.75}} \left(\frac{b_y}{h_y} \right)^3 \sqrt{\frac{k_z}{k_y}} \tag{48}$$

This becomes in real units;

$$A = \frac{1}{2743262799} \frac{h_z^{0.75} k_y^{1.5}}{L_w^{0.17} k_z^{0.5}} \left(\frac{h_y}{b_y} \right)^3 \frac{t_{er-ss1i}}{\phi \mu c_t} \tag{49}$$

The dimensionless expression for the time of intersection between early radial flow regime and second case of steady-state period is:

$$t_{DA_{er-ss2i}} = 644854.44 \frac{L_w^{0.17} b_y^{3.07}}{h_z^{0.72} h_y^{2.95}} \sqrt{\frac{k_z}{k_y}} \tag{50}$$

After replacing the dimensionless parameters in the above expression, it yields:

$$A = \frac{1}{2445409329} \frac{h_z^{0.72} h_y^{2.95}}{L_w^{0.17} b_y^{3.07}} \frac{k_y^{1.5} t_{er-ss2i}}{\phi \mu c_t k_z^{0.5}} \tag{51}$$

The intersection time between early radial flow and the third case of steady-state period is governed by:

$$t_{DA_{er-ss3i}} = \frac{1}{121.47} \left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}} \right) \sqrt{\frac{k_z}{k_y}} \tag{52}$$

This becomes in real units;

$$h_y = \left[\frac{1}{31.2191} \left(\frac{h_z^{0.75}}{h_x^{0.2} L_w^{0.85}} \right) \frac{k_y^{1.5} t_{er-ss3i}}{\phi \mu c_t k_z^{0.5}} \right]^{0.4} \tag{53}$$

The time of intersection between early radial flow regime and the fourth case of steady-state period leads to obtain the following dimensionless expression:

$$t_{DA_{er-ss4i}} = \frac{1}{234.062} \left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}} \right) \sqrt{\frac{k_z}{k_y}} \tag{54}$$

in real units, this becomes:

$$h_y = \left[\frac{1}{16.2016} \left(\frac{h_z^{0.75}}{h_x^{0.2} L_w^{0.85}} \right) \frac{k_y^{1.5} t_{er-ss4i}}{\phi \mu c_t k_z^{0.5}} \right]^{0.4} \tag{55}$$

The intersection time between early radial flow and fifth case of steady-state period is represented by:

$$t_{DA_{er-ss5i}} = \frac{1}{2042.8993} \left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}} \right) \sqrt{\frac{k_z}{k_y}} \tag{56}$$

After replacing the dimensionless parameters in the above expression, it yields:

$$h_y = \left[\frac{1}{1.8563} \left(\frac{h_z^{0.75}}{h_x^{0.2} L_w^{0.85}} \right) \frac{k_y^{1.5} t_{er-ss5i}}{\phi \mu c_t k_z^{0.5}} \right]^{0.4} \tag{57}$$

The intersecting time between early radial flow and pseudosteady-state period provides:

$$(t_{DA_{er-pssi}}) = \frac{1}{4\pi} \sqrt{\frac{k_y}{k_z}} \left(\frac{h_z}{L_w} \right) \tag{58}$$

in real units, it becomes:

$$A = \frac{1}{301.7727} \left(\frac{L_w}{h_z} \right) \frac{k_y^{0.5} k_z^{0.5} t_{er-pssi}}{\phi \mu c_t} \tag{59}$$

The time of intersection between the first case of steady-state period and early linear flow regime gives:

$$(t_{D_{el-ss1i}})^{1.5} = 204066.921 \frac{L_w^{0.17}}{h_z^{0.25}} \left(\frac{b_y}{h_y} \right)^3 \frac{A}{r_w^{\ 3}} \tag{60}$$

This becomes in real units;

$$A = \frac{1}{47654884040} \frac{h_z^{0.25}}{L_w^{0.17}} \left(\frac{h_y}{b_y} \right)^3 \left(\frac{k_y t_{el-ss1i}}{\phi \mu c_t} \right)^{1.5} \tag{61}$$

The time of intersection between the first case of steady-state period and elliptical flow regime gives in both dimensionless form is given by Equation (62) and after replacing the dimensionless quantities allows finding reservoir area:

$$\left(t_{D_{Ell-ss1i}} \right)^{1.36} = 469771.5923 \frac{h_z^{0.25} A}{r_w^{2.72} L_w^{0.11}} \left(\frac{k_x}{k_y} \right)^{0.14} \left(\frac{b_y}{h_y} \right)^3 \tag{62}$$

$$A = \frac{1}{34608037630} \left(\frac{h_y}{b_y} \right)^3 \frac{L_w^{0.11} k_y^{1.5}}{h_z^{0.25} k_x^{0.14}} \left(\frac{t_{Ell-ss1i}}{\phi \mu c_t} \right)^{1.36} \tag{63}$$

As in the former case, the intersection time between the first case of steady state and pseudorradial flow regime leads to:

$$t_{D_{pr-ss1i}} = 723398.4 \frac{h_z^{0.25}}{L_w^{0.83}} \left(\frac{b_y}{h_y} \right)^3 \frac{A}{r_w^2} \sqrt{\frac{k_x}{k_y}} \tag{64}$$

$$A = \frac{1}{2743262799} \frac{L_w^{0.83}}{h_z^{0.25}} \left(\frac{h_y}{b_y} \right)^3 \frac{k_y^{1.5} t_{pr-ss1i}}{\phi \mu c_t k_x^{0.5}} \tag{65}$$

The time of intersection between the first case of steady-state period and late linear flow regime gives:

$$(t_{D_{ll-ss1i}})^{1.5} = 204066.921 \frac{h_z^{0.25} b_y^3}{L_w^{0.83} r_w^3} \left(\frac{h_x}{h_y} \right)^2 \tag{66}$$

After replacing the dimensionless quantities and solving for reservoir length:

$$h_y = \left[47654884040 \frac{h_z^{0.25} b_y^3 h_x^2}{L_w^{0.83}} \left(\frac{\phi \mu c_t}{k_y t_{ll-ss1i}} \right)^{1.5} \right]^{0.5} \tag{67}$$

The time of intersection between the first case of steady-state period and parabolic flow regime provide Equation (68) which allows obtaining reservoir area one the dimensionless parameters are replaced there:

$$(t_{D_{PB-ss1i}})^{0.5} = 64980.18 \frac{b_y^{0.7} A}{h_z^{0.13} L_w^{0.45} h_y^{1.95} r_w^{0.75}} \tag{68}$$

$$A = \frac{1}{4001527.862} \frac{h_z^{0.13} L_w^{0.45} h_y^{1.95}}{b_y^{0.7} r_w^{0.25}} \left(\frac{k_y t_{PB-ss1i}}{\phi \mu c_t} \right)^{0.5} \tag{69}$$

Other expressions useful to find reservoir area or reservoir length use the time of intersection of the second case of steady- state period with early linear flow regime - Equation (70), elliptical flow regime -Equation (72), with

