VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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EXPERT FUZZY MODELING OF DYNAMIC PROPERTIES OF COMPLEX SYSTEMS

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ABSTRACT

The purpose of this article is to represent an extension of the classical notion of fuzzy sets to estimate the condition of complex systems, which is based on the dynamic fuzzy sets concept. In this article, the authors prove the necessity to use dynamic fuzzy sets to estimate the condition of a complex system. Firstly, we present the basic definition and construction method of dynamic fuzzy sets. There are described analytical and graphical representations of dynamic fuzzy sets, which are represented by dynamic membership functions constructed in a three-dimensional coordinate system, where one of the axes in the graph captures the time variation of the properties of the economic objects expressed by a set of numbers with fuzzy function. We develop the simulation of dynamic membership functions by expressions of expert assessments on multilevel fuzzy description of the complex indicators, which are made in strict sequence from a reference point and bifurcation points, selection of functional dependencies, to build a static membership functions, and then the dynamic membership functions on the whole interval assessment. Finally, we develop a few software systems to interactive construction dynamic membership function. Thus, this paper can provide interactive process of expert assessment of complex dynamic systems, which include quantitative and qualitative indicators.

Keywords: decision making, dynamic fuzzy sets, membership function, defining points, point of bifurcation, dynamic fuzzy modeling.

1. INTRODUCTION

According to the decision theory, description of properties of complex systems can be represented as a set of fuzzy numbers with the membership function.

In the theory of Lotfi A. Zadeh (Zadeh, 1965; Zadeh, 1973), a fuzzy set is represented by a static set:

$$\left\langle U\ ,\ M\ ,\ \mu_{\tilde{A}} \right\rangle,\ x \in X\ ,$$

where U - universe of discourse, $\mu_{\tilde{A}}(x): X \to \mathrm{M}$, and $\mu_{\tilde{A}}(x)$ is the grade of X membership of x in A, M - number in the interval [0, 1].

But when we need to estimate dynamic complex system, we also use membership function without revaluation in time.

The purpose of this article is to represent an extension of the classical notion of fuzzy sets to estimate the condition of complex systems, which is based on the dynamic fuzzy sets concept.

Many scholars studied fuzzy sets theory and applied it to static decision making. Since the early 2000s, there have attempts in the scientific works to develop dynamic decision making models (Fanzhang, 2008; Xu, 2008; Zhang, 2012).

This paper provides a mathematical apparatus of dynamic fuzzy sets based on the account of the time factor in the construction of membership functions.

2. BASIC DEFINITIONS

We suppose that the problem of processing fuzzy information about the object, described by qualitative and quantitative parameters, can solved by using dynamic fuzzy sets.

We established two groups of reasons that affect the changes of membership functions. Objective reason. Objective reasons relate primarily to the variability of the studied objects or phenomena, as well as the volatility of the environment in which they are treated. For example, expert assessment of speed of information processing changes in response to scientific and technical progress; demands of consumers changed due to the introduction of new products and services, etc.

The plots of the membership functions of the fuzzy set "High speed information processing" throughout the development of scientific and technical progress shown in Figure-1.

Expert assessment of speed of information processing has changed with the development of scientific and technological progress, due to the emergence of new equipment. During the production of computers on electronic lamps, execution of tens of thousands of operations per second was considered the highest performance, as evidenced by the value of the membership function. After 20 years of production and application of second generation machines, the degree of belonging to fuzzy set "High processing speed" at the value x = 10^4 to become zero. On the segment between points A and B (graph in the center, Figure-1), the experts have expressed vague opinion: increasing productivity along the axis, the

values of the membership function gradually increases", and with values x = 10^8 and above is equal to unity.

