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RATE-TRANSIENT ANALYSIS FOR OFF-CENTERED HORIZONTAL WELLS IN HOMOGENEOUS ANISOTROPIC HYDROCARBON RESERVOIRS WITH CLOSED AND OPEN BOUNDARIES

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ABSTRACT

Some low and ultra-low permeability reservoirs are tested by recording the flow rate variation while the well-flowing pressure is kept constant. A typical situation occurs in shale formations which permeability goes to the order of nanodarcies. The mentioned procedure is normally referred as rate transient analysis and the interpretation of the reciprocal of the flow rate is very similar to the interpretation of a pressure test. The application of the *TDS* Technique for transient-rate analysis of horizontal wells in oil and gas homogeneous anisotropic reservoirs is presented here. New governing equations for hemi linear flow, parabolic flow, several situations of steady-state and pseudosteady-state flows are introduced and used to generate practical analytical expressions for the determination of well and reservoir parameters. The developed equations were successfully tested with synthetic examples.

Keywords: transient-rate analysis, horizontal well flow regimes, anisotropy, *TDS* Technique, hemi linear flow, Parabolic Flow, close and open boundaries.

1. INTRODUCTION

Nowadays, with the understandable overpowering from horizontal wells over vertical wells and the constant struggling with unusual-shaped reservoirs with different sceneries of boundaries have become necessary to find out a way to analyze transient-rate tests in horizontal wells, and then, provide representative estimation of the different reservoir and well parameters of the reservoir. There have been some recent publications on rate-transient analysis using either conventional analysis of the *TDS* Technique, Tiab (1993). For instance, Escobar, Rojas and Cantillo (2012) published an article about straight-line conventional rate-transient analysis for vertical wells in long homogeneous and heterogeneous reservoirs. Given the practicability of *TDS* technique over the conventional technique when analyzing pressure testing, Escobar, Rojas and Bonilla (2012) worked on rate-transient-rate for vertical wells in long homogeneous and naturally fractured reservoirs using the *TDS* technique. Then, few years later, Escobar, Castro and Mosquera (2014) worked on rate-transient analysis for hydraulically fractured vertical oil and gas wells and, more recently, Escobar, Montenegro and Bernal (2014) developed a TDS methodology for the interpretation of rate-transient analysis for hydraulically-fractured gas shale wells using the concept of the induced permeability field.

When talking about transient-rate analysis interpretation for horizontal wells in hydrocarbon homogeneous anisotropic reservoirs with closed and open boundaries there are currently no publications on that matter. There are some close-related researches on that topic by Escobar, Rojas and Ghisays-Ruiz (2015) in which they worked on rate-transient analysis for hydraulically-fractured horizontal wells in naturally-fractured shale gas reservoirs, increasing the application of the *TDS* Technique to the heyday of hydrocarbon industry.

In this work an application of the *TDS* technique to analyze rate-transient tests that were run in horizontal wells in oil and gas homogeneous anisotropic reservoirs is presented. The original work of its kind by Engler and Tiab (1996a, 1996b) -which were based on the model by Goode and Thambynayagam (1987) - only goes until late linear flow regime. Then, in this work -which is based on these just mentioned- were added different reservoir geometries and models which give form to several flow regimes according to its boundary configuration and to the position of the well inside the reservoir.

Following the formulation proposed by Engler and Tiab (1996b) who presented the *TDS* technique for interpreting pressure tests in horizontal wells in homogeneous and anisotropic porous media; the governing equations of such flow regimes as early radial, early linear, pseudo radial and late linear were transformed from pressure-transient analysis to rate-transient analysis. A similar treatment was performed on the elliptical flow regime given by Martinez, Escobar and Bonilla (2012), in which they proposed a reformulation of the elliptical flow governing equation for a more complete well test data interpretation for horizontal wells for pressure-transient analysis.

Following the ideas exposed by Escobar, Hernandez and Hernandez (2007) on long homogeneous reservoirs, we also performed some analysis on off-centered wells in long homogeneous anisotropic reservoirs to provide the formation of hemi linear flow regime, parabolic flow regime, pseudo steady state flow and five cases of steady-state flow in rate-transient testing according to the reservoir boundaries and dimensions. Equations for each flow regime were developed and successfully tested with synthetic examples. Only ine example is presented due to space-saving reasons.

For the extension to gas wells, the pseudo functions were also used: the one from Agarwal (1949)

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introduced the pseudotime function to account for the dependence on time of the gas viscosity and total system compressibility and the other one from Hussainy *et al.* (1966) who presented the gas pseudo pressure concept.

2. MATHEMATICAL FORMULATION

2.1. Mathematical model

When working with rate-transient analysis, the dimensionless rate inside a reservoir has an approximate performance equal to the inverse of the dimensionless pressure. This is:

$$P_D \approx \frac{1}{q_D} \tag{1}$$

This approach is not always reliable; therefore, for several studied flow regimes in rate-transient analysis, the pressure equations were rewritten and some constants that accomplish linearization were added.

The dimensionless variables proposed for rate-transient analysis were obtained by rewriting all dimensionless variables of Tiab and Engler (1996b) to allow for the conversion from dimensionless to dimensional values:

$$(1/q_D) = \frac{k_y L_w \Delta P}{141.2\mu B}(1/q) \tag{2}$$

$$[t_D * (1/q_D)'] = \frac{k_y L_w \Delta P}{141.2\mu B}[t * (1/q)'] \tag{3}$$

$$t_D = \frac{0.0002637 k_y t}{\phi \mu c_t r_w^2} \tag{4}$$

$$t_{DA} = t_D \frac{r_w^2}{A} = \frac{0.0002637 k_y t}{\phi \mu c_t A} \tag{5}$$

$$Y_D = \frac{2b_y}{h_y} \tag{6}$$

Goode and Thambynayagam (1987) presented the mathematical solution for horizontal well pressure reservoir pressure behavior in both homogeneous and heterogeneous reservoirs. This formulation was used by Engler and Tiab (1996a, 1996b). This mathematical solution was also adopted in this work.

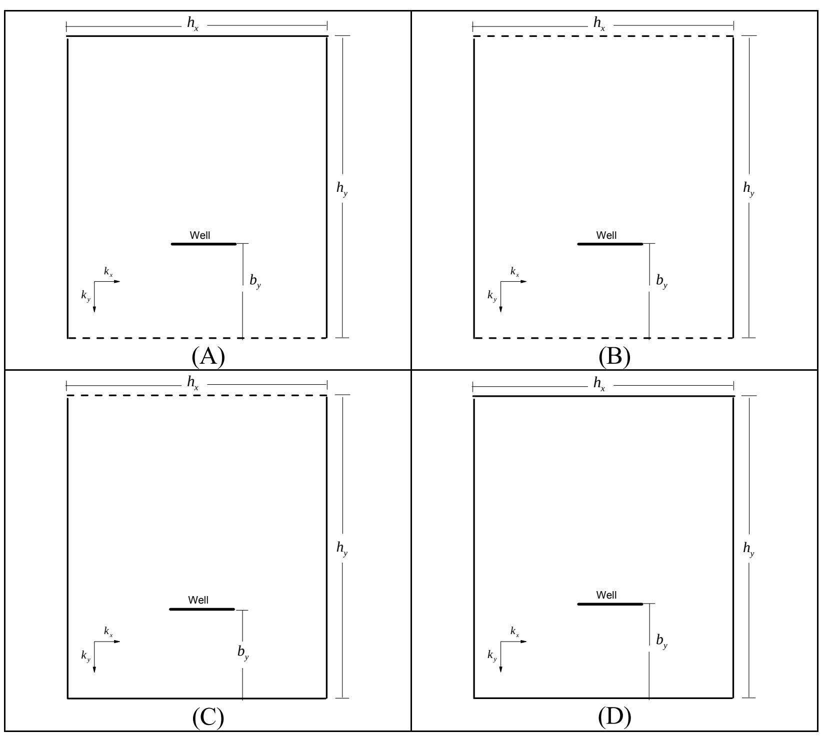

Figure-1. Reservoir geometry cases for off-centered horizontal wells in the reservoir. (A) Well located near the open boundary and the other boundary is closed. (B) Well is located within two open boundaries. (C) Well located near the closed boundary and the other boundary is open. (D) All closed boundaries. After Pabon and Cortes (2016)

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2.2. TDS technique

The governing flow regime models presented by Engler and Tiab (1996b) for pressure-transient analysis were taken and with the use of the statement given in Equation (1) a reformulation were made to find the expressions corresponding to the reciprocal rate and reciprocal rate derivative for each flow regime:

For early radial flow:

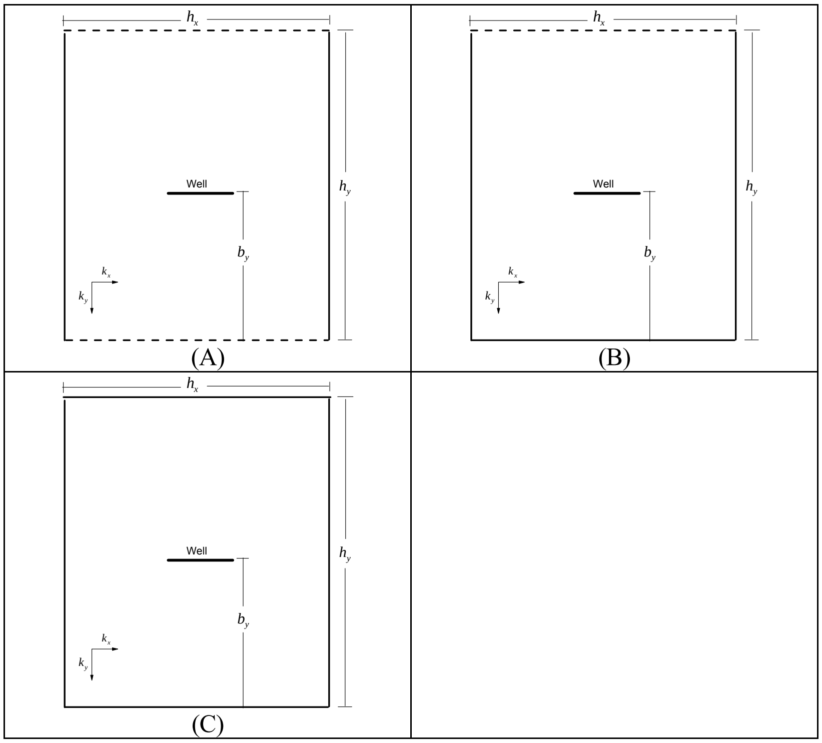

Figure-2. Reservoir geometry cases for centered horizontal wells in the reservoir (A) The reservoir has both open boundaries. (B) The reservoir has one open boundary. (C) The reservoir has all closed boundaries. After Pabon and Cortes (2016).