ARPN Journal of Engineering and Applied Sciences

pseudorradial flow regime, Equation (74) and with late linear flow –Equation (76). Then;

$$(t_{D_{el-ss2i}})^{1.5} = 181910.078 \frac{h_z^{0.28} L_w^{0.17} b_y^{3.07} A}{h_y^{2.95} r_w^3} \quad (70)$$

$$A = \frac{1}{42480690310} \frac{h_y^{2.95}}{h_z^{0.28} L_w^{0.17} b_y^{3.07}} \left(\frac{k_y t_{el-ss2i}}{\phi \mu c_t} \right)^{1.5} \quad (71)$$

$$(t_{D_{Ell-ss2i}})^{1.36} = 418765.5038 \frac{h_z^{0.28} A}{r_w^{2.72} L_w^{0.11}} \left(\frac{k_x}{k_y} \right)^{0.14} \left(\frac{b_y^{3.07}}{h_y^{2.95}} \right) \quad (72)$$

$$A = \frac{1}{30850422960} \frac{L_w^{0.11} h_y^{2.95} k_y^{1.5}}{h_z^{0.28} b_y^{3.07} k_x^{0.14}} \left(\frac{t_{Ell-ss2i}}{\phi \mu c_t} \right)^{1.36} \quad (73)$$

$$t_{D_{pr-ss2i}} = 644854.4364 \frac{b_y^{3.07} h_z^{0.28}}{h_y^{2.95} L_w^{0.83}} \frac{A}{r_w^2} \sqrt{\frac{k_x}{k_y}} \quad (74)$$

$$A = \frac{1}{2445409315} \frac{h_y^{2.95} L_w^{0.83}}{b_y^{3.07} h_z^{0.28}} \frac{k_y^{1.5} t_{pr-ss2i}}{\phi \mu c_t k_x^{0.5}} \quad (75)$$

$$(t_{D_{ll-ss2i}})^{1.5} = 181910.078 \frac{h_z^{0.28} b_y^{3.07} h_x^2}{L_w^{0.83} h_y^{1.95} r_w^3} \quad (76)$$

$$h_y = \left[42480690310 \frac{h_z^{0.28} b_y^{3.07} h_x^2}{L_w^{0.83}} \left(\frac{\phi \mu c_t}{k_y t_{ll-ss2i}} \right)^{1.5} \right]^{1/1.95} \quad (77)$$

Also, reservoir area or reservoir length can be found from the intersection of the second case of steady-state period and parabolic flow regimes. Then:

$$(t_{D_{PB-ss2i}})^{0.5} = 57924.8726 \frac{b_y^{0.77} A}{h_z^{0.1} h_y^{1.9} L_w^{0.45} r_w^{0.75}} \quad (78)$$

$$A = \frac{1}{3567056.779} \frac{h_z^{0.1} h_y^{1.9} L_w^{0.45}}{b_y^{0.77} r_w^{0.25}} \left(\frac{k_y t_{PB-ss2i}}{\phi \mu c_t} \right)^{0.5} \quad (79)$$

The time of intersection between the third case of steady-state period with early linear flow gives Equation (80), with elliptical flow gives Equation (82), with pseudorradial equation (84), with late linear flow gives Equation (86) and with hemilinear flow gives Equation (88). Once the dimensionless time is replaced in these resulting expressions, reservoir length is solved for:

$$(t_{D_{el-ss3i}})^{1.5} = \frac{1}{430.5999} \left(\frac{h_y^{2.5} L_w^{0.85} h_z^{0.25} h_x^{0.2}}{r_w^3} \right) \quad (80)$$

$$h_y = \left[\frac{1}{542.3265} \left(\frac{1}{h_x^{0.2} L_w^{0.85} h_z^{0.25}} \right) \left(\frac{k_y t_{el-ss3i}}{\phi \mu c_t} \right)^{1.5} \right]^{0.4} \quad (81)$$

$$(t_{D_{Ell-ss3i}})^{1.36} = \frac{1}{187.0509} \left(\frac{k_x}{k_y} \right)^{0.14} \left(\frac{h_z^{0.25} h_y^{1.5} L_w^{0.57}}{h_x^{0.8}} \right) \frac{A}{r_w^{2.72}} \quad (82)$$

$$h_y = \left[\frac{1}{393.8496} \frac{k_y^{1.5}}{h_z^{0.25} L_w^{0.57} h_x^{0.2} k_x^{0.14}} \left(\frac{t_{Ell-ss3i}}{\phi \mu c_t} \right)^{1.36} \right]^{0.4} \quad (83)$$

$$t_{D_{pr-ss3i}} = \frac{1}{121.47} \left(\frac{h_y^{2.5} h_z^{0.25} h_x^{0.2}}{L_w^{0.15} r_w^2} \right) \sqrt{\frac{k_x}{k_y}} \quad (84)$$

$$h_y = \left[\frac{1}{31.2191} \left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{0.2}} \right) \frac{k_y^{1.5} t_{pr-ss3i}}{\phi \mu c_t k_x^{0.5}} \right]^{0.4} \quad (85)$$

$$(t_{D_{ll-ss3i}})^{1.5} = \frac{1}{430.5999} \left(\frac{h_z^{0.25} h_y^{2.5} h_x^{1.2}}{L_w^{0.15} r_w^3} \right) \quad (86)$$

$$h_y = \left[\frac{1}{542.3266} \left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{1.2}} \right) \left(\frac{k_y t_{ll-ss3i}}{\phi \mu c_t} \right)^{1.5} \right]^{0.4} \quad (87)$$

$$(t_{D_{hl-ss3i}})^{1.5} = \frac{1}{861.1999} \left(\frac{h_y^{2.5} h_z^{0.25} h_x^{1.2}}{L_w^{0.15} r_w^3} \right) \quad (88)$$

$$h_y = \left[\frac{1}{271.1633} \left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{1.2}} \right) \left(\frac{k_y t_{hl-ss3i}}{\phi \mu c_t} \right)^{1.5} \right]^{0.4} \quad (89)$$

Reservoir length is also found from the time of intersection formed by the fourth case of steady-state period with early linear flow -Equation (90) - with elliptical flow -Equation (92) - with pseudorradial flow -Equation (94) – and with late linear flow -Equation (96).