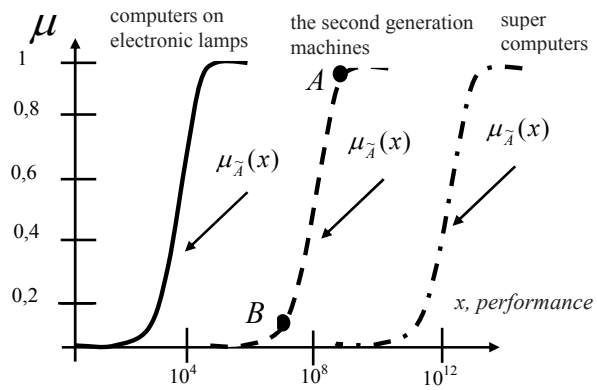

Figure-1. Membership functions of the fuzzy set "High speed information processing".

The second group of reasons that affect the changes of membership functions is the subjective reasons, which are caused by changes in the internal judgments of experts on the expiration time, the peculiarities of the perception of the analyzed objects, and the environment in which they are located. For example, expert assessment may be different if the definition of concepts such as "paid work", "prestige area", "good stay", etc. (Tereliansky and Kostikova, 2012).

Let X be a space of objects, with generic elements of X denoted by x. Thus, $X = \{x\}$. Let T be a set of time segments. Thus $\grave{O} = \{t_n\}$

Dynamic fuzzy set $\tilde{A}_t$ in X is characterized by the set of ordered pairs $\tilde{A}_t = \{x_t, \mu_{\tilde{A}_t}(x,t)\}$, $x \in X$ and X is changed in time $t \in T$ with lots of grades of membership $\mu_{\tilde{A}}(x,t)$.

Dynamic membership function $\mu_{\tilde{A}_t} : \tilde{O} \times \grave{O} \to [0,1]$ associates with each point in X the degree of fuzzy dynamic set in interval [0, 1] at any point of time $\grave{O} = \{t_n\}$.

If we imagine the data of Figure-1 in a three-dimensional coordinate system, we get a dynamic fuzzy set, graphically represented by a dynamic membership function (Figure-2).

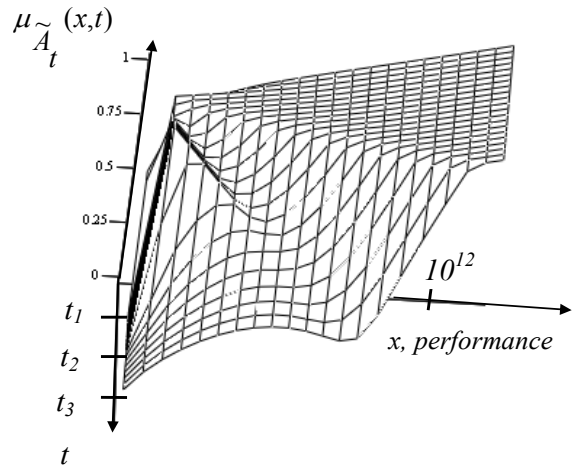

Figure-2. Dynamic membership functions of the fuzzy set "High speed information processing".

Any point of dynamic membership functions will be determined by the coordinates: abscissa – the time period; on the y – axis is the degree of fuzzy dynamic set, along the axis of the applicator is the domain of fuzzy values (Tereliansky and Kostikova, 2012)..

In practice, it is convenient to use those membership functions that admit analytic representation in the form of some simple mathematical functions. This simplifies not only the corresponding numerical calculations, but also reduces the computing resources required to store individual values of these membership functions.

The dynamic membership function can be viewed as a set of functions determined at different time points. In this case the shape of the dynamic Membership functions (DMF) will be based on the type of one-dimensional membership functions. The second criterion affecting the shape of the DMF is the range of values of the object, which is considered the property expressed by the dynamic fuzzy set. On this basis we identified four types of dynamic membership functions.

Let at time t = 0 and the membership function of the fuzzy set adopted a triangular form. When you change the time parameter ($t+i$, where i - is the dynamic step function) and compliance with the terms of the conservation functions, you will get a sequence $\{F_n\}$ where for each n: $F_n \subseteq Fun(n)$, (Figure-3, a). Thus, we can conclude that there exists a family of triangular functions $f_\Delta = (f_1, f_2, ..., f_n)$ specified by the time parameter $t_i, t_{i+1}, ..., t_k$.