$$(1/q_D)_{er} = \frac{1}{2}\sqrt{\frac{k_y}{k_z}}\left[\ln\left(\sqrt{\frac{k_z}{k_y}}t_{D_{er}}\right) + 0.80907 + 2s_m\right] \quad (7)$$

$$[t_D * (1/q_D)']_{er} = \frac{1}{2}\sqrt{\frac{k_y}{k_z}} \quad (8)$$

Replacing the dimensionless derivative given by Equation (3) into Equation (8), it yields:

$$\sqrt{k_y k_z} = \frac{70.6\mu B}{L_w \Delta P[t*(1/q)']_{er}} \quad (9)$$

Dividing Equation (7) by Equation (8) and, then, substituting Equations (2), (3) and (4) into the resulting value, we obtain:

$$s_m = \frac{1}{2}\left[\frac{(1/q)_{er}}{[t*(1/q)]_{er}} - \ln\left(\frac{\sqrt{k_y k_z}\, t_{er}}{\phi\mu c_t r_w^{\,2}}\right) + 7.43\right] \quad (10)$$

For early linear flow:

$$(1/q_D)_{el} = \frac{236}{100}\left(\frac{r_w}{h_z}\right)\sqrt{\pi t_{D_{el}}} + \sqrt{\frac{k_y}{k_z}}(s_z + s_m) \quad (11)$$

$$[t_D * (1/q_D)']_{el} = \frac{118}{100}\frac{r_w}{h_z}\sqrt{\pi t_{D_{el}}} \quad (12)$$

Replacing the dimensionless variables given by Equation (3) and (4) into Equation (12), it yields:

$$L_w = \frac{4.7956B}{h_z \Delta P[t*(1/q)']_{el}}\sqrt{\frac{\mu t_{el}}{k_y \phi c_t}} \quad (13)$$

$$k_y = \left(\frac{4.7956B}{h_z \Delta P[t*(1/q)']_{el}}\right)^2 \frac{\mu t_{el}}{L_w^{\,2}\phi c_t} \quad (14)$$

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Dividing Equation (11) by Equation (12) and, then, substituting Equation (2), (3) and (4) into the result, we obtain:

$$s_m + s_z = \frac{1}{29.443 h_z}\sqrt{\frac{k_z t_{el}}{\phi\mu c_t}}\left[\frac{(1/q)_{el}}{[t*(1/q)']_{el}} - 2.0\right] \tag{15}$$

The governing equation for pseudo radial flow is:

$$(1/q_D)_{pr} = \frac{L_w}{2h_z}\sqrt{\frac{k_y}{k_x}}\left[\ln\left(\frac{16k_x r_w^2}{k_y L_w^{\,2}} t_{D_{pr}}\right) + 0.80907\right] + \frac{k_y}{\sqrt{k_x k_z}}(s_m + s_z) \tag{16}$$

$$[t_D*(1/q_D)']_{pr} = \frac{1}{2}\frac{L_w}{h_z}\sqrt{\frac{k_y}{k_x}} \tag{17}$$

Replacing the dimensionless quantity given by Equation (3) into Equation (17), it yields:

$$\sqrt{k_x k_y} = \left(\frac{70.6\mu B}{h_z \Delta P[t*(1/q)']_{pr1}}\right) \tag{18}$$

Dividing Equation (16) by Equation (17) and, then, substituting Equations (2), (3) and (4) into the result, we obtain:

$$s_m + s_z = \frac{L_w}{2h_z}\sqrt{\frac{k_z}{k_y}}\left(\frac{(1/q)_{pr}}{[t*(1/q)']_{pr}} - \ln\left(\frac{k_x t_{pr}}{\phi\mu c_t L_w^{\,2}}\right) + 4.659\right) \tag{19}$$

For late linear flow:

$$(1/q_D)_{ll} = \frac{236}{100}\left(\frac{L_w r_w}{h_x h_z}\right)\sqrt{\pi t_{D_{ll}}} + \sqrt{\frac{k_y}{k_z}}(s_x + s_z + s_m) \tag{20}$$

$$[t_D*(1/q_D)']_{ll} = \frac{118}{100}\left(\frac{L_w r_w}{h_x h_z}\right)\sqrt{\pi t_{D_{ll}}} \tag{21}$$

Replacing the dimensionless reciprocal derivative and time given by Equations (3) and (4) into Equation (21), it yields:

$$h_x = \frac{4.7956B}{\Delta P h_z [t*(1/q)']_{ll}}\sqrt{\frac{\mu t_{ll}}{\phi k_y c_t}} \tag{22}$$

$$k_y = \frac{\mu t_{ll}}{\phi c_t}\left[\frac{4.7956B}{\Delta P h_z h_x [t*(1/q)']_{ll}}\right]^2 \tag{23}$$

Dividing Equation (20) by Equation (21) and, then, substituting Equation (2), (3) and (4) into the result, we obtain:

$$(s_x + s_z + s_m) = \frac{1}{29.443}\left[\left(\frac{L_w}{h_x h_z}\right)\sqrt{\frac{k_z t_{ll}}{\phi\mu c_t}}\right]\left[\frac{(1/q)_{ll}}{[t*(1/q)']_{ll}} - 2\right] \tag{24}$$

For the elliptical flow regime, Martinez *et al*. (2012) presented the pressure and pressure derivative dimensionless governing equations for transient-pressure analysis which traduced to rate-transient analysis:

$$(1/q_D) = \frac{2.6562 r_w^{0.72} L_w^{0.28}}{h_z}\left(\frac{k_y}{k_x}\right)^{0.14} t_{Dell}^{0.36} + s_{Ell} \tag{25}$$

$$[t_D*(1/q_D)']_{Ell} = \frac{0.95621522 r_w^{0.72} L_w^{0.28}}{h_z}\left(\frac{k_y}{k_x}\right)^{0.14} t_{Dell}^{0.36} \tag{26}$$

Replacing the dimensionless quantities given by Equations (3) and (4) into Equation (26), it yields:

$$L_w = \left[6.9501\frac{\mu^{0.64} B}{\Delta P k_y^{0.5} k_x^{0.14} h_z [t*(1/q)']_{Ell}}\left(\frac{t_{Ell}}{\phi c_t}\right)^{0.36}\right]^{1/0.72} \tag{27}$$

$$k_y = \left[6.9501\frac{\mu^{0.64} B}{\Delta P L_w^{0.72} k_x^{0.14} h_z [t*(1/q)']_{Ell}}\left(\frac{t_{Ell}}{\phi c_t}\right)^{0.36}\right]^{1/0.5} \tag{28}$$

Dividing Equation (25) by Equation (26) and, then, substituting Equation (2), (3) and (4) into the result, we obtain:

$$s_{Ell} = \frac{1}{20.3163}\left[\frac{(1/q)_{Ell}}{[t*(1/q)']_{Ell}} - 2.777\right]\left[\frac{L_w^{0.28} k_y^{0.5}}{h_z k_x^{0.14}}\left(\frac{t_{Ell}}{\phi\mu c_t}\right)^{0.36}\right] \tag{29}$$

For vertical wells, Escobar *et al.* (2007) differentiated between linear flow (dual linear flow) when there exist two linear flows at both sides of the well and single linear (hemi linear flow) in which a linear flow takes place at only one lateral side of the well. This definition is used by Pabon and Cortes (2016). Hemilinear flow occurs within the reservoir when the horizontal well is located near a closed boundary, regardless the far boundary is closed (late pseudosteady - state period) or open (steady-state). Reservoir geometries of this flow regime are shown in Figure-1 (C) and (D). The governing equation of the reciprocal rate derivative for hemi linear flow regime was empirically obtained by Pabon and Cortes (2016). Then, the governing reciprocal rate equation was obtained by integration:

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$$(1/q_D)_{hl} = \frac{764\sqrt{\pi}L_w}{100h_z}\left[\left(\frac{r_w}{h_x}\right)^2 t_{D_{hl}}\right]^{0.5} + \sqrt{\frac{k_y}{k_z}}(s_x + s_z + s_m + s_{hl}) \tag{30}$$

The dimensionless governing reciprocal rate derivative is:

$$[t_D * (1/q_D)']_{hl} = \frac{382\sqrt{\pi}L_w}{100h_z}\left[\left(\frac{r_w}{h_x}\right)^2 t_{D_{hl}}\right]^{0.5} \tag{31}$$

Replacing the dimensionless quantities given by Equations (3) and (4) into Equation (31), it yields:

$$k_y = \left(\frac{\mu t_{hl}}{\phi c_t}\right)\left[\frac{15.5249B}{h_z h_x \Delta P[t*(1/q)']_{hl}}\right]^2 \tag{32}$$

$$h_x = \frac{15.5249B}{h_z \Delta P[t*(1/q)']_{hl}}\sqrt{\frac{\mu t_{hl}}{k_y \phi c_t}} \tag{33}$$

Dividing Equation (30) by Equation (31) and, then, substituting Equations (2), (3) and (4) into the resulting expression, it yields:

$$(s_x + s_z + s_m + s_{hl}) = \frac{L_w}{9.0951h_z h_x}\left(\frac{k_z t_{hl}}{\phi \mu c_t}\right)^{0.5}\left[\frac{(1/q)_{hl}}{[t*(1/q)']_{hl}} - 2\right] \tag{34}$$