ARPN Journal of Engineering and Applied Sciences

$$(t_{D_{el-ss4i}})^{1.5} = \frac{1}{829.7282}\left(\frac{h_y^{2.5}L_w^{0.85}h_z^{0.25}h_x^{0.2}}{r_w^3}\right) \quad (90)$$

$$h_y = \left[\frac{1}{281.4485}\left(\frac{1}{h_x^{0.2}L_w^{0.85}h_z^{0.25}}\right)\left(\frac{k_y t_{el-ss4i}}{\phi\mu c_t}\right)^{1.5}\right]^{0.4} \quad (91)$$

$$(t_{D_{Ell-ss4i}})^{1.36} = \frac{1}{360.4306}\left(\frac{k_x}{k_y}\right)^{0.14}\left(\frac{h_z^{0.25}h_y^{1.5}L_w^{0.57}}{h_x^{0.8}}\right)\frac{A}{r_w^{2.72}} \quad (92)$$

$$h_y = \left[\frac{1}{204.3942}\frac{k_y^{1.5}}{h_z^{0.25}L_w^{0.57}h_x^{0.2}k_x^{0.14}}\left(\frac{t_{Ell-ss4i}}{\phi\mu c_t}\right)^{1.36}\right]^{0.4} \quad (93)$$

$$t_{D_{pr-ss4i}} = \frac{1}{234.062}\left(\frac{h_y^{2.5}h_z^{0.25}h_x^{0.2}}{L_w^{0.15}r_w^2}\right)\sqrt{\frac{k_x}{k_y}} \quad (94)$$

$$h_y = \left[\frac{1}{16.2016}\left(\frac{L_w^{0.15}}{h_z^{0.25}h_x^{0.2}}\right)\frac{k_y^{1.5}t_{pr-ss4i}}{\phi\mu c_t k_x^{0.5}}\right]^{0.4} \quad (95)$$

$$(t_{D_{ll-ss4i}})^{1.5} = \frac{1}{829.7282}\left(\frac{h_z^{0.25}h_y^{2.5}h_x^{1.2}}{L_w^{0.15}r_w^3}\right) \quad (96)$$

$$h_y = \left[\frac{1}{281.4485}\left(\frac{L_w^{0.15}}{h_z^{0.25}h_x^{1.2}}\right)\left(\frac{k_y t_{ll-ss4i}}{\phi\mu c_t}\right)^{1.5}\right]^{0.4} \quad (97)$$

Reservoir length is also found from the intersection time points of the fifth case of steady-state period with early linear flow -Equation (98)- with elliptical flow -Equation (100)- with pseudorradial flow -Equation (102)- and with late linear flow (104):

$$(t_{D_{el-ss5i}})^{1.5} = \frac{1}{7241.8895}\left(\frac{h_y^{2.5}L_w^{0.85}h_z^{0.25}h_x^{0.2}}{r_w^3}\right) \quad (98)$$

$$h_y = \left[\frac{1}{32.2465}\left(\frac{1}{h_x^{0.2}L_w^{0.85}h_z^{0.25}}\right)\left(\frac{k_y t_{el-ss5i}}{\phi\mu c_t}\right)^{1.5}\right]^{0.4} \quad (99)$$

$$(t_{D_{Ell-ss5i}})^{1.36} = \frac{1}{3145.8481}\left(\frac{k_x}{k_y}\right)^{0.14}\left(\frac{h_z^{0.25}h_y^{1.5}L_w^{0.57}}{h_x^{0.8}}\right)\frac{A}{r_w^{2.72}} \quad (100)$$

$$h_y = \left[\frac{1}{23.4181}\frac{k_y^{1.5}}{h_z^{0.25}L_w^{0.57}h_x^{0.2}k_x^{0.14}}\left(\frac{t_{Ell-ss5i}}{\phi\mu c_t}\right)^{1.36}\right]^{0.4} \quad (101)$$

$$t_{D_{pr-ss5i}} = \frac{1}{2042.8993}\left(\frac{h_y^{2.5}h_z^{0.25}h_x^{0.2}}{L_w^{0.15}r_w^2}\right)\sqrt{\frac{k_x}{k_y}} \quad (102)$$

$$h_y = \left[\frac{1}{1.8563}\left(\frac{L_w^{0.15}}{h_z^{0.25}h_x^{0.2}}\right)\frac{k_y^{1.5}t_{pr-ss5i}}{\phi\mu c_t k_x^{0.5}}\right]^{0.4} \quad (103)$$

$$(t_{D_{ll-ss5i}})^{1.5} = \frac{1}{7241.8895}\left(\frac{h_z^{0.25}h_y^{2.5}h_x^{1.2}}{L_w^{0.15}r_w^3}\right) \quad (104)$$

$$h_y = \left[\frac{1}{32.2465}\left(\frac{L_w^{0.15}}{h_z^{0.25}h_x^{1.2}}\right)\left(\frac{k_y t_{ll-ss5i}}{\phi\mu c_t}\right)^{1.5}\right]^{0.4} \quad (105)$$

The late pseudosteady-state period also intersects with such other flow regimes as early linear -Equation (106)- elliptical -Equation (108)- pseudorradial -Equation (110)- late linear -Equation (112)- hemilinear -Equation (114). All of these intercepts allow finding reservoir area:

$$(t_D)_{el-pssi}^{0.5} = \frac{A}{2\sqrt{\pi}r_w L_w} \quad (106)$$

$$A = \frac{L_w}{17.3716}\left(\frac{k_y t_{el-pssi}}{\phi\mu c_t}\right)^{0.5} \quad (107)$$

$$(t_{D_{Ell-pssi}})^{0.64} = \frac{1}{8.1605}\frac{A}{L_w^{0.72}r_w^{1.28}}\left(\frac{k_y}{k_x}\right)^{0.14} \quad (108)$$

$$A = \frac{1}{23.9206}L_w^{0.72}k_x^{0.14}k_y^{0.5}\left(\frac{t_{Ell-pssi}}{\phi\mu c_t}\right)^{0.64} \quad (109)$$

$$t_{D_{pr-pssi}} = \frac{1}{4\pi}\frac{A}{r_w^2}\sqrt{\frac{k_y}{k_x}} \quad (110)$$

VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

$$A=\frac{\sqrt{k_x k_y}\,t_{pr-pssi}}{301.7727\phi\mu c_t} \tag{111}$$

$$(t_{D_{ll-pssi}})^{0.5}=\left(\frac{A}{2\pi h_x r_w}\right)\sqrt{\pi} \tag{112}$$

$$A=\left(\frac{h_x}{17.3716}\right)\left(\frac{k_y t_{ll-pssi}}{\phi\mu c_t}\right)^{0.5} \tag{113}$$

$$(t_{D_{hl-pssi}})^{0.5}=\frac{A}{\sqrt{\pi}r_w h_x} \tag{114}$$

$$A=34.7432h_x\left(\frac{k_y t_{hl-pssi}}{\phi\mu c_t}\right)^{0.5} \tag{115}$$

Based upon the works of Engler and Tiab (1996b) and Martinez *et al.* (2012), Pabon and Cortes (2016) also developed all the equations for the interpretation of pressure tests in horizontal gas wells using the *TDS* Technique. These equations are reported in Appendix A.

3. SYNTHETIC EXAMPLE

The simulated test reported in Figure-4 was run for a homogeneous, isotropic, oil reservoir which geometry is shown in Figure-1 (A), with the information given below;

B = 1.2 bbl/STB	q = 600 STB/D
h_z = 150 ft	μ = 1.2 cp
L_w = 500 ft	c_t = 1x10^{-6} psi^{-1}
C = 0 STB/psi	ϕ = 15 %
k = 30 md	r_w = 0.4
h_x = 12000 ft	h_y = 320000 ft
b_y = 20000 ft	A = 3840000000 ft^2

Estimate permeability (k), horizontal well length (L_w), eccentricity well within the reservoir in y-axis (b_y), Reservoir width (h_x), reservoir length (h_y), Reservoir area (A) product using the *TDS* Technique.