As usual, dynamic membership functions of triangular type $\forall t = const,\ npu\ t \in [0, \infty]$ can be defined analytically by the following expression:

ARPN Journal of Engineering and Applied Sciences

$$f_{I_\Delta}(x,t;a,b,c)=\begin{cases} 0, x\le a \\ \frac{x-a}{b-a}, a\le x\le b \\ \frac{c-x}{c-b}, b\le c\le x \\ 0, c\le x \end{cases}$$

where *a, b, c* - some numeric parameters that accept an arbitrary valid values and ordered with respect: $a\le b\le c$. The parameters *a* and *c* characterize the base of the triangle and the parameter *b* characterize the top of the triangle.

Similarly, we have identified a family of trapezoidal membership functions:

$$f_{I_T}(x,t;a,b,c,d)=\begin{cases} 0, x\le a \\ \frac{x-a}{b-a}, a\le x\le b \\ 1, b\le x\le c \\ \frac{d-x}{d-c}, c\le x\le d \\ 0, d\le x \end{cases}$$

where *a, b, c, d*– some numeric parameters that accept an arbitrary valid values and ordered by: $a \le b \le c \le d$ The parameters *a* and *d* characterize the bottom of the trapezoid and the parameters *b* and *c* – the top of the trapezoid. Furthermore, this membership function generates a normal convex fuzzy set with the carrier – the interval (*a, d*), boundaries (*a, b*), (*c, d*) and core [*b, c*].

Membership functions to dynamic membership functions of triangular and trapezoidal types are shown in Figure-3.

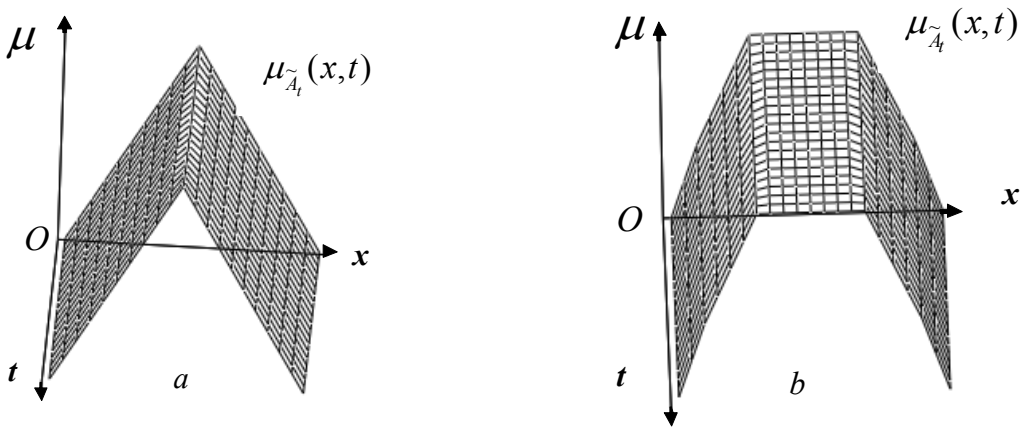

Figure-3. Dynamic membership functions of the first type of triangular (a) and trapezoidal (b) types.

The second type of dynamic membership functions can be built if you will modify the interval that defines the value of the parameter *x*. And the values of the function at the *x*-axis at different time periods do not overlap. Thus, the kind of one-dimensional membership functions remains unchanged. Visually, the plot of the membership functions becomes broken (Figure-4 *a*).

To find a coefficient of proportionality linking the adjacent functions with each other we proceed as way:

For example we will use the triangular type of DMF:

$$f_{II_\Delta}(x,t;a_t,b_t,c_t),$$

where a_t, b_t, c_t – some numeric parameters that accept an arbitrary valid values in different times and ordered: $a_t \le b_t \le c_t$. Thus the proportionality factor between the model constructions will be as follows: $R(t) = (R(t_0); R_1(t_1); R_2(t_2),...., R_k(t_k))$.