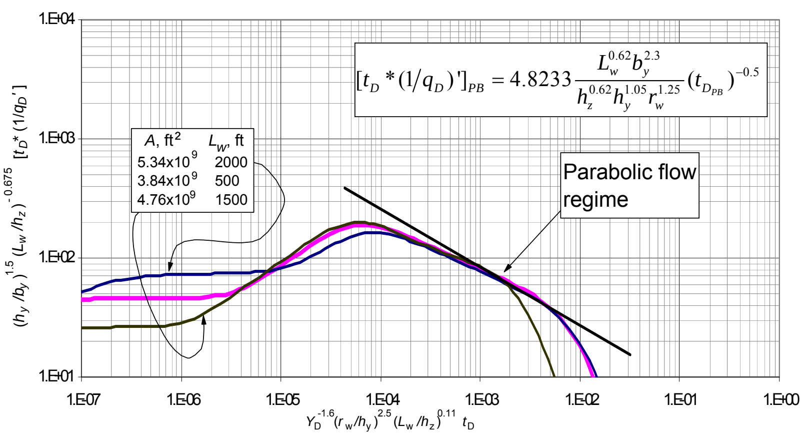

Figure-3. Dimensionless $Y_D^{-1.6}(r_w/h_y)^{2.5}(L_w/h_z)^{0.11} t_D$ vs $(h_y/b_y)^{1.5}(L_w/h_z)^{0.675} t_D*P_D'$ log-log plot with unified parabolic flow regime. After Pabon and Cortes (2016)

The occurrence of parabolic flow within the reservoir results when the horizontal well is located near an open boundary regardless the far boundary is either closed or open to flow. Reservoir geometries of this flow regime are shown in Figure-1 (A) and (B).). The governing equation of the reciprocal rate derivative for parabolic flow regime was empirically obtained from Figure-3 by Pabon and Cortes (2016). Then, the governing rate-transient equation was obtained by integration:

The dimensionless governing reciprocal rate is:

$$(1/q_D)_{PB} = \frac{-9.6466L_w^{0.62}b_y^{2.3}}{h_z^{0.62}h_y^{1.05}r_w^{1.25}}t_{D_{PB}}^{-0.5} + \sqrt{\frac{k_y}{k_z}}(s_x + s_z + s_m + s_{PB}) \tag{35}$$

The dimensionless governing reciprocal rate derivative is:

$$[t_D * (1/q_D)']_{PB} = \frac{4.8233L_w^{0.62}b_y^{2.3}}{h_z^{0.62}h_y^{1.05}r_w^{1.25}}t_{D_{PB}}^{-0.5} \tag{36}$$

Replacing the dimensionless variables given by Equations (3) and (4) into Equation (36), it yields:

$$b_y = \left[\frac{\frac{1}{41939.60115}\frac{\Delta P k_y^{1.5}L_w^{0.38}h_z^{0.62}h_y^{1.05}r_w^{0.25}}{\mu^{1.5}B}}{[t*(1/q)']_{PB}\left(\frac{t_{PB}}{\phi c_t}\right)^{0.5}}\right]^{1/2.3} \tag{37}$$

Dividing Equation (35) by Equation (36) and, then, substituting Equations (2), (3) and (4) into the result, we obtain:

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$$s_x + s_z + s_m + s_{PB} = \left[\frac{297.0224 L_w^{0.62} b_y^{2.3}}{k_y h_z^{0.62} h_y^{1.05} r_w^{0.25}} \sqrt{\frac{k_z \phi \mu c_t}{t_{PB}}} \right] \left[\frac{(1/q)_{PB}}{[t*(1/q)']_{PB}} + 2 \right] \quad (38)$$

For the first case of steady-state, the reservoir geometry is shown in Figure-1 (B). The dimensionless governing reciprocal rate derivative equation is:

$$[t_D*(1/q_D)']_{ss1} = 2417844.769 \frac{L_w^{0.05}}{h_z^{1.2}} \left(\frac{b_y}{h_y}\right)^{2.7} \left(\frac{h_x}{r_w}\right)^{0.01} t_{DA_{ss1}}^{-1} \quad (39)$$

Replacing the dimensionless quantities given by Equations (3) and (4) into Equation (39), it yields:

$$h_y = \left[1.2946518 \times 10^{12} \frac{\phi B c_t b_y^{2.7} \mu^2 h_x^{1.01}}{k_y^2 L_w^{0.95} \Delta P h_z^{1.2} r_w^{0.01} [t*(1/q)']_{ss1} t_{ss1}} \right]^{1/1.7} \quad (40)$$

For the second case of steady-state period, the reservoir geometry is shown in Figure-1 (A). The dimensionless governing reciprocal rate derivative expression is:

$$[t_D*(1/q_D)']_{ss2} = \frac{915637.772 L_w^{0.1} b_y^{2.92}}{h_z^{0.96} h_y^{2.8}} t_{DA_{ss2}}^{-1} \quad (41)$$

Replacing the dimensionless parameters given by Equations (3) and (4) into Equation (41), it yields:

$$A = \frac{1}{4.902846167 \times 10^{11}} \frac{k_y^2 L_w^{0.9} \Delta P h_z^{0.96} h_y^{2.8}}{\mu^2 b_y^{2.92}} [t*(1/q)']_{ss2} \left(\frac{t_{ss2}}{\phi B c_t}\right) \quad (42)$$

The third case of steady-state is sketched by the reservoir geometry of Figure-1 (C). The dimensionless governing reciprocal rate derivative equation is:

$$[t_D*(1/q_D)']_{ss3} = \frac{1}{817.45774} \frac{L_w^{0.85} h_y^{1.7}}{h_z^{0.86} h_x} t_{DA_{ss3}}^{-1} \quad (43)$$

Replacing Equations (3) and (4) into Equation (43), it yields:

$$h_y = \left[\frac{k_y^2 L_w^{0.15} \Delta P h_z^{0.86} t_{ss3}}{655.027 \mu^2 B \phi c_t} [t*(1/q)']_{ss3} \right]^{1/2.7} \quad (44)$$

The fourth case of steady-state behavior is represented by the reservoir geometry shown in Figure-2 (B). The dimensionless governing reciprocal rate derivative equation is:

$$[t_D*(1/q_D)']_{ss4} = \frac{L_w^{0.85} h_y^{1.5}}{1076 h_z^{0.75} h_x^{0.8}} (t_{DA})_{ss4}^{-1} \quad (45)$$

Replacing the dimensionless parameters given by Equations (3) and (4) into Equation (45), it yields:

$$h_y = \left[\frac{k_y^2 L_w^{0.15} \Delta P h_z^{0.75}}{497.636 \mu^2 h_x^{0.2}} [t*(1/q)']_{ss4} \left(\frac{t_{ss4}}{\phi B c_t}\right) \right]^{0.4} \quad (46)$$

For the fifth case of steady-state behavior, the reservoir geometry is shown in Figure-2 (A) and the dimensionless equation of the governing reciprocal rate derivative is given by:

$$[t_D*(1/q_D)']_{ss5} = \frac{L_w^{0.85} h_y^{1.5}}{6200.5952 h_z^{0.75} h_x^{0.8}} (t_{DA})_{ss5}^{-1} \quad (47)$$

Replacing the dimensionless quantities given by Equations (3) and (4) into Equation (47), it yields:

$$h_y = \left[\frac{k_y^2 L_w^{0.15} \Delta P h_z^{0.75}}{86.3557 \mu^2 h_x^{0.2}} [t*(1/q)']_{ss5} \left(\frac{t_{ss5}}{\phi B c_t}\right) \right]^{0.4} \quad (48)$$

For the pseudosteady-state period, the reservoir geometry is shown in Figure-1 (D) and Figure-2 (C). The dimensionless governing reciprocal rate derivative equation is:

$$[t_D*(1/q_D)']_{pss} = \frac{55}{10} \pi \left(\frac{L_w}{h_z}\right) (t_{DA})_{pss} \quad (49)$$

Again, replacing Equations (3) and (4) into Equation (49), and solving for reservoir area, it yields:

$$A = \frac{1}{1.5543} \left(\frac{B}{h_z \Delta P [t*(1/q)']_{pss}} \right) \left(\frac{t_{pss}}{\phi c_t} \right) \quad (50)$$

2.2.1. Intersection points between flow regimes

Pabon and Cortes (2016) intercepted several reciprocal rate derivative equations to obtain useful intersection point expression, starting with the intersection times between early radial and other flows with different slopes. For instance, the intersection time between early radial and early linear flow regimes is given by:

$$t_{D_{er-eli}}=\frac{1}{17.4974}\left(\frac{k_y}{k_z}\right)\left(\frac{h_z}{r_w}\right)^2 \tag{51}$$

After replacing the dimensionless time and solving for the vertical permeability, we obtain:

$$k_z=216.7285\frac{\phi\mu c_t h_z^2}{t_{er-eli}} \tag{52}$$

The intersection time between the early radial and elliptical flow regimes is given by:

$$t_{D_{er-Elli}}^{0.36}=\frac{k_y^{0.36}k_x^{0.14}h_z}{1.91243044k_z^{0.5}r_w^{0.72}L_w^{0.28}} \tag{53}$$

After replacing the dimensionless parameters in the above expression and solving for the vertical permeability, it yields:

$$k_z=\left[10.158152\frac{k_x^{0.14}h_z}{L_w^{0.28}}\left(\frac{\phi\mu c_t}{t_{er-Elli}}\right)^{0.36}\right]^2 \tag{54}$$

The obtained expression for the intersection time between the early radial and late linear flow regimes is:

$$t_{D_{er-lli}}=\frac{1}{17.4974}\frac{k_y}{k_z}\left(\frac{h_x h_z}{L_w r_w}\right)^2 \tag{55}$$

In real units, the vertical permeability is brought from the above equation:

$$k_z=\frac{216.7285\phi\mu c_t}{t_{er-lli}}\left(\frac{h_x h_z}{L_w}\right)^2 \tag{56}$$