Solution

The Characteristic points were read from Figure-4. They are reported in Tables 1 and -2.

Table-1. Characteristic points from Figure-4.

Time (hr)		**Pressure** (psi)		**Pressure derivative** (psi)	
t_{er}	0.02	$(\Delta P)_{er}$	38.2	$(t^*\Delta P')_{er}$	3.84
t_{Ell}	0.254	$(\Delta P)_{Ell}$	50.1	$(t^*\Delta P')_{Ell}$	6.8
t_{pr}	50.6	$(\Delta P)_{pr}$	111.6	$(t^*\Delta P')_{pr}$	13.5
t_{ll}	1009	$(\Delta P)_{ll}$	160.7	$(t^*\Delta P')_{ll}$	26.8
t_{PB}	127128	$(\Delta P)_{PB}$	348.8	$(t^*\Delta P')_{PB}$	20.7
t_{ss2}	2014849	$(\Delta P)_{ss2}$	386.9	$(t^*\Delta P')_{ss2}$	8.4

Table-2. Intersection points from Figure-4.

Time intersection (hr)	
ter-ss2i	4200000
tpr-ss2i	1133032.871
tll-ss2i	71489.541
tPB-ss2i	4510685.103

Use of Equations (6), (17), (11) and (14) allow finding and verifying reservoir permeability:

$$\sqrt{k_y k_z}=\frac{70.6(600)(1.2)(1.2)}{(500)(3.8)}=32.1\text{ md}$$

$$k_y=\left(5.5962\frac{(600)(1.2)^{0.64}(1.2)}{(500)^{0.72}(30)^{0.14}(150)(6.8)}\left[\frac{0.254}{(0.15)(10^{-6})}\right]^{0.36}\right)^2$$

$$k_y=30.14\text{ ft}$$

$$\sqrt{k_x k_y}=\frac{70.6(600)(1.2)(1.2)}{(150)(13.5)}=30.12\text{ md}$$

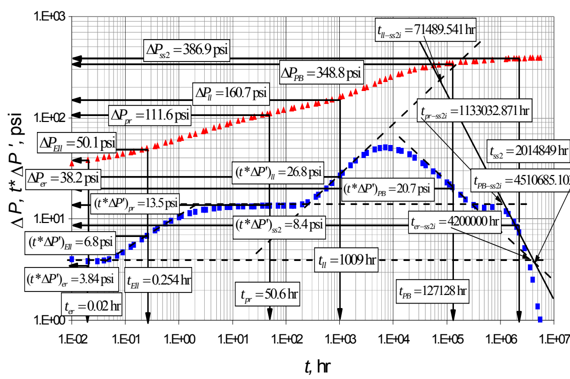

Figure-4. Pressure and pressure derivative versus time log-log plot for the synthetic example.

$$k_y=\left(\frac{4.064(600)(1.2)}{(150)(26.8)}\right)^2\frac{(1.2)(1009)}{(12000)^2(0.15)(10^{-6})}=29.7\text{ md}$$

Determine horizontal well length with Equation (16):

$$L_w = \left[5.5962 \frac{(600)(1.2)^{0.64}(1.2)}{(30)^{0.5}(30)^{0.14}(150)(6.8)} \left(\frac{0.254}{(0.15)(10^{-6})} \right)^{0.36} \right]^{1/0.72}$$

$$L_w = 501.7 \text{ ft}$$

Equation (13) is use to find the reservoir width (h_x):

$$h_x = \frac{4.064(600)(1.2)}{(150)(26.8)} \sqrt{\frac{(1.2)(1009)}{(30)(0.15)(10^{-6})}} = 11939 \text{ ft}$$

Equation (77) is use to find the reservoir length (h_y):

$$h_y = \left[\frac{42480690310(150)^{0.28}(20000)^{3.07}(12000)^2}{(500)^{0.83}} \left(\frac{(0.15)(1.2)(10^{-6})}{(30)(71489.541)} \right)^{1.5} \right]^{1/1.95}$$

$$h_y = 324067.699 \text{ ft}$$

Equation (26) is use to find the (b_y) :

$$b_y = \left[\frac{(30)^{1.5}(500)^{0.38}(150)^{0.62}(320000)^{1.05}(0.4)^{0.25}(20.7)(127128)^{0.5}}{48403.5189(600)(1.2)^{1.5}(1.2)(0.15)^{0.5}(10^{-6})^{0.5}} \right]^{1/2.3}$$

$$b_y = 19961 \text{ ft}$$

Use of Equations (31), (51) and (75) allow finding and verifying reservoir area:

$$A = \frac{(30)^2(500)^{0.83}(150)^{0.72}(320000)^{2.95}(8.4)(2014849)}{1.726458976 \times 10^{11}(1.2)^2(20000)^{3.07}(600)(1.2)(0.15)(10^{-6})}$$

$$A = 3951690732 \text{ ft}^2$$

$$A = \frac{1}{2445409329} \frac{(150)^{0.72}(320000)^{2.95}}{(500)^{0.17}(20000)^{3.07}} \frac{(30)^{1.5}(4200000)}{(0.15)(1.2)(10^{-6})(30)^{0.5}}$$

$$A = 3987839130 \text{ ft}^2$$

$$A = \frac{1}{2445409315} \frac{(320000)^{2.95}(500)^{0.83}}{(20000)^{3.07}(150)^{0.28}} \frac{(30)^{1.5}(1133032.871)}{(0.15)(1.2)(10^{-6})(30)^{0.5}}$$

$$A = 3585994320 \text{ ft}^2$$

4. CONCLUSIONS

a) *TDS* technique was extended to characterize rectangular homogeneous anisotropic reservoirs for horizontal oil and gas wells. New reliable equations were developed to calculate such well and reservoir parameter as permeability in *x*-, *y*- and *z*- directions, horizontal wellbore length, skin factor, reservoir area, etc. When the wellbore is off-centered along the reservoir length some new flow regimes can show up, such as: hemilinear flow, parabolic flow, five cases of steady state and pseudosteady state. The expressions for such flow regimes were developed to allow solving for reservoir parameters which were successfully tested with simulated examples (although, only one is presented for space-saving reasons) providing acceptable results compared to the input-simulated values.

b) New expressions are introduced to estimate and verify reservoir area, reservoir length, among others, using time intersection points.

ACKNOWLEDGEMENTS

The authors thank Universidad Surcolombiana for supporting the development of this research.