We suppose that in period t_0, the proportionality factor is defined as $R(t_0) = (1,1,1)$. Then the next period the proportionality factor may be expressed as $R(t_1) = \left(\frac{a_2}{a_1}; \frac{b_2}{b_1}; \frac{c_2}{c_1}\right)$.

Using this notation, any numeric parameter $a_{t+1}, b_{t+1}, c_{t+1}$ may be expressed as $a_t = R(t)\cdot a_{t-1}, b_t = R(t)\cdot b_{t-1}, c_t = R(t)\cdot c_{t-1}$.

Then, we may have for a discrete number of slices:

$$f_{II_\Delta}(x,t;a_t,b_t,c_t)=\left\{\frac{x-R(t)\cdot a_{t-1}}{b_{t-1}\cdot R(t)-a_{t-1}\cdot R(t)}\right\},\ t=0,1,2,\ldots,k$$

We identify the third type of dynamic membership functions when the shape of the graph at different points in time has changed, and the analyzed interval of values of the parameter X remains constant (Tereliansky and Kostikova, 2013).

For example, at time moment t_o membership function was triangular and at time t_1 membership function takes the trapezoidal shape. Then the required dynamic membership function can be written:

$$f_{III}(x,t;a,b,c,d)=f_{I_\Delta}(x,t;a,b,c)\cdot f_{I_T}(x,t;a,b,c,d))$$

The fourth type of dynamic membership functions are formed by changing of both factors: the type of one-dimensional membership functions and value range of the object. Then we may have

$$f_{IV}(x,t;a_1,\ldots,a_n,b_1,\ldots,b_n)=f_I(x,t;a_1,\ldots,a_n)\cdot f_I(x,t;b_1,\ldots,b_n)$$