The expression for the intersection time between the early radial and hemi linear flow regimes is:

$$t_{D_{er-hli}}=\frac{k_y}{183.3735k_z}\left(\frac{h_x h_z}{r_w L_w}\right)^2 \tag{57}$$

From this, the vertical permeability is obtained:

$$k_z=\frac{20.6801\phi\mu c_t}{t_{er-hli}}\left(\frac{h_x h_z}{L_w}\right)^2 \tag{58}$$

The intersection time between the early radial and parabolic flow regimes is given by:

$$(t_D)_{er-PBi}^{-0.5}=\frac{h_z^{0.62}h_y^{1.05}r_w^{1.25}}{9.6466L_w^{0.62}b_y^{2.3}}\sqrt{\frac{k_y}{k_z}} \tag{59}$$

The use of Equation (3) into Equation (59) allows obtaining:

$$b_y=\left[\frac{k_y h_z^{0.62}h_y^{1.05}r_w^{0.25}}{594.0448k_z^{0.5}L_w^{0.62}}\left(\frac{t_{er-PBi}}{\phi\mu c_t}\right)^{0.5}\right]^{1/2.3} \tag{60}$$

The time of intersection between the early radial flow regime and the first case of steady-state period is governed by:

$$t_{DA_{er-ss1i}}^{-1}=\frac{h_z^{1.2}}{4835689.538L_w^{0.05}}\left(\frac{h_y}{b_y}\right)^{2.7}\left(\frac{r_w}{h_x}\right)^{0.01}\sqrt{\frac{k_y}{k_z}} \tag{61}$$

After plugging the dimensionless time, we can obtain:

$$h_y=\left[1.8337844\times10^{10}\left(\frac{\phi\mu c_t}{t_{er-ss1i}}\right)\frac{k_z^{0.5}h_x^{1.01}L_w^{0.05}b_y^{2.7}}{k_y^{1.5}h_z^{1.2}r_w^{0.01}}\right]^{1/1.7} \tag{62}$$

The dimensionless expression for the time of intersection between the early radial flow regime and second case of steady-state period is:

$$t_{DA_{er-ss2i}}^{-1}=\frac{h_z^{0.96}h_y^{2.8}}{1831275.544L_w^{0.1}b_y^{2.92}}\sqrt{\frac{k_y}{k_z}} \tag{63}$$

After replacing the dimensionless parameters in the above expression, and solving for reservoir area, it yields:

$$A=\frac{1}{6944541312}\left(\frac{t_{er-ss2i}}{\phi\mu c_t}\right)\frac{k_y^{1.5}h_z^{0.96}h_y^{2.8}}{k_z^{0.5}L_w^{0.1}b_y^{2.92}} \tag{64}$$

The intersection time between the early radial flow regime and the third case of steady-state period is governed by:

$$t_{DA_{er-ss3i}}^{-1}=408.7288\sqrt{\frac{k_y}{k_z}}\frac{h_z^{0.86}h_x}{L_w^{0.85}h_y^{1.7}} \tag{65}$$

After replacing the dimensionless time and solving for reservoir length, we obtain:

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$$h_y = \left[\frac{1}{9.278} \frac{k_y^{1.5} h_z^{0.86}}{k_z^{0.5} L_w^{0.85}} \left(\frac{t_{er-ss3i}}{\phi \mu c_t r_w} \right) \right]^{1/2.7} \quad (66)$$

The time of intersection between the early radial flow regime and the fourth case of steady-state period leads to obtain the following dimensionless expression:

$$(t_{DA})^{-1}_{er-ss4i} = 538 \sqrt{\frac{k_y}{k_z}} \frac{h_z^{0.75} h_x^{0.8}}{L_w^{0.85} h_y^{1.5}} \quad (67)$$

Again, after replacing the dimensionless time and solving for reservoir length, we obtain:

$$h_y = \left[\frac{k_y^{1.5} h_z^{0.75}}{7.048676 k_z^{0.5} L_w^{0.85} h_x^{0.2}} \left(\frac{t_{er-ss4i}}{\phi \mu c_t} \right) \right]^{1/2.5} \quad (68)$$

The intersection time between the early radial flow regime and the fifth case of steady-state period is represented by:

$$(t_{DA})^{-1}_{er-ss5i} = 3100.2976 \sqrt{\frac{k_y}{k_z}} \frac{h_z^{0.75} h_x^{0.8}}{L_w^{0.85} h_y^{1.5}} \quad (69)$$

After replacing the dimensionless time and solving for reservoir length, it yields:

$$h_y = \left[\frac{1}{1.22317} \frac{k_y^{1.5} h_z^{0.75}}{k_z^{0.5} L_w^{0.85} h_x^{0.2}} \left(\frac{t_{er-ss5i}}{\phi \mu c_t} \right) \right]^{0.4} \quad (70)$$

The intersecting time between the early radial flow and pseudosteady state period provides:

$$t_{DA_{er-pssi}} = \frac{1}{34.5575} \sqrt{\frac{k_y}{k_z}} \left(\frac{h_z}{L_w} \right) \quad (71)$$

After replacing Equation (5) in Equation (71) and solving for the area, it becomes:

$$A = \frac{1}{109.7356} \frac{t_{er-pssi}}{\phi \mu c_t} \sqrt{k_y k_z} \left(\frac{L_w}{h_z} \right) \quad (72)$$

The time of intersection between the first case of steady-state period and early linear flow regime gives:

$$(t_D)^{1.5}_{el-ss1i} = \frac{1156036.299 L_w^{0.05} h_x^{1.01} b_y^{2.7}}{h_y^{1.7} h_z^{0.2} r_w^{2.99}} \quad (73)$$

This becomes in real units once the dimensionless time is substituted in the above expression;

$$h_y = \left[\frac{2.69964 L_w^{0.05} h_x^{1.01} b_y^{2.7} r_w^{0.01}}{h_z^{0.2}} \left(\frac{\phi \mu c_t}{k_y t_{el-ss1i}} \right)^{1.5} \right]^{1/1.7} \quad (74)$$

The time of intersection between the first case of steady-state period and elliptical flow regime gives in dimensionless form is given by Equation (75) and after replacing the dimensionless quantities allows finding reservoir length:

$$(t_D)^{1.36}_{Ell-ss1i} = \frac{2528557.085 b_y^{2.7} h_x^{1.01}}{h_y^{1.7} h_z^{0.2} r_w^{2.73} L_w^{0.275}} \left(\frac{k_x}{k_y} \right)^{0.14} \quad (75)$$

from which reservoir length is obtained once the dimensionless time is substituted:

$$h_y = \left[\frac{1.8315444 \times 10^{11} b_y^{2.7} h_x^{1.01} k_x^{0.14}}{h_z^{0.2} r_w^{0.01} L_w^{0.275} k_y^{1.5}} \left(\frac{\phi \mu c_t}{t_{Ell-ss1i}} \right)^{1.36} \right]^{1/1.7} \quad (76)$$

As for the former case, the intersection time between the first case of steady-state and pseudo radial flow regime leads to:

$$t_{DA_{pr-ss1i}} = \frac{4835689.538}{h_z^{0.2} L_w^{0.95}} \left(\frac{b_y}{h_y} \right)^{2.7} \left(\frac{h_x}{r_w} \right)^{0.01} \sqrt{\frac{k_x}{k_y}} \quad (77)$$

which leads finding;

$$h_y = \left[\frac{1.83378443 \times 10^{10} k_x^{0.5} \phi \mu c_t h_x^{1.01} b_y^{2.7}}{h_z^{0.2} L_w^{0.95} k_y^{1.5} r_w^{0.01} t_{pr-ss1i}} \right]^{1/1.7} \quad (78)$$

The time of intersection between the first case of steady-state period and the late linear flow regime gives:

$$(t_D)^{1.5}_{ll-ss1i} = 1156036.299 \frac{h_x^{2.01} b_y^{2.7}}{L_w^{0.95} h_z^{0.2} r_w^{3.01} h_y^{1.7}} \quad (79)$$

After replacing the dimensionless time and solving for reservoir length:

$$h_y = \left[\frac{2.69964 h_x^{2.01} b_y^{2.7}}{L_w^{0.95} h_z^{0.2} r_w^{0.01}} \left(\frac{\phi \mu c_t}{k_y t_{ll-ss1i}} \right)^{1.5} \right]^{1/1.7} \quad (80)$$

The time of intersection between the first case of steady-state period and parabolic flow regime provides

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Equation (81) and after replacing the dimensionless quantities allows finding reservoir length:

$$(t_D)^{0.5}_{PB-ss1i} = \frac{501284.3425 b_y^{0.4} h_x^{1.01}}{h_z^{0.58} h_y^{0.65} r_w^{0.76} L_w^{0.57}} \tag{81}$$

$$h_y = \left[\frac{30869463.01 b_y^{0.4} h_x^{1.01} r_w^{0.24}}{h_z^{0.58} L_w^{0.57}}\left(\frac{\phi\mu c_t}{k_y t_{PB-ss1i}}\right)^{0.5}\right]^{1/0.65} \tag{82}$$