Nomenclature

A	Reservoir area, ft^2
B	Volume factor, rb/STB
b_y	Eccentricity well within the reservoir in y-axis, ft
C	Wellbore Storage
c_t	Total system compressibility, psi^{-1}
h_x	Reservoir width, ft
h_y	Reservoir length, ft
h_z	Reservoir thickness, ft
k	Reservoir horizontal permeability, md
L_w	Horizontal well length, ft
P	Pressure, psi
P_i	Initial reservoir pressure, psi
q	Flow rate, BPD
r_w	Wellbore radius , ft
s	Skin factor
s_m	Mechanical skin factor
s_{Ell}	Elliptical pseudoskin factor
s_{hl}	Hemilinear pseudoskin factor
s_{PB}	Parabolic pseudoskin factor
s_x	x-direction pseudoskin factor
s_z	z-direction pseudoskin factor
T	Reservoir temperature, °R
t	Time, hr
$t_a(P)$	Pseudotime function, (hr)(psi)/cp
t_D	Dimensionless time coordinate
$t_{Da}(P)$	Dimensionless pseudotime function
t_D*P_D'	Dimensionless pressure derivative
$(t*\Delta P')$	Pressure derivative, psi
$t*\Delta m(P)'$	Pseudopressure derivative function, psi2/cp
$t_D*m(P)_D'$	Dimensionless pseudopressure derivative function

Greeks

Δ	Change, drop
ϕ	Porosity, fraction
μ	Viscosity, cp

Suffices

D	Dimensionless
el	Early linear flow period
el-pssi	Intercept of pseudosteady-state and early linear flow
el-ss1i	Intercept of first case of steady state and early linear flow
el-ss2i	Intercept of second case of steady state and early linear flow
e-ss3i	Intercept of third case of steady state and early linear flow
el-ss4i	Intercept of fourth case of steady state and early linear flow
el-ss5i	Intercept of fifth case of steady state and early linear flow
Ell	Elliptical flow period
Ell-pssi	Intercept of pseudosteady-state and elliptical flow
Ell-ss1i	Intercept of first case of steady state and elliptical flow
Ell-ss2i	Intercept of second case of steady state and elliptical flow
Ell-ss3i	Intercept of third case of steady state and elliptical flow
Ell-ss4i	Intercept of fourth case of steady state and elliptical flow
Ell-ss5i	Intercept of fifth case of steady state and elliptical flow
er	Early radial flow period
er-eli	Intercept of early radial and early linear flow
er-Elli	Intercept of early radial and elliptical flow
er-hli	Intercept of early radial and hemilinear flow
er-lli	Intercept of early radial and late linear flow
er-PBi	Intercept of early radial and parabolic flow
er-pri	Intercept of early radial and pseudorradial flow
er-pssi	Intercept of early radial and pseudosteady-state
er-ss1i	Intercept of early radial and first case of steady state
er-ss2i	Intercept of early radial and second case of steady state
er-ss3i	Intercept of early radial and third case of steady state
er-ss4i	Intercept of early radial and fourth case of steady state
er-ss5i	Intercept of early radial and fifth case of steady state
g	Gas
hl	Hemilinear flow period
hl-pssi	Intercept of pseudosteady-state and hemilinear flow
hl-ss3i	Intercept of third case of steady state and hemilinear flow
i	Intersection
ll	Late linear flow period
ll-pssi	Intercept of pseudosteady-state and late linear flow
ll-ss1i	Intercept of first case of steady state and late linear flow
ll-ss2i	Intercept of second case of steady state and late linear flow
ll-ss3i	Intercept of third case of steady state and late linear flow
ll-ss4i	Intercept of fourth case of steady state and late linear flow
ll-ss5i	Intercept of fifth case of steady state and late linear flow
PB	Parabolic flow period
PB-ss1i	Intercept of first case of steady state and parabolic flow
PB-ss2i	Intercept of second case of steady state and parabolic flow
pr	Pseudorradial flow period
pr-pssi	Intercept of pseudosteady-state and pseudorradial flow
pr-ss1i	Intercept of first case of steady state and pseudorradial flow
pr-ss2i	Intercept of second case of steady state and pseudorradial flow
pr-ss3i	Intercept of third case of steady state and pseudorradial flow
pr-ss4i	Intercept of fourth case of steady state and pseudorradial flow
pr-ss5i	Intercept of fifth case of steady state and pseudorradial flow
pr-pssi	Intercept of pseudosteady-state and pseudorradial flow
pss	Pseudosteady state
ss	Steady state
ss1	First case of steady state
ss2	Second case of steady state
ss3	Third case of steady state
ss4	Fourth case of steady state
ss5	Fifth case of steady state
t	Total
w	Well
x	x-direction index
y	y-direction index
z	z-direction index

REFERENCES

Agarwal R. G. 1979, January 1. Real Gas Pseudo-Time - A New Function for Pressure Buildup Analysis of MHF Gas Wells. Society of Petroleum Engineers. doi:10.2118/8279-MS.

Al-Hussainy R., Ramey H.J. and Crawford P.B. 1966. The flow of real gases through porous media. Journal of Petroleum Technology. May. pp. 624-636, Trans. AIME, 237. Society of Petroleum Engineers. doi:10.2118/1243-A-PA.

Bernal K.M., Escobar F.H. and Ghisays-Ruiz A.G. 2014. Pressure and Pressure Derivative Analysis for Hydraulically-Fractured Shale Formations Using the

Concept of Induced Permeability Field. Journal of Engineering and Applied Sciences. ISSN 1819-6608. 9(10): 1952-1958.

Chacon A., Djebrouni A. and Tiab D. 2004. Determining the Average Reservoir Pressure from Vertical and Horizontal Well Test Analysis Using Tiab's Direct Synthesis Technique. Society of Petroleum Engineers. doi:10.2118/88619-MS.

Clonts M.D. and Ramey Jr., H.J. 1986. Pressure transient analysis for wells with horizontal drainholes. Society of Petroleum Engineers. doi:10.2118/15116-MS.

Cortes N.M and Pabon O.D. 2016. Aplicación de la metología *TDS* (Tiab's Direct Synthesis) en pruebas de presión y pruebas de caudal para pozos horizontales en yacimientos homogéneos anisotrópicos de hidrocarburos. BSc. Thesis. Universidad Surcolombiana.

Daviau F., Mouronval G., Bourdarot G. and Curutchet P. 1988, December 1. Pressure Analysis for Horizontal Wells. Society of Petroleum Engineers. doi:10.2118/14251-PA.

Goode P. A. and Thambynayagam R. K. 1987. Pressure Drawdown and Buildup Analysis of Horizontal Wells in Anisotropic Media. SPE Formation Evaluation. pp. 683-697. doi:10.2118/14250-PA.

Engler T.W. and Tiab D. 1996a. Analysis of Pressure and Pressure Derivate Without Type-Curve Matching. 6. Horizontal Well Tests in Anisotropic Reservoirs. Journal of Petroleum Science and Engineering. 15: 153-168.

Engler T. W. and Tiab D. 1996b. Analysis of Pressure and Pressure Derivatives without Type-Curve Matching. 5- Horizontal Well Tests in Naturally Fractured Reservoirs. Journal of Petroleum Science and Engineering. 15: 139-151.

Escobar F.H., Muñoz O.F. and Sepulveda J.A. 2004. Horizontal Permeability Determination from the Elliptical Flow Regime for Horizontal Wells. CT&F – Ciencia, Tecnología and Futuro. 2(5): 83-95.