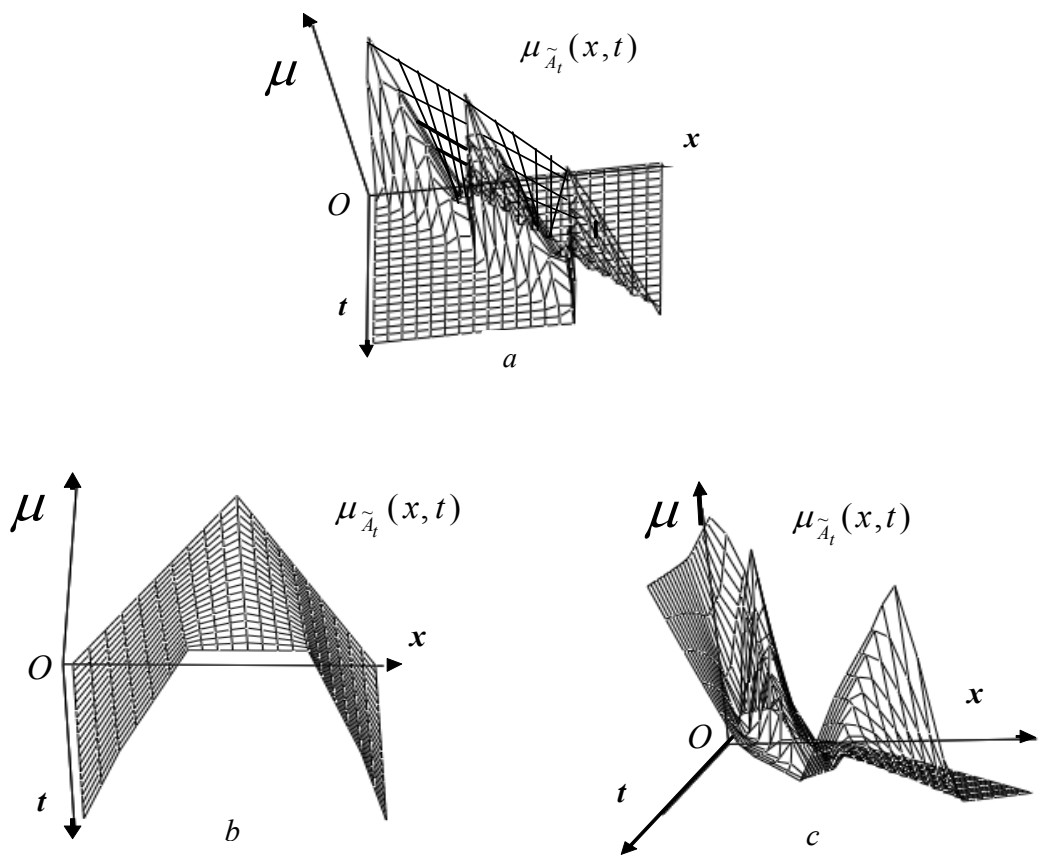

Figure-4. Dynamic membership functions of the second type (*a*), third (*b*) and forth (*c*) types.

3. OPERATIONS ON DYNAMIC FUZZY SETS

The union of dynamic fuzzy sets $\tilde{A}_t=\left\{x,\mu_{\tilde{A}_t}(x,t)\right\}$ and $\tilde{B}_t\left\{x,\mu_{\tilde{B}_t}(x,t)\right\}$ is denoted $\tilde{A}_t\cup\tilde{B}_t$ is defined by

$$\mu_{\tilde{A}_t \cup \tilde{B}_t}(x,t) = \mu_{\tilde{N}_t}(\tilde{o},t) = \max\left\{\mu_{\tilde{A}_t}(\tilde{o},t),\ \mu_{\tilde{B}_t}(\tilde{o},t)\right\}, x \in \tilde{O}, t \in T \quad (1)$$

The union corresponds to the connective or. Dynamic fuzzy Set $\tilde{C}_t$, contains either only elements that belong to the lot or lots , or elements that belong to both sets. The union of dynamic fuzzy sets is shown in Figure-5.

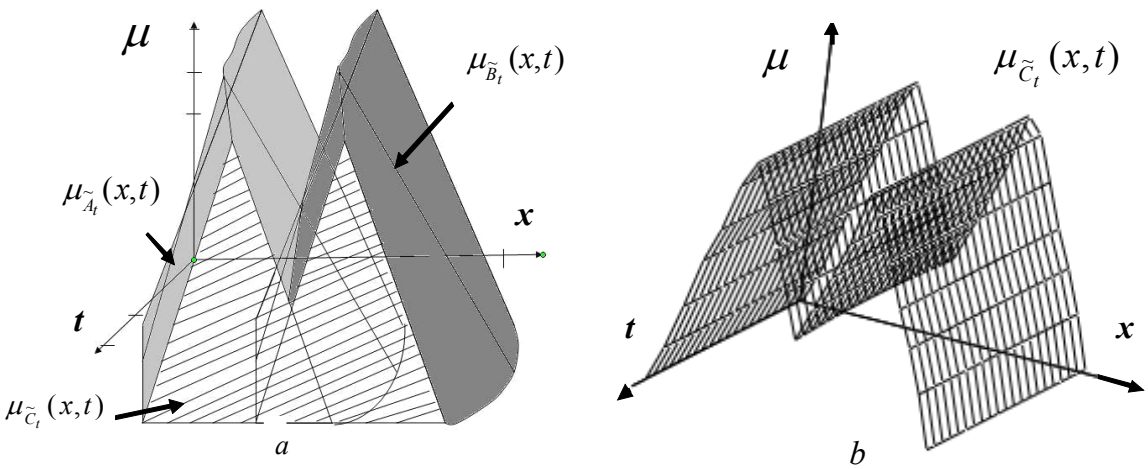

Figure-5. The union of dynamic fuzzy sets $\tilde{A}_t$ and $\tilde{B}_t$ *(a)* and the output dynamic fuzzy set $\tilde{C}_t$ (b).