Other useful expressions to find reservoir area or reservoir length use the time of intersection of the second case of steady- state period with early linear flow regime -Equation (83), elliptical flow regime -Equation (85), with pseudo radial flow regime, Equation (87) and with late linear flow regime –Equation (89). Then;

$$(t_D)^{1.5}_{el-ss2i} = 437790.9265\frac{L_w^{0.1} b_y^{2.92} h_z^{1.04} A}{h_y^{2.8} r_w^3} \tag{83}$$

$$A = \frac{1.0223546\times10^{11} h_y^{2.8}}{L_w^{0.1} b_y^{2.92} h_z^{1.04}}\left(\frac{k_y t_{el-ss2i}}{\phi\mu c_t}\right)^{1.5} \tag{84}$$

$$(t_D)^{1.36}_{Ell-ss2i} = \frac{957564.524 b_y^{2.92} h_z^{0.04} A}{h_y^{2.8} L_w^{0.18} r_w^{2.72}}\left(\frac{k_x}{k_y}\right)^{0.14} \tag{85}$$

$$A = \frac{h_y^{2.8} L_w^{0.18} k_y^{1.5}}{7.0543706\times10^{10} b_y^{2.92} h_z^{0.04} k_x^{0.14}}\left(\frac{t_{Ell-ss2i}}{\phi\mu c_t}\right)^{1.36} \tag{86}$$

$$t_{DA_{pr-ss2i}} = \frac{1831275.544 b_y^{2.92} h_z^{0.04}}{h_y^{2.8} L_w^{0.9}}\sqrt{\frac{k_x}{k_y}} \tag{87}$$

$$A = \frac{1}{6944541312}\frac{t_{pr-ss2i}}{\phi\mu c_t}\frac{k_y^{1.5} h_y^{2.8} L_w^{0.9}}{k_x^{0.5} b_y^{2.92} h_z^{0.04}} \tag{88}$$

$$(t_D)^{1.5}_{ll-ss2i} = \frac{437790.9265 b_y^{2.92} h_z^{0.04}}{h_y^{1.8} L_w^{0.9}}\frac{h_x^2}{r_w^3} \tag{89}$$

$$h_y = \left[1.0223546\times10^{11}\frac{b_y^{2.92} h_z^{0.04} h_x^2}{L_w^{0.9}}\left(\frac{\phi\mu c_t}{k_y t_{ll-ss2i}}\right)^{1.5}\right]^{1/1.8} \tag{90}$$

Also, reservoir area or reservoir length can be found from the intersection of the second case of steady-state period and parabolic flow regimes. Then:

$$(t_D)^{0.5}_{PB-ss2i} = \frac{189836.3718 A b_y^{0.62}}{h_z^{0.34} h_y^{1.75} L_w^{0.52} r_w^{0.75}} \tag{91}$$

$$A = \frac{h_z^{0.34} h_y^{1.75} L_w^{0.52}}{11690265.11 r_w^{0.25} b_y^{0.62}}\sqrt{\frac{k_y t_{PB-ss2i}}{\phi\mu c_t}} \tag{92}$$

The time of intersection between the third case of steady-state period with early linear flow gives Equation (93), with elliptical flow gives Equation (95), with pseudo radial equation (97), with late linear flow gives Equation (99) and with hemilinear flow gives Equation (101). Once the dimensionless time is replaced in these resulting expressions, reservoir length is solved for:

$$(t_D)^{1.5}_{el-ss3i} = \frac{L_w^{0.85} h_y^{2.7} h_z^{0.14}}{1709.7092 r_w^3} \tag{93}$$

$$h_y = \left[\frac{1}{136.588 L_w^{0.85} h_z^{0.14}}\left(\frac{k_y t_{el-ss3i}}{\phi\mu c_t}\right)^{1.5}\right]^{1/2.7} \tag{94}$$

$$(t_D)^{1.36}_{Ell-ss3i} = \frac{L_w^{0.57} h_y^{2.7} h_z^{0.14}}{781.6655 r_w^{2.72}}\left(\frac{k_x}{k_y}\right)^{0.14} \tag{95}$$

$$h_y = \left[\frac{1}{94.24738}\left(\frac{t_{Ell-ss3i}}{\phi\mu c_t}\right)^{1.36}\frac{k_y^{1.5}}{L_w^{0.57} h_z^{0.14} k_x^{0.14}}\right]^{1/2.7} \tag{96}$$

$$t_{DA_{pr-ss3i}} = \frac{h_y^{1.7} h_z^{0.14}}{408.72887 L_w^{0.15} h_x}\sqrt{\frac{k_x}{k_y}} \tag{97}$$

$$h_y = \left[\frac{k_y^{1.5} t_{pr-ss3i}}{9.278\phi\mu c_t k_x^{0.5}}\frac{L_w^{0.15}}{h_z^{0.14}}\right]^{1/2.7} \tag{98}$$

$$(t_D)^{1.5}_{ll-ss3i} = \frac{1}{1709.7092}\frac{h_y^{2.7} h_z^{0.14} h_x}{r_w^3 L_w^{0.15}} \tag{99}$$

$$h_y = \left[\left(\frac{k_y t_{ll-ss3i}}{\phi\mu c_t}\right)^{1.5}\frac{L_w^{0.15}}{136.588 h_z^{1.14} h_x}\right]^{1/2.7} \tag{100}$$

$$(t_D)^{1.5}_{hl-ss3i} = \frac{1}{5534.821}\frac{h_y^{2.7} h_z^{0.14}}{L_w^{0.15}}\frac{h_x}{r_w^3} \tag{101}$$

$$h_y = \left[\frac{1}{42.1921} \left(\frac{k_y t_{hl-ss3i}}{\phi \mu c_t} \right)^{1.5} \frac{L_w^{0.15}}{h_z^{0.14} h_x} \right]^{1/2.7} \quad (102)$$

Reservoir length is also found from the time of intersection formed by the fourth case of steady-state period with early linear flow - Equation (103) - with elliptical flow -Equation (105) - with pseudo radial flow - Equation (107) – and with late linear flow -Equation (109).

$$(t_D)_{el-ss4i}^{1.5} = \frac{1}{2250.449} \frac{L_w^{0.85} h_y^{2.5} h_z^{0.25} h_x^{0.2}}{r_w^3} \quad (103)$$

$$h_y = \left[\frac{1}{103.7685 L_w^{0.85} h_z^{0.25} h_x^{0.2}} \left(\frac{k_y t_{el-ss4i}}{\phi \mu c_t} \right)^{1.5} \right]^{0.4} \quad (104)$$

$$(t_D)_{Ell-ss4i}^{1.36} = \frac{L_w^{0.57} h_y^{2.5} h_z^{0.25} h_x^{0.2}}{1028.8876 r_w^{2.72}} \left(\frac{k_x}{k_y} \right)^{0.14} \quad (105)$$

$$h_y = \left[\frac{k_y^{1.5}}{71.60153 L_w^{0.57} h_z^{0.25} h_x^{0.2} k_x^{0.14}} \left(\frac{t_{Ell-ss4i}}{\phi \mu c_t} \right)^{1.36} \right]^{0.4} \quad (106)$$

$$t_{DA_{pr-ss4i}} = \frac{1}{538} \frac{h_y^{1.5} h_z^{0.25}}{h_x^{0.8} L_w^{0.15}} \sqrt{\frac{k_x}{k_y}} \quad (107)$$

$$h_y = \left[\frac{k_y^{1.5} t_{pr-ss4i}}{7.04867675 \phi \mu c_t} \frac{L_w^{0.15}}{h_x^{0.2} h_z^{0.25} k_x^{0.5}} \right]^{0.4} \quad (108)$$

$$(t_D)_{ll-ss4i}^{1.5} = \frac{h_y^{2.5} h_x^{1.2} h_z^{0.25}}{2250.449 L_w^{0.15} r_w^3} \quad (109)$$

$$h_y = \left[\frac{1}{103.7685} \left(\frac{k_y t_{ll-ss4i}}{\phi \mu c_t} \right)^{1.5} \frac{L_w^{0.15}}{h_x^{1.2} h_z^{0.25}} \right]^{0.4} \quad (110)$$

Reservoir length is also found from the intersection time points of the fifth case of steady-state period with early linear flow -Equation (111)- with elliptical flow -Equation (113)- with pseudo radial flow - Equation (115)- and with late linear flow (117).

$$(t_D)_{el-ss5i}^{1.5} = \frac{1}{12968.51723} \frac{L_w^{0.85} h_y^{2.5} h_x^{0.2} h_z^{0.25}}{r_w^3} \quad (111)$$

$$h_y = \left[\frac{1}{18} \frac{1}{L_w^{0.85} h_x^{0.2} h_z^{0.25}} \left(\frac{k_y t_{el-ss5i}}{\phi \mu c_t} \right)^{1.5} \right]^{0.4} \quad (112)$$

$$(t_D)_{Ell-ss5i}^{1.36} = \frac{1}{5929.1035} \frac{L_w^{0.57} h_y^{2.5} h_z^{0.25} h_x^{0.2}}{r_w^{2.72}} \left(\frac{k_x}{k_y} \right)^{0.14} \quad (113)$$

$$h_y = \left[\frac{k_y^{1.5}}{12.425137 L_w^{0.57} h_z^{0.25} h_x^{0.2} k_x^{0.14}} \left(\frac{t_{Ell-ss5i}}{\phi \mu c_t} \right)^{1.36} \right]^{0.4} \quad (114)$$

$$t_{DA_{pr-ss5i}} = \frac{1}{3100.2976} \frac{h_z^{0.25} h_y^{1.5}}{L_w^{0.15} h_x^{0.8}} \sqrt{\frac{k_x}{k_y}} \quad (115)$$

$$h_y = \left[\frac{t_{pr-ss5i}}{1.22316906 \phi \mu c_t} \frac{L_w^{0.15} k_y^{1.5}}{h_x^{0.2} h_z^{0.25} k_x^{0.5}} \right]^{0.4} \quad (116)$$

$$(t_D)_{ll-ss5i}^{1.5} = \frac{1}{12968.51723} \frac{h_y^{2.5} h_x^{1.2} h_z^{0.25}}{L_w^{0.15} r_w^3} \quad (117)$$

$$h_y = \left[\frac{1}{18} \left(\frac{k_y t_{ll-ss5i}}{\phi \mu c_t} \right)^{1.5} \frac{L_w^{0.15}}{h_x^{1.2} h_z^{0.25}} \right]^{0.4} \quad (118)$$