Escobar F. H. and Montealegre M. 2007, January 1. Conventional Analysis for the Determination of the Horizontal Permeability from the Elliptical Flow of Horizontal Wells. Society of Petroleum Engineers. doi:10.2118/105928-MS.

Escobar F.H., Hernandez Y.A. and Hernandez C.M. 2007. Pressure Transient Analysis for Long Homogeneous Reservoirs Using *TDS* Technique. Journal of Petroleum Science and Engineering. 58: 68-82.

Escobar F.H., Hernandez Y.A. and Tiab D. 2010. Determination of Reservoir Drainage Area for Constant Pressure Systems Using Well Test Data. CT&F - Ciencia, Tecnología y Futuro. 4(1): 51-62.

Escobar F.H., Montealegre-M. M. and Cantillo J.H. 2006. Conventional Analysis for Characterization of Bi-Radial (Elliptical) Flow in Infinite-Conductivity Vertical Fractured Wells. CT&F - Ciencia, Tecnología and Futuro. 3(2): 141-147. ISSN 0122-5383. Dic. 2006.

Escobar F.H., Bernal K.M. and Olaya-Marin G. 2014. Pressure and Pressure Derivative Analysis for Fractured Horizontal Wells in Unconventional Shale Reservoirs Using Dual-Porosity Models in the Stimulated Reservoir Volume. Journal of Engineering and Applied Sciences. ISSN 1819-6608, 9(12): 2650-2669.

Escobar F.H., Zhao Y.L. and Zhang L.H. 2014. Interpretation of Pressure Tests in Horizontal Wells in Homogeneous and Heterogeneous Reservoirs with Threshold Pressure Gradient. Journal of Engineering and Applied Sciences. 9(11): 2220-2228.

Issaka M. B., Zaoral K., Ambastha A. K. and Mattar L. 2000. Determination of Horizontal Permeability Anisotropy from Horizontal Well Tests. SPE Saudi Arabia Section Technical Symposium, Dhahran, Saudi Arabia, 21-23 October.

Martinez J., Escobar F.H. and Bonilla L.F. 2012. Reformulation of the Elliptic Flow Governing Equation for a More Complete Well Test Data Interpretation In Horizontal Wells, Journal of Engineering and Applied Sciences. 7(3).

Tiab D. 1993a. Analysis of Pressure and Pressure Derivative without Type-Curve Matching: 1- Skin and Wellbore Storage. Journal of Petroleum Science and Engineering. 12: 171-181.

Tiab D. 1993b, January 1. Analysis of Pressure and Pressure Derivative without Type-Curve Matching - III. Vertically Fractured Wells in Closed Systems. Society of Petroleum Engineers. doi:10.2118/26138-MS.

Appendix A. Gas reservoir equations

Dimensionless pressure derivative:

$$[t_D * m(P)_D\,']_{er} = \frac{1}{2}\sqrt{\frac{k_y}{k_z}} \tag{A.1}$$

$$[t_D * m(P)_D\,']_{el} = \frac{r_w}{h_z}\sqrt{\pi t_{Da_{el}}} \tag{A.2}$$

ARPN Journal of Engineering and Applied Sciences

$$[t_D * m(P)_D\,']_{Ell} = 0.76994694 \frac{r_w^{0.72} L_w^{0.28}}{h_z} \left(\frac{k_y}{k_x}\right)^{0.14} (t_{Da_{Ell}})^{0.36} \quad (A.3)$$

$$[t_D * m(P)_D\,']_{pr} = \frac{1}{2}\frac{L_w}{h_z}\sqrt{\frac{k_y}{k_x}} \quad (A.4)$$

$$[t_D * m(P)_D\,']_{ll} = \left(\frac{L_w r_w}{h_x h_z}\right)\sqrt{\pi t_{Da_{ll}}} \quad (A.5)$$

$$[t_D * m(P)_D\,']_{hl} = \frac{2L_w}{h_z}\frac{r_w}{h_x}\sqrt{\pi t_{Da_{hl}}} \quad (A.6)$$

$$[t_D * m(P)_D\,']_{PB} = 5.5663 \frac{L_w^{0.62} b_y^{2.3}}{h_z^{0.62} h_y^{1.05} r_w^{1.25}} (t_{Da_{PB}})^{-0.5} \quad (A.7)$$

$$[t_D * m(P)_D\,']_{ss1} = 361699.2 \frac{L_w^{0.17}}{h_z^{0.75}}\left(\frac{b_y}{h_y}\right)^3 t_{DaA_{ss1}}^{-1} \quad (A.8)$$

$$[t_D * m(P)_D\,']_{ss2} = 322427.2182 \frac{L_w^{0.17} b_y^{3.07}}{h_z^{0.72} h_y^{2.95}} t_{DaA_{ss2}}^{-1} \quad (A.9)$$

$$[t_D * m(P)_D\,']_{ss3} = \frac{1}{242.94}\left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}}\right) t_{DaA_{ss3}}^{-1} \quad (A.10)$$

$$[t_D * m(P)_D\,']_{ss4} = \frac{1}{468.124}\left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}}\right)(t_{DaA})_{ss4}^{-1} \quad (A.11)$$

$$[t_D * m(P)_D\,']_{ss5} = \frac{1}{4085.7986}\left(\frac{h_y^{1.5} L_w^{0.85}}{h_x^{0.8} h_z^{0.75}}\right)(t_{DaA})_{ss5}^{-1} \quad (A.12)$$

$$[t_D * m(P)_D\,']_{pss} = 2\pi\left(\frac{L_w}{h_z}\right)(t_{DaA})_{pss} \quad (A.13)$$

TDS tecnique:

$$\sqrt{k_z k_y} = \frac{711.26 qT}{L_w [t * \Delta m(P)']_{er1}} \quad (A.14)$$

$$k_y = \left(40.94 \frac{qT}{L_w h_z [t * \Delta m(P)']_{el}}\right)^2 \left(\frac{t_a(P)_{el}}{\phi}\right) \quad (A.15)$$

$$L_w = \left[56.3792 \frac{qT}{k_y^{0.5} k_x^{0.14} h_z [t * \Delta m(P)']_{Ell}} \left(\frac{t_a(P)_{Ell}}{\phi}\right)^{0.36}\right]^{1/0.72} \quad (A.16)$$

$$\sqrt{k_x k_y} = 711.26 \frac{qT}{h_z [t * \Delta m(P)']_{pr1}} \quad (A.17)$$

$$h_x = 40.94 \frac{qT}{h_z [t * \Delta m(P)']_{ll}} \sqrt{\frac{t_a(P)_{ll}}{k_y \phi}} \quad (A.18)$$

$$h_x = 81.88 \frac{qT}{h_z [t * \Delta m(P)']_{hl}} \sqrt{\left(\frac{t_a(P)_{hl}}{k_y \phi}\right)} \quad (A.19)$$