The intersection of dynamic fuzzy sets $\tilde{A}_t = \left\{x, \mu_{\tilde{A}_t}(x,t)\right\}$ and $\tilde{B}_t \left\{x, \mu_{\tilde{B}_t}(x,t)\right\}$ is denoted $\tilde{A}_t \cap \tilde{B}_t$ and is defined by

$$\mu_{\tilde{A}_t \cap \tilde{B}_t}(x,t) = \mu_{\tilde{C}_t}(x,t) = \min\left\{\mu_{\tilde{A}_t}(x,t),\ \mu_{\tilde{B}_t}(x,t)\right\}, x \in X, t \in T \quad (2)$$

As a result we obtained the dynamic fuzzy set $\tilde{C}_t$, which is contained simultaneously in $\tilde{A}_t$ and $\tilde{B}_t$ and $\tilde{C}_t$ includes only those elements that belong to both sets.

The intersection of dynamic fuzzy sets is shown in Figure-6.

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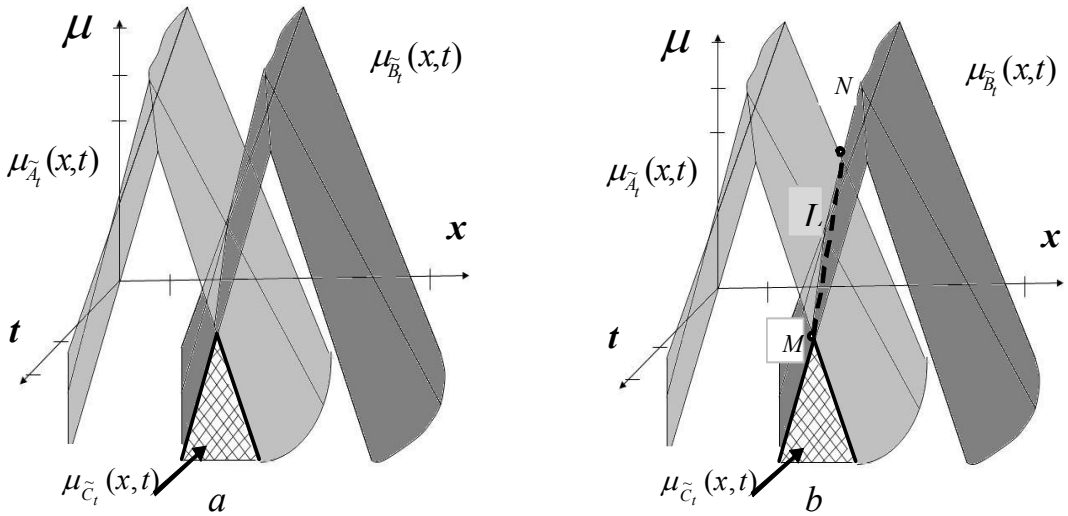

Figure-6. The intersection of dynamic fuzzy sets.

When this operation is performed, the membership functions of dynamic fuzzy sets $\widetilde{A}_t$ и $\widetilde{B}_t$ intersect in a line *L*. In this case, the line of intersection of two dynamic and fuzzy sets (Figure-6, b) is straight line; in other cases, it can take the form of broken one.

The boundaries of the line *L* are shown by points *M* и *N*, which belong simultaneously to both surfaces of dynamic fuzzy sets $\widetilde{A}_t$ and $\widetilde{B}_t$. The coordinates of the points are determined by dropping perpendicular on the axis of *Ot*, *O* μ , *Ox*.

Then we search the equation of the line is lying on it coordinates of the known points.

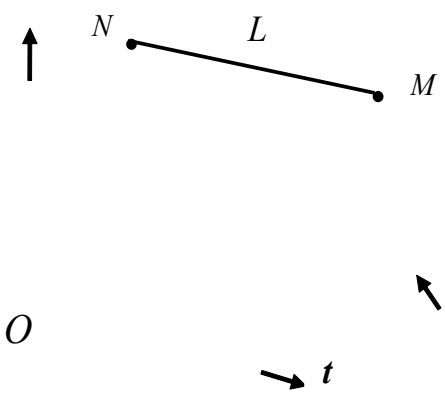

Figure-7. The result of intersection of dynamic fuzzy sets.