The late pseudosteady-state period also intersects with such other flow regimes as early linear –Equation (119)- elliptical –Equation (121)- pseudo radial –Equation (123)- late linear –Equation (125)- and hemilinear – Equation (127). All of these intercepts allow finding reservoir area:

$$(t_D)_{el-pssi}^{0.5} = \frac{1}{8.26143744} \frac{A}{L_w r_w} \sqrt{\pi} \quad (119)$$

$$A = \frac{L_w}{7.454} \left(\frac{k_y t_{el-pssi}}{\phi \mu c_t} \right)^{0.5} \quad (120)$$

$$(t_D)_{Ell-pssi}^{0.64} = \frac{1}{18.07} \frac{A}{L_w^{0.72} r_w^{1.28}} \left(\frac{k_y}{k_x} \right)^{0.14} \quad (121)$$

$$A = \frac{L_w^{0.72} k_y^{0.5} k_x^{0.14}}{10.802676} \left(\frac{t_{Ell-pssi}}{\phi \mu c_t} \right)^{0.64} \quad (122)$$

$$t_{DA_{pr-pssi}} = \frac{1}{34.5575} \sqrt{\frac{k_y}{k_x}} \quad (123)$$

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$$A = \frac{1}{109.735}\frac{t_{pr-pssi}}{\phi\mu c_t}(k_y k_x)^{0.5} \tag{124}$$

$$(t_D)^{0.5}_{ll-pssi} = \frac{1}{8.26143744}\frac{A}{r_w h_x} \tag{125}$$

$$A = \frac{h_x}{7.454}\left(\frac{k_y t_{ll-pssi}}{\phi\mu c_t}\right)^{0.5} \tag{126}$$

$$(t_D)^{0.5}_{hl-pssi} = \frac{1}{2.551962}\frac{A}{r_w h_x} \tag{127}$$

$$A = \frac{h_x}{24.13074}\left(\frac{k_y t_{hl-pssi}}{\phi\mu c_t}\right)^{0.5} \tag{128}$$

Based upon the works of Engler and Tiab (1996b) and Martinez *et al*. (2012), Pabon and Cortes (2016) also developed all the equations for the interpretation of pressure tests in horizontal gas wells using the *TDS* Technique. These equations are reported in Appendix A.

3. SYNTHETIC EXAMPLE

The simulated test reported in Figure-4 was run for a homogeneous, isotropic, oil reservoir which geometry is shown in Figure-1 (A), with the information given below;

B = 1 bbl/STB	ΔP= 300 psia
h_z = 100 ft	μ = 1 cp
L_w = 2000 ft	c_t = 3x10^{-6} psi^{-1}
C = 0 STB/psi	ϕ = 10 %
k = 50 md	r_w = 0.3 ft
h_x = 20000 ft	A = 3840000000 ft^2
h_y = 267000 ft	

Estimate permeability (k), horizontal well length (L_w), Reservoir width (h_x), reservoir length (h_y) using the *TDS* Technique.

Solution

The Characteristic points were read from Figure-4. They are reported in Tables-1 and -2.

Table-1. Characteristic points from Figure-4.

Time, hr		Reciprocal rate, 1/BPD		Reciprocal rate derivative, 1/BPD	
t_{el}	0.04	$(1/q)_{el}$	2.636×10^{-5}	$[t*(1/q)']_{el}$	4.003×10^{-6}
t_{Ell}	2.524	$(1/q)_{Ell}$	7.742×10^{-5}	$[t*(1/q)']_{Ell}$	2.478×10^{-5}
t_{pr}	200.475	$(1/q)_{pr}$	2.447×10^{-4}	$[t*(1/q)']_{pr}$	4.309×10^{-5}
t_{ll}	798.105	$(1/q)_{ll}$	3.091×10^{-4}	$[t*(1/q)']_{ll}$	5.808×10^{-5}
t_{hl}	20047.48	$(1/q)_{hl}$	1.304×10^{-3}	$[t*(1/q)']_{hl}$	9.048×10^{-4}
t_{ss3}	634000	$(1/q)_{ss3}$	7.292×10^{-3}	$[t*(1/q)']_{ss3}$	1.111×10^{-3}

Table-2. Intersection points from Figure-4.

Time intersection (hr)	
$t_{el-ss3i}$	120000
$t_{Ell-ss3i}$	480000
$t_{pr-ss3i}$	19000000
$t_{ll-ss3i}$	400000
$t_{hl-ss3i}$	250000

Table-3. Characteristic maximum points from Figure-4.

Time (hr)		**Transient-rate derivative**	
t_{X3}	252000	$[t*(1/q)']_{X3}$	2.373×10^{-3}

Use of Equations (14), (18), (23), (28) and (32) allow finding and verifying reservoir permeability:

$$k_y = \left(\frac{4.7956(1)}{(100)(300)(4.003\times10^{-6})}\right)^2 \left(\frac{(1)(0.04)}{(2000)^2(0.1)(3\times10^{-6})}\right) = 53.156 \text{ md}$$

$$\sqrt{k_x k_y} = \left(\frac{70.6(1)(1)}{(100)(300)(4.309\times10^{-5})}\right) = 54.614 \text{ md}$$

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$$k_y = \frac{(1)(798.105)}{(0.1)(3\times10^{-6})}\left[\frac{(4.7956)(1)}{(300)(100)(20000)(5.808\times10^{-5})}\right]^2 = 50.381\text{ md}$$

$$k_y = \left[6.9501\frac{(1)^{0.64}(1)}{(300)(2000)^{0.72}(50)^{0.14}(100)(2.478\times10^{-5})}\left(\frac{2.524}{(0.1)(3\times10^{-6})}\right)^{0.36}\right]^{1/0.5}$$

k_y = 49.925 md

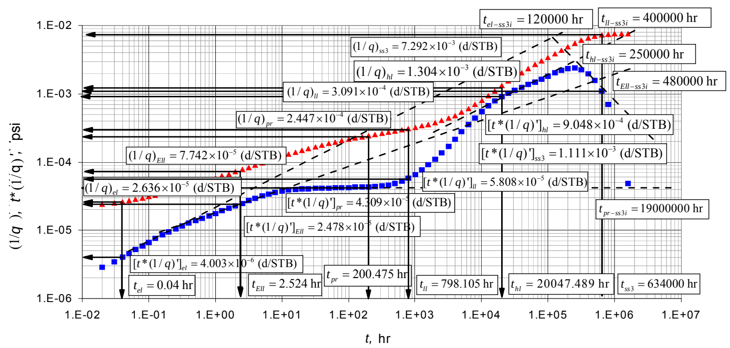

Figure-4. Reciprocal rate and reciprocal rate derivative versus time log-log plot for the synthetic example.

$$k_y = \left(\frac{(1)(20047.48)}{(0.1)(3\times10^{-6})}\right)\left[\frac{15.5249(1)}{(100)(20000)(300)(9.048\times10^{-4})}\right]^2$$

k_y = 54.65 md

Determine horizontal well length with Equations (13) and (27):

$$L_w = \frac{4.7956(1)}{(100)(300)()h_z\Delta P(4.003\times10^{-6})}\sqrt{\frac{(1)(0.04)}{(50)(0.1)(3\times10^{-6})}}$$

L_w = 2062.15 ft

$$L_w = \left[6.9501\frac{(1)^{0.64}(1)}{(300)(50)^{0.5}(50)^{0.14}(100)(2.478\times10^{-5})}\left(\frac{2.524}{(0.1)(3\times10^{-6})}\right)^{0.36}\right]^{1/0.72}$$

$$L_w = 1997.92\text{ ft}$$

Equations (22) and (33) are use to find the reservoir width (h_x):

$$h_x = \frac{4.7956(1)}{(300)(100)(5.808\times10^{-5})}\sqrt{\frac{(1)(798.105)}{(0.1)(50)(3\times10^{-6})}}$$

h_x = 20076.1 ft

$$h_x = \frac{15.5249(1)}{(100)(300)(9.048\times10^{-4})}\sqrt{\frac{(1)(20047.48)}{(50)(0.1)(3\times10^{-6})}}$$

h_x = 20909.29 ft

Equations (44), (94), (96), (98), (100) and (102) are employed to find the reservoir length (h_y):

$$h_y = \left[\frac{(50)^2(2000)^{0.15}(300)(100)^{0.86}(634000)}{655.027(1)^2(1)(0.1)(3\times10^{-6})}(1.111\times10^{-3})\right]^{1/2.7}$$

h_y= 265424.03 ft

$$h_y = \left[\frac{1}{136.588(2000)^{0.85}(100)^{0.14}}\left(\frac{(50)(120000)}{(0.1)(1)(3\times10^{-6})}\right)^{1.5}\right]^{1/2.7}$$

h_y= 285529.33 ft

$$h_y = \left[\frac{1}{94.24738}\left(\frac{480000}{(0.1)(1)(3\times10^{-6})}\right)^{1.36}\frac{(50)^{1.5}}{(2000)^{0.57}(100)^{0.14}(50)^{0.14}}\right]^{1/2.7}$$

h_y= 295964.25 ft

$$h_y = \left[\frac{(50)^{1.5}(19000000)}{9.278(0.1)(1)(3\times10^{-6})(50)^{0.5}}\frac{(2000)^{0.15}}{(100)^{0.14}}\right]^{1/2.7}$$

h_y= 289957.44 ft

$$h_y = \left[\left(\frac{(50)(400000)}{(0.1)(1)(3\times10^{-6})}\right)^{1.5}\frac{(2000)^{0.15}}{136.588(100)^{1.14}(20000)}\right]^{1/2.7}$$

h_y= 324067.7 ft

$$h_y = \left[\frac{1}{42.1921}\left(\frac{(50)(250000)}{(0.1)(1)(3\times10^{-6})}\right)^{1.5}\frac{(2000)^{0.15}}{(100)^{0.14}(20000)}\right]^{1/2.7}$$

h_y= 282699.23 ft

4. COMMENTS ON THE RESULTS

As seen in the detailed example -with one single exception for the estimation of reservoir length- all the estimated parameters agree quite well with the simulated input results. The deviation may be due to the lacking of accuracy on the reading or the approximation is not exact but its result can be considered acceptable.

5. CONCLUSIONS

TDS technique was extended to characterize reciprocal rate and reciprocal rate derivative data in rectangular homogeneous anisotropic reservoirs for horizontal oil and gas wells. Equations were developed to calculate directional permeabilities, horizontal wellbore length, skin factor, reservoir area, etc. For off-centered wellbores along the reservoir length, some flow regimes develop: hemi linear, parabolic, five cases of steady-state and pseudosteady state. The expressions for such flow regimes were developed so they can be used to find reservoir parameters. Because of space-saving reasons, only one synthetic example is presented to demonstrate the accuracy and practicability of the proposed equations.