$$b_y = \left[\frac{\frac{1}{487606.9931}\frac{k_y^{1.5} L_w^{0.38} h_z^{0.62} h_y^{1.05} r_w^{0.25}}{qT}}{\sqrt{\frac{t_a(P)_{PB}}{\phi}}[t * \Delta m(P)']_{PB}}\right]^{1/2.3} \quad (A.20)$$

$$A = \frac{k_y^2 L_w^{0.83} h_z^{0.75} [t * \Delta m(P)']_{ss1}}{1.9511731 \times 10^{12}} \left(\frac{h_y}{b_y}\right)^3 \left(\frac{t_a(P)_{ss1}}{qT\phi}\right) \quad (A.21)$$

$$A = \frac{k_y^2 L_w^{0.83} h_z^{0.72} h_y^{2.95} [t * \Delta m(P)']_{ss2}}{1.739322 \times 10^{12} qT b_y^{3.07}} \left(\frac{t_a(P)_{ss2}}{\phi}\right) \quad (A.22)$$

$$h_y = \left[\frac{k_y^2 L_w^{0.15} h_z^{0.75} [t * \Delta m(P)']_{ss3}}{22204.9206 h_x^{0.2}} \left(\frac{t_a(P)_{ss3}}{qT\phi}\right)\right]^{0.4} \quad (A.23)$$

$$h_y = \left[\frac{k_y^2 L_w^{0.15} h_z^{0.75} [t * \Delta m(P)']_{ss4}}{11523.578 h_x^{0.2}} \left(\frac{t_a(P)_{ss4}}{qT\phi}\right)\right]^{0.4} \quad (A.24)$$

$$h_y = \left[\frac{k_y^2 L_w^{0.15} h_z^{0.75} [t * \Delta m(P)']_{ss5}}{1320.2959 h_x^{0.2}} \left(\frac{t_a(P)_{ss5}}{qT\phi}\right)\right]^{0.4} \quad (A.25)$$

$$A = 2.35694 \left(\frac{t_a(P)_{pss}}{h_z \phi}\right) \frac{qT}{[t * \Delta m(P)']_{pss}} \quad (A.26)$$

A.1. Intersections

VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$$k_z = 301.7727\phi \frac{h_z^2}{t_a(P)_{er-eli}} \quad (A.27)$$

$$k_z = \left[12.6156 \frac{h_z k_x^{0.14}}{L_w^{0.28}} \left(\frac{\phi}{t_a(P)_{er-Elli}}\right)^{0.36}\right]^2 \quad (A.28)$$

$$k_z = 301.7727 \left(\frac{h_x h_z}{L_w}\right)^2 \frac{\phi}{t_a(P)_{er-lli}} \quad (A.29)$$

$$k_z = 75.4432 \left(\frac{h_z h_x}{L_w}\right)^2 \left(\frac{\phi}{t_a(P)_{ier-hli}}\right) \quad (A.30)$$

$$b_y = \left[\frac{h_z^{0.62} h_y^{1.05} r_w^{0.25}}{685.5538 L_w^{0.62}} \sqrt{\frac{k_y}{k_z}} \left(\frac{k_y t_a(P)_{er-PBi}}{\phi}\right)^{0.5}\right]^{1/2.3} \quad (A.31)$$

$$A = \frac{1}{2743262799} \frac{h_z^{0.75} k_y^{1.5}}{L_w^{0.17} k_z^{0.5}} \left(\frac{h_y}{b_y}\right)^3 \frac{t_a(P)_{er-ss1i}}{\phi} \quad (A.32)$$

$$A = \frac{1}{2445409329} \frac{h_z^{0.72} h_y^{2.95}}{L_w^{0.17} b_y^{3.07}} \frac{k_y^{1.5} t_a(P)_{er-ss2i}}{\phi k_z^{0.5}} \quad (A.33)$$

$$h_y = \left[\frac{1}{31.2191} \left(\frac{h_z^{0.75}}{h_x^{0.2} L_w^{0.85}}\right) \frac{k_y^{1.5} t_a(P)_{er-ss3i}}{\phi k_z^{0.5}}\right]^{0.4} \quad (A.34)$$

$$h_y = \left[\frac{1}{16.2016} \left(\frac{h_z^{0.75}}{h_x^{0.2} L_w^{0.85}}\right) \frac{k_y^{1.5} t_a(P)_{er-ss4i}}{\phi k_z^{0.5}}\right]^{0.4} \quad (A.35)$$

$$h_y = \left[\frac{1}{1.8563} \left(\frac{h_z^{0.75}}{h_x^{0.2} L_w^{0.85}}\right) \frac{k_y^{1.5} t_a(P)_{er-ss5i}}{\phi k_z^{0.5}}\right]^{0.4} \quad (A.36)$$

$$h_y = \frac{1}{301.7727} \left(\frac{L_w}{h_z}\right) \frac{k_y^{0.5} k_z^{0.5} t_a(P)_{er-pssi}}{\phi} \quad (A.37)$$

$$A = \frac{1}{47654884040} \frac{h_z^{0.25}}{L_w^{0.17}} \left(\frac{h_y}{b_y}\right)^3 \left(\frac{k_y t_a(P)_{el-ss1i}}{\phi}\right)^{1.5} \quad (A.38)$$

$$A = \frac{1}{34608037630} \left(\frac{h_y}{b_y}\right)^3 \frac{L_w^{0.11} k_y^{1.5}}{h_z^{0.25} k_x^{0.14}} \left(\frac{t_a(P)_{Ell-ss1i}}{\phi}\right)^{1.36} \quad (A.39)$$

$$A = \frac{1}{2743262799} \frac{L_w^{0.83}}{h_z^{0.25}} \left(\frac{h_y}{b_y}\right)^3 \frac{k_y^{1.5} t_a(P)_{pr-ss1i}}{\phi k_x^{0.5}} \quad (A.40)$$

$$h_y = \left[47654884040 \frac{h_z^{0.25} b_y^3 h_x^2}{L_w^{0.83}} \left(\frac{\phi}{k_y t_a(P)_{ll-ss1i}}\right)^{1.5}\right]^{0.5} \quad (A.41)$$

$$A = \frac{1}{4001527.862} \frac{h_z^{0.13} L_w^{0.45} h_y^{1.95}}{b_y^{0.7} r_w^{0.25}} \left(\frac{k_y t_a(P)_{PB-ss1i}}{\phi}\right)^{0.5} \quad (A.42)$$

$$A = \frac{1}{42480690310} \frac{h_y^{2.95}}{h_z^{0.28} L_w^{0.17} b_y^{3.07}} \left(\frac{k_y t_a(P)_{el-ss2i}}{\phi}\right)^{1.5} \quad (A.43)$$

$$A = \frac{1}{30850422960} \frac{L_w^{0.11} h_y^{2.95} k_y^{1.5}}{h_z^{0.28} b_y^{3.07} k_x^{0.14}} \left(\frac{t_a(P)_{Ell-ss2i}}{\phi}\right)^{1.36} \quad (A.44)$$