Let $M\ (x_M, \mu_M, t_M)$ and $N\ (x_N, \mu_N, t_N)$ be points at straight line.

The equation of the straight line passing through the point M is defined as:

$$y - y_M = k(x - x_M) \tag{3}$$

where *k* –unknown coefficient

In case of the straight line *L* also passes through the point $N\ (x_N, \mu_N, t_N)$, the coordinates of the point *N* must satisfy the equation (4):

$$y_N - y_M = k(x_N - x_M) \tag{4}$$

Then, $k = \dfrac{y_N - y_M}{x_N - x_M}$

And the equation of the straight line is expressed:

$$\frac{y - y_M}{y_N - x_M} = \frac{x - x_M}{x_N - x_M} \tag{5}$$

The complement of $\widetilde{A}$ is denoted $\neg\ \widetilde{A}$ and is defined by

$$\mu_{\neg \widetilde{A}_t}(\tilde{o}, t) = 1 - \mu_{\widetilde{A}_t}(\tilde{o}, t),\ \tilde{o} \in \tilde{O}, t \in T. \tag{6}$$

The operation of complementation corresponds to negation.

The operation of complementation are shown in Figure-8.

VOL. 11, NO. 17, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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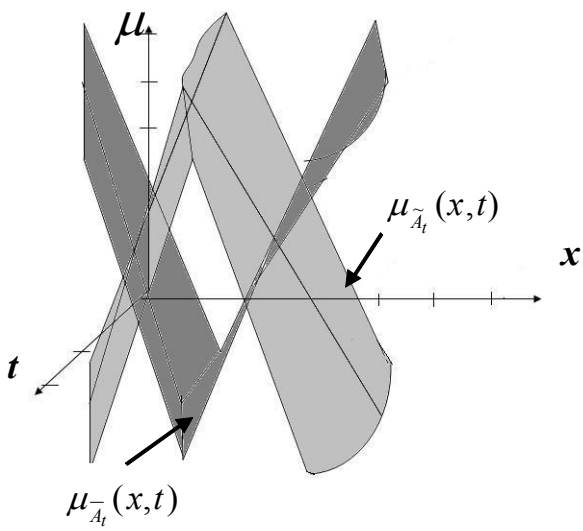

Figure-8. The operation of complementation.

In addition to the basic operations just defined, there are others that are described in [].

4. DYNAMIC FUZZY MODEL

Generally, dynamic fuzzy model is the information-logical model of the system constructed on the basis of the dynamic characteristics of the objects described by means of dynamic fuzzy sets.

Membership function in fuzzy set theory is a basic element that affects multi-criteria evaluation. The issue of modeling membership functions is one of the most important, for the reason that future calculations depend entirely on correctness of its construction.

At first, it is necessary to give a statement of the problem subject area, as it features the study of the subject set the basic features for modeling reflect semantic load of the task, the basis for the definition of evaluation criteria and the constraints in the problem (Kostikova and Tereliansky, 2014).

In general, the expert should determine the type of the membership function and set the base point for its formation.

The simulation algorithm is the following:

- firstly, we determine a reference and bifurcation points on the entire horizon of constructing the model. Due to objective reasons, the expert is often a known point at the initial moment of time, several important for expert points at the interval (this is usually the bifurcation point of the system), and points in a finite time. The reference point is the point at which the system state is recorded and accurately measure baselines. In a state of instability in the system, the conditions for a dramatic change in the way of its development, there are points of bifurcation.

- secondly, we formed arrays of experimental data from the indicated expert points describing the degree of belonging of each studied parameter to the given fuzzy set in different times;

Next, we carried out search of the unknown point by applying the algorithms of approximation;

Finally we get the dynamic fuzzy sets and their graphical representation in the form of creation of dynamic membership functions.

The operation of modeling is shown in Figure-9.