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Nomenclature

A	Reservoir area, ft^2
B	Volume factor, rb/STB
b_y	Eccentricity well within the reservoir in y-axis, ft
C	Wellbore Storage
c_t	Total system compressibility, psi^{-1}
h_x	Reservoir width, ft
h_y	Reservoir length, ft
h_z	Reservoir thickness, ft
k	Reservoir horizontal permeability, md
L_w	Horizontal well length, ft
P	Pressure, psi
P_i	Initial reservoir pressure, psi
q	Flow rate, BPD
r_w	Wellbore radius , ft
s	Skin factor
s_m	Mechanical skin factor
s_{Ell}	Elliptical pseudoskin factor
s_{hl}	Hemilinear pseudoskin factor
s_{PB}	Parabolic pseudoskin factor

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s_x	x-direction pseudoskin factor
s_z	z-direction pseudoskin factor
T	Reservoir temperature, °R
t	Time, hr
$t_a(P)$	Pseudotime function, (hr)(psi)/cp
t_D	Dimensionless time coordinate
$t_{Da}(P)$	Dimensionless pseudotime function
t_D*P_D'	Dimensionless pressure derivative
$(t*\Delta P')$	Pressure derivative, psi
$t*\Delta m(P)'$	Pseudopressure derivative function, psi2/cp
$t_D*m(P)_D$'	Dimensionless pseudopressure derivative function

Greeks

Δ	Change, drop
ϕ	Porosity, fraction
μ	Viscosity, cp

Suffices

D	Dimensionless
el	Early linear flow period
el-pssi	Intercept of pseudosteady-state and early linear flow
el-ss1i	Intercept of first case of steady state and early linear flow
el-ss2i	Intercept of second case of steady state and early linear flow
el-ss3i	Intercept of third case of steady state and early linear flow
el-ss4i	Intercept of fourth case of steady state and early linear flow
el-ss5i	Intercept of fifth case of steady state and early linear flow
Ell	Elliptical flow period
Ell-pssi	Intercept of pseudosteady-state and elliptical flow
Ell-ss1i	Intercept of first case of steady state and elliptical flow
Ell-ss2i	Intercept of second case of steady state and elliptical flow
Ell-ss3i	Intercept of third case of steady state and elliptical flow
Ell-ss4i	Intercept of fourth case of steady state and elliptical flow
Ell-ss5i	Intercept of fifth case of steady state and elliptical flow
er	Early radial flow period
er-eli	Intercept of early radial and early linear flow
er-Elli	Intercept of early radial and elliptical flow
er-hli	Intercept of early radial and hemi linear flow
er-lli	Intercept of early radial and late linear flow
er-PBi	Intercept of early radial and parabolic flow
er-pri	Intercept of early radial and pseudo radial flow
er-pssi	Intercept of early radial and pseudosteady-state
er-ss1i	Intercept of early radial and first case of steady state
er-ss2i	Intercept of early radial and second case of steady state
er-ss3i	Intercept of early radial and third case of steady state
er-ss4i	Intercept of early radial and fourth case of steady state
er-ss5i	Intercept of early radial and fifth case of steady state
g	Gas
hl	Hemilinear flow period
hl-pssi	Intercept of pseudosteady-state and hemi linear flow
hl-ss3i	Intercept of third case of steady state and hemi linear flow
i	Intersection
ll	Late linear flow period
ll-pssi	Intercept of pseudosteady-state and late linear flow
ll-ss1i	Intercept of first case of steady state and late linear flow
ll-ss2i	Intercept of second case of steady state and late linear flow
ll-ss3i	Intercept of third case of steady state and late linear flow
ll-ss4i	Intercept of fourth case of steady state and late linear flow
ll-ss5i	Intercept of fifth case of steady state and late linear flow
PB	Parabolic flow period
PB-ss1i	Intercept of first case of steady state and parabolic flow
PB-ss2i	Intercept of second case of steady state and parabolic flow
pr	Pseudoradial flow period
pr-pssi	Intercept of pseudosteady-state and pseudo radial flow
pr-ss1i	Intercept of first case of steady state and pseudo radial flow
pr-ss2i	Intercept of second case of steady state and pseudo radial flow
pr-ss3i	Intercept of third case of steady state and pseudo radial flow
pr-ss4i	Intercept of fourth case of steady state and pseudo radial flow
pr-ss5i	Intercept of fifth case of steady state and pseudo radial flow
pr-pssi	Intercept of pseudosteady-state and pseudo radial flow
pss	Pseudosteady state
ss	Steady-state
ss1	First case of steady-state
ss2	Second case of steady-state
ss3	Third case of steady-state
ss4	Fourth case of steady-state
ss5	Fifth case of steady-state
t	Total
w	Well

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x	x-direction index
y	y-direction index
z	z-direction index

Appendix A. Gas reservoir equations

Dimensionless pressure derivative:

$$[t_D * m(1/q_D)']_{er} = \frac{1}{2}\sqrt{\frac{k_y}{k_z}} \quad (A.1)$$

$$[t_D * m(1/q_D)']_{el} = \frac{118 r_w}{100 h_z}\sqrt{\pi t_{Da_{el}}} \quad (A.2)$$

$$[t_D * m(1/q_D)']_{Ell} = 0.95621522 \frac{r_w^{0.72} L_w^{0.28}}{h_z} \left(\frac{k_y}{k_x}\right)^{0.14} (t_{Da_{Ell}} \quad (A.3)$$

$$[t_D * m(1/q_D)']_{pr} = \frac{1}{2}\frac{L_w}{h_z}\sqrt{\frac{k_y}{k_x}} \quad (A.4)$$

$$[t_D * m(1/q_D)']_{ll} = \frac{118}{100}\left(\frac{L_w r_w}{h_x h_z}\right)\sqrt{\pi t_{Da_{ll}}} \quad (A.5)$$

$$[t_D * m(1/q_D)']_{hl} = \frac{382\sqrt{\pi} L_w}{100 h_z}\left[\left(\frac{r_w}{h_x}\right)^2 t_{Da_{hl}}\right]^{0.5} \quad (A.6)$$

$$[t_D * m(1/q_D)']_{PB} = 4.8233 \frac{L_w^{0.62} b_y^{2.3}}{h_z^{0.62} h_y^{1.05} r_w^{1.25}} (t_{Da_{PB}})^{-0.5} \quad (A.7)$$

$$[t_D * m(1/q_D)']_{ss1} = 2417844.769 \frac{L_w^{0.05}}{h_z^{1.2}} \left(\frac{b_y}{h_y}\right)^{2.7} \left(\frac{h_x}{r_w}\right)^{0.01} t_{DaA_{ss1}}^{-1} \quad (A.8)$$

$$[t_D * m(1/q_D)']_{ss2} = 915637.772 \frac{L_w^{0.1} b_y^{2.92}}{h_z^{0.96} h_y^{2.8}} t_{DaA_{ss2}}^{-1} \quad (A.9)$$

$$[t_D * m(1/q_D)']_{ss3} = \frac{1}{817.45774} \frac{L_w^{0.85} h_y^{1.7}}{h_z^{0.86} h_x} t_{DaA_{ss3}}^{-1} \quad (A.10)$$

$$[t_D * m(1/q_D)']_{ss4} = \frac{1}{1076} \frac{L_w^{0.85} h_y^{1.5}}{h_z^{0.75} h_x^{0.8}} (t_{DaA})_{ss4}^{-1} \quad (A.11)$$

$$[t_D * m(1/q_D)']_{ss5} = \frac{1}{6200.5952} \frac{L_w^{0.85} h_y^{1.5}}{h_z^{0.75} h_x^{0.8}} (t_{DaA})_{ss5}^{-1} \quad (A.12)$$

$$[t_D * m(1/q_D)']_{pss} = \frac{55}{10}\pi\left(\frac{L_w}{h_z}\right)(t_{DaA})_{pss} \quad (A.13)$$

TDS technique

$$\sqrt{k_y k_z} = \frac{711.26T}{L_w \Delta P [t * m(1/q)']_{er1}} \quad (A.14)$$

$$k_y = \left(\frac{t_a(P)_{el}}{\phi}\right)\left[\frac{48.3137T}{L_w \Delta P h_z [t * m(1/q)']_{el}}\right]^2 \quad (A.15)$$

$$L_w = \left[70.0186 \frac{T}{k_y^{0.5} k_x^{0.14} \Delta P h_z [t * m(1/q)']_{Ell}} \left[\frac{t_a(P)_{Ell}}{\phi}\right]^{0.36}\right]^{1/0.72} \quad (A.16)$$

$$\sqrt{k_y k_x} = \frac{711.26TL_w}{h_z L_w \Delta P [t * m(1/q)']_{pr}} \quad (A.17)$$

$$h_x = \left(\frac{48.3137T}{h_z \Delta P [t * m(1/q)']_{ll}}\right)\sqrt{\left(\frac{t_a(P)_{ll}}{k_y \phi}\right)} \quad (A.18)$$

$$h_x = \frac{156.4054T}{h_z \Delta P [t * m(1/q)']_{hl}}\sqrt{\frac{t_a(P)_{hl}}{k_y \phi}} \quad (A.19)$$

$$b_y = \left[\frac{\frac{1}{422520.3114}\frac{\Delta P k_y^{1.5} L_w^{0.38} h_z^{0.62} h_y^{1.05} r_w^{0.25}}{T}}{[t * m(1/q)']_{PB}\left(\frac{t_a(P)_{PB}}{\phi}\right)^{0.5}}\right]^{1/2.3} \quad (A.20)$$

$$h_y = \left[\frac{1.3043 \times 10^{13} \phi T b_y^{2.7} h_x^{1.01}}{k_y^2 L_w^{0.95} \Delta P h_z^{1.2} r_w^{0.01} [t * m(1/q)']_{ss1} t_a(P)_{ss1}}\right]^{1/1.7} \quad (A.21)$$

$$A = \frac{k_y^2 L_w^{0.9} \Delta P [t * m(1/q)']_{ss2}}{4.93937445 \times 10^{12} T} \frac{h_z^{0.96} h_y^{2.8}}{b_y^{2.92}}\left[\frac{t_a(P)_{ss2}}{\phi}\right] \quad (A.22)$$

$$h_y = \left[\frac{k_y^2 L_w^{0.15} \Delta P h_z^{0.86} t_a(P)_{ss3}}{6599.07313 T \phi}[t * m(1/q)']_{ss3}\right]^{1/2.7} \quad (A.23)$$

$$h_y = \left[\frac{k_y^2 L_w^{0.15} \Delta P h_z^{0.75} [t*m(1/q)']_{ss4}}{5013.44183 h_x^{0.2}} \left(\frac{t_a(P)_{ss4}}{\phi T} \right) \right]^{0.4} \quad (A.24)$$

$$h_y = \left[\frac{k_y^2 L_w^{0.15} \Delta P h_z^{0.75} [t*m(1/q)']_{ss5}}{869.9912 h_x^{0.2}} \left(\frac{t_a(P)_{ss5}}{\phi T} \right) \right]^{0.4} \quad (A.25)$$

$$A = \frac{1}{6.48158} \left(\frac{T}{h_z \Delta P [t*m(1/q)']_{pss}} \right) \left(\frac{t_a(P)_{pss}}{\phi} \right) \quad (A.26)$$