$$A = \frac{1}{2445409315} \frac{h_y^{2.95} L_w^{0.83}}{b_y^{3.07} h_z^{0.28}} \frac{k_y^{1.5} t_a(P)_{pr-ss2i}}{\phi k_x^{0.5}} \quad (A.45)$$

$$h_y = \left[42480690310 \frac{h_z^{0.28} b_y^{3.07} h_x^2}{L_w^{0.83}} \left(\frac{\phi}{k_y t_a(P)_{ll-ss2i}}\right)^{1.5}\right]^{1/1.95} \quad (A.46)$$

$$A = \frac{1}{3567056.779} \frac{h_z^{0.1} h_y^{1.9} L_w^{0.45}}{b_y^{0.77} r_w^{0.25}} \left(\frac{k_y t_a(P)_{PB-ss2i}}{\phi}\right)^{0.5} \quad (A.47)$$

$$h_y = \left[\frac{1}{542.3265} \left(\frac{1}{h_x^{0.2} L_w^{0.85} h_z^{0.25}}\right) \left(\frac{k_y t_a(P)_{el-ss3i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.48)$$

$$h_y = \left[\frac{1}{393.8496} \frac{k_y^{1.5}}{h_z^{0.25} L_w^{0.57} h_x^{0.2} k_x^{0.14}} \left(\frac{t_a(P)_{Ell-ss3i}}{\phi}\right)^{1.36}\right]^{0.4} \quad (A.49)$$

$$h_y = \left[\frac{1}{31.2191} \left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{0.2}}\right) \frac{k_y^{1.5} t_a(P)_{pr-ss3i}}{\phi k_x^{0.5}}\right]^{0.4} \quad (A.50)$$

$$h_y = \left[\frac{1}{542.3266} \left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{1.2}}\right) \left(\frac{k_y t_a(P)_{ll-ss3i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.51)$$

$$h_y = \left[\frac{1}{271.1633} \left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{1.2}}\right) \left(\frac{k_y t_a(P)_{hl-ss3i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.52)$$

$$h_y = \left[\frac{1}{281.4485}\left(\frac{1}{h_x^{0.2} L_w^{0.85} h_z^{0.25}}\right)\left(\frac{k_y t_a(P)_{el-ss4i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.53)$$

$$h_y = \left[\frac{1}{204.3942}\frac{k_y^{1.5}}{h_z^{0.25} L_w^{0.57} h_x^{0.2} k_x^{0.14}}\left(\frac{t_a(P)_{Ell-ss4i}}{\phi}\right)^{1.36}\right]^{0.4} \quad (A.54)$$

$$h_y = \left[\frac{1}{16.2016}\left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{0.2}}\right)\frac{k_y^{1.5} t_a(P)_{pr-ss4i}}{\phi k_x^{0.5}}\right]^{0.4} \quad (A.55)$$

$$h_y = \left[\frac{1}{281.4485}\left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{1.2}}\right)\left(\frac{k_y t_a(P)_{ll-ss4i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.56)$$

$$h_y = \left[\frac{1}{32.2465}\left(\frac{1}{h_x^{0.2} L_w^{0.85} h_z^{0.25}}\right)\left(\frac{k_y t_a(P)_{el-ss5i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.57)$$

$$h_y = \left[\frac{1}{23.4181}\frac{k_y^{1.5}}{h_z^{0.25} L_w^{0.57} h_x^{0.2} k_x^{0.14}}\left(\frac{t_a(P)_{Ell-ss5i}}{\phi}\right)^{1.36}\right]^{0.4} \quad (A.58)$$

$$h_y = \left[\frac{1}{1.8563}\left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{0.2}}\right)\frac{k_y^{1.5} t_a(P)_{pr-ss5i}}{\phi k_x^{0.5}}\right]^{0.4} \quad (A.59)$$

$$h_y = \left[\frac{1}{32.2465}\left(\frac{L_w^{0.15}}{h_z^{0.25} h_x^{1.2}}\right)\left(\frac{k_y t_a(P)_{ll-ss5i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.60)$$

$$A = \frac{L_w}{17.3716}\left(\frac{k_y t_a(P)_{el-pssi}}{\phi}\right)^{0.5} \quad (A.61)$$

$$A = \frac{1}{23.9206} L_w^{0.72} k_x^{0.14} k_y^{0.5}\left(\frac{t_a(P)_{Ell-pssi}}{\phi}\right)^{0.64} \quad (A.62)$$

$$A = \frac{\sqrt{k_x k_y}\, t_a(P)_{pr-pssi}}{301.7727\phi} \quad (A.63)$$

$$A = \left(\frac{h_x}{17.3716}\right)\left(\frac{k_y t_a(P)_{ll-pssi}}{\phi}\right)^{0.5} \quad (A.64)$$

$$A = 34.7432 h_x\left(\frac{k_y t_a(P)_{hl-pssi}}{\phi}\right)^{0.5} \quad (A.65)$$

A.2. Skin factors

$$(s_z + s_m') = \frac{1}{34.74321 h_z}\left[\frac{\Delta m(P)_{el}}{[t*\Delta m(P)']_{el}} - 2\right]\sqrt{\left(\frac{k_z t_a(P)_{el}}{\phi}\right)} \quad (A.66)$$

$$s_{Ell} = \left[\frac{1}{25.2313}\frac{L_w^{0.28} k_y^{0.5}}{h_z k_x^{0.14}}\left(\frac{t_a(P)_{Ell}}{\phi}\right)^{0.36}\right]\left[\frac{\Delta m(P)_{Ell}}{[t*\Delta m(P)']_{Ell}} - 2.778\right] \quad (A.67)$$

$$(s_z + s_m') = \frac{L_w}{2h_z}\sqrt{\frac{k_z}{k_y}}\left[\frac{\Delta m(P)_{pr}}{[t*\Delta m(P)']_{pr}} - \left[\ln\left(\frac{k_x}{L_w^2 \phi} t_a(P)_{pr}\right) + 4.659\right]\right] \quad (A.68)$$

$$(s_x + s_z + s_m') = \left[\frac{\Delta m(P)_{ll}}{[t*\Delta m(P)']_{ll}} - 2\right]\left[\left(\frac{L_w}{h_x h_z}\right)\sqrt{\left(\frac{k_z}{1207.1\phi}\right) t_a(P)_{ll}}\right] \quad (A.69)$$

$$(s_x + s_z + s_m' + s_{hl}) = \left[\frac{L_w}{17.37 h_z h_x}\sqrt{\frac{k_z t_a(P)_{hl}}{\phi}}\right]\left[\frac{\Delta m(P)_{hl}}{[t*\Delta m(P)']_{hl}} - 2\right] \quad (A.70)$$

$$(s_x + s_z + s_m' + s_{PB}) = \left[\frac{5.5663 L_w^{0.62} b_y^{2.3}}{h_z^{0.62} h_y^{1.05} r_w^{1.25}}\sqrt{\frac{k_z}{k_y}}(t_{Da_{PB}})^{-0.5}\right] \quad (A.71)$$

$$\left[\frac{m(P)_D}{[t_D * m(P)_D']} + 2\right]$$