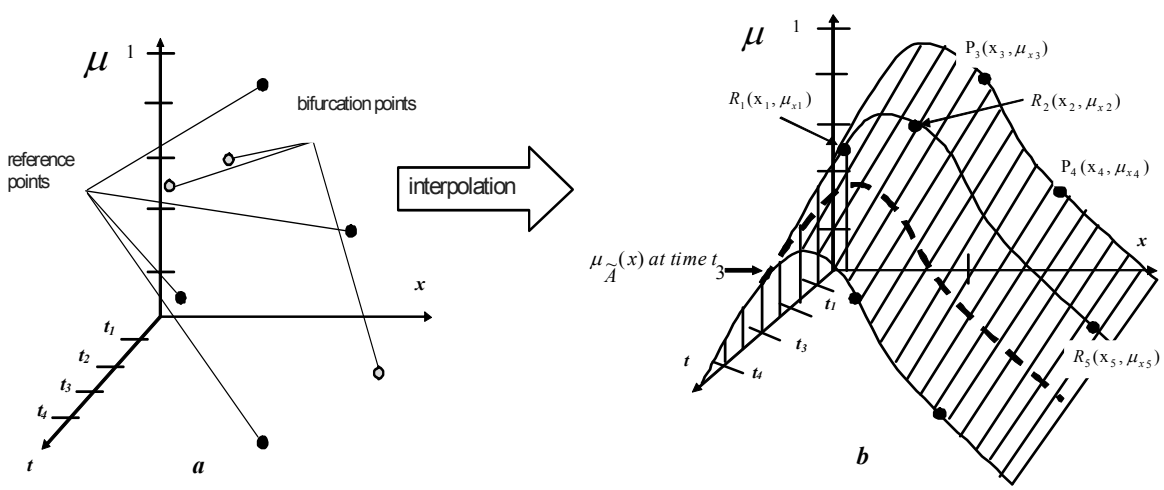

Figure-9. The operation of modeling.

Based on this theoretical and applied research, we have developed software tools for dynamic fuzzy modeling, registered in the state register of computer programs. The program "Calculation, approximation and visualization of dynamic fuzzy sets" is designed for an interactive process of expert assessment of the properties

of complex systems; it visualizes three-dimensional graphs. The software allows editing and inputting data; building a static membership functions for a certain time; building approximation of the missing values; building a dynamic membership functions (Kostikova and Gagarin, 2014).

In order to work with multiple experts at the same time, we have developed another program “Program system for the calculation of the generalized membership functions taking into account the dynamics of expert estimates” (Kostikova and Gagarin, 2014). Several experts may have disagreements, and it will be necessary to implement the algorithm of formation of a collective evaluation based on the source of distinct values. To obtain a generalized dynamics, membership functions were chosen as the methods based on the operations of intersection and merging and the method of calculation of the average.

CONCLUSIONS

As was stated in the introduction, the purpose of this article is to illustrate the necessity to use dynamic fuzzy sets to estimate the condition of dynamic complex systems. We have created a model for dynamic expert fuzzy knowledge processing about different complex objects. The aim is to build an integrated spatial and temporal estimate of quantitative and qualitative characteristics of complex systems and study the orientation dynamics of their changes. modernization of the mathematical tools of fuzzy set theory necessary to overcome restrictions in decision making due to use of static membership functions for the description and estimation of dynamic objects. the theory of formal languages can be extended to fuzzy languages. Conclusion. Created a model for dynamic processing fuzzy expert knowledge about different economic objects aimed at building an integrated spatial and temporal estimate of quantitative and qualitative characteristics of socio-economic systems and study the orientation dynamics of their changes.

The developed mathematical apparatus and software systems do not limit the expert in the choice of subject area, the number of studied parameters and the planning horizon. Interactive process of expert assessment of socio-economic indicators is provided by special software that helps significantly reduce the complexity of the formation of fuzzy sets in a dynamic environment and increase the efficiency of the solution with their help of practical tasks. Conclusion. Created a model for dynamic processing fuzzy expert knowledge about different economic objects aimed at building an integrated spatial and temporal estimate of quantitative and qualitative characteristics of socio-economic systems and study the orientation dynamics of their changes.

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