A.1. Intersections

$$k_z = 301.7727 \phi \frac{h_z^2}{t_a(P)_{er-eli}} \quad (A.27)$$

$$k_z = \left[10.158152 \frac{k_x^{0.14} h_z}{L_w^{0.28}} \left(\frac{\phi}{t_a(P)_{er-Elli}} \right)^{0.36} \right]^2 \quad (A.28)$$

$$k_z = 216.7285 \frac{\phi}{t_a(P)_{er-lli}} \left(\frac{h_x h_z}{L_w} \right)^2 \quad (A.29)$$

$$k_z = 20.6801 \frac{\phi}{t_a(P)_{er-hli}} \left(\frac{h_x h_z}{L_w} \right)^2 \quad (A.30)$$

$$b_y = \left[\frac{1}{594.0448} \frac{k_y h_z^{0.62} h_y^{1.05} r_w^{0.25}}{k_z^{0.5} L_w^{0.62}} \left(\frac{t_a(P)_{er-PBi}}{\phi} \right)^{0.5} \right]^{1/2.3} \quad (A.31)$$

$$h_y = \left[\left(\frac{1.8337844 \times 10^{10} \phi}{t_a(P)_{er-ss1i}} \right) \frac{k_z^{0.5} h_x^{1.01} L_w^{0.05} b_y^{2.7}}{k_y^{1.5} h_z^{1.2} r_w^{0.01}} \right]^{1/1.7} \quad (A.32)$$

$$A = \left(\frac{t_a(P)_{er-ss2i}}{6944541312 \phi} \right) \frac{k_y^{1.5} h_z^{0.96} h_y^{2.8}}{k_z^{0.5} L_w^{0.1} b_y^{2.92}} \quad (A.33)$$

$$h_y = \left[\frac{1}{9.278} \frac{k_y^{1.5} h_z^{0.86}}{k_z^{0.5} L_w^{0.85}} \left(\frac{t_a(P)_{er-ss3i}}{\phi r_w} \right) \right]^{1/2.7} \quad (A.34)$$

$$h_y = \left[\frac{1}{7.048676} \frac{k_y^{1.5} h_z^{0.75}}{k_z^{0.5} L_w^{0.85} h_x^{0.2}} \left(\frac{t_a(P)_{er-ss4i}}{\phi} \right) \right]^{1/2.5} \quad (A.35)$$

$$h_y = \left[\frac{1}{1.22317} \frac{k_y^{1.5} h_z^{0.75}}{k_z^{0.5} L_w^{0.85} h_x^{0.2}} \left(\frac{t_a(P)_{er-ss5i}}{\phi} \right) \right]^{0.4} \quad (A.36)$$

$$A = \frac{1}{109.7356} \frac{t_a(P)_{er-pssi}}{\phi} \sqrt{k_y k_z} \left(\frac{L_w}{h_z} \right) \quad (A.37)$$

$$h_y = \left[\frac{3.1855783 \times 10^{11} \dfrac{L_w^{0.05} h_x^{1.01} b_y^{2.7} r_w^{0.01}}{h_z^{0.2}}}{\left(\dfrac{\phi}{k_y t_a(P)_{el-ss1i}} \right)^{1.5}} \right]^{1/1.7}$$

(A.38)

$$h_y = \left[\frac{1.8315444 \times 10^{11} b_y^{2.7} h_x^{1.01} k_x^{0.14}}{h_z^{0.2} r_w^{0.01} L_w^{0.275} k_y^{1.5}} \left(\frac{\phi \mu c_t}{t_{Ell-ss1i}} \right)^{1.36} \right]^{1/1.7} \quad (A.39)$$

$$h_y = \left[\frac{1.83378443 \times 10^{10} k_x^{0.5} \phi \mu c_t h_x^{1.01} b_y^{2.7}}{h_z^{0.2} L_w^{0.95} k_y^{1.5} r_w^{0.01} t_{pr-ss1i}} \right]^{1/1.7}$$

(A.40)

$$h_y = \left[2.69964 \frac{h_x^{2.01} b_y^{2.7}}{L_w^{0.95} h_z^{0.2} r_w^{0.01}} \left(\frac{\phi \mu c_t}{k_y t_{ll-ss1i}} \right)^{1.5} \right]^{1/1.7} \quad (A.41)$$

$$h_y = \left[\frac{\dfrac{30869463.01 b_y^{0.4} h_x^{1.01} r_w^{0.24}}{h_z^{0.58} L_w^{0.57}}}{\left(\dfrac{\phi}{k_y t_a(P)_{PB-ss1i}} \right)^{0.5}} \right]^{1/0.65} \quad (A.42)$$

$$A = \frac{1}{1.206378 \times 10^{11}} \left(\frac{k_y t_a(P)_{el-ss2i}}{\phi} \right)^{1.5} \frac{h_y^{2.8}}{L_w^{0.1} b_y^{2.92} h_z^{1.04}} \quad (A.43)$$

$$A = \frac{1}{7.0543706 \times 10^{10}} \left(\frac{t_a(P)_{Ell-ss2i}}{\phi} \right)^{1.36} \frac{h_y^{2.8} L_w^{0.18} k_y^{1.5}}{b_y^{2.92} h_z^{0.04} k_x^{0.14}} \quad (A.44)$$

$$A = \frac{1}{6944541312} \frac{t_a(P)_{pr-ss2i}}{\phi} \frac{k_y^{1.5} h_y^{2.8} L_w^{0.9}}{k_x^{0.5} b_y^{2.92} h_z^{0.04}} \quad (A.45)$$

$$h_y = \left[1.0223546 \times 10^{11} \frac{b_y^{2.92} h_z^{0.04} h_x^2}{L_w^{0.9}} \left(\frac{\phi}{k_y t_a(P)_{ll-ss2i}} \right)^{1.5} \right]^{1/1.8} \quad (A.46)$$

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$$A=\frac{1}{11690265.11}\left(\frac{k_y t_a(P)_{PB-ss2i}}{\phi}\right)^{0.5}\frac{h_z^{0.34}h_y^{1.75}L_w^{0.52}}{r_w^{0.25}b_y^{0.62}} \quad (A.47)$$

$$h_y=\left[\frac{1}{161.1738553L_w^{0.85}h_z^{0.14}}\left(\frac{k_y t_a(P)_{el-ss3i}}{\phi}\right)^{1.5}\right]^{1/2.7} \quad (A.48)$$

$$h_y=\left[\frac{1}{94.24738}\left(\frac{t_a(P)_{Ell-ss3i}}{\phi}\right)^{1.36}\frac{k_y^{1.5}}{L_w^{0.57}h_z^{0.14}k_x^{0.14}}\right]^{1/2.7} \quad (A.49)$$

$$h_y=\left[\frac{1}{9.278}\frac{k_y^{1.5}t_a(P)_{pr-ss3i}}{\phi k_x^{0.5}}\frac{L_w^{0.15}}{h_z^{0.14}}\right]^{1/2.7} \quad (A.50)$$

$$h_y=\left[\frac{1}{136.588}\left(\frac{k_y t_a(P)_{ll-ss3i}}{\phi}\right)^{1.5}\frac{L_w^{0.15}}{h_z^{1.14}h_x}\right]^{1/2.7} \quad (A.51)$$

$$h_y=\left[\frac{1}{42.1921}\left(\frac{k_y t_a(P)_{hl-ss3i}}{\phi}\right)^{1.5}\frac{L_w^{0.15}}{h_z^{0.14}h_x}\right]^{1/2.7} \quad (A.52)$$

$$h_y=\left[\frac{1}{122.447L_w^{0.85}h_z^{0.25}h_x^{0.2}}\left(\frac{k_y t_a(P)_{el-ss4i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.53)$$

$$h_y=\left[\frac{1}{71.60153}\left(\frac{t_a(P)_{Ell-ss4i}}{\phi}\right)^{1.36}\frac{k_y^{1.5}}{L_w^{0.57}h_z^{0.25}h_x^{0.2}k_x^{0.14}}\right]^{0.4} \quad (A.54)$$

$$h_y=\left[\frac{1}{7.04867675}\frac{k_y^{1.5}t_a(P)_{pr-ss4i}}{\phi}\frac{L_w^{0.15}}{h_x^{0.2}h_z^{0.25}k_x^{0.5}}\right]^{0.4} \quad (A.55)$$

$$h_y=\left[\frac{1}{103.7685}\left(\frac{k_y t_a(P)_{ll-ss4i}}{\phi}\right)^{1.5}\frac{L_w^{0.15}}{h_x^{1.2}h_z^{0.25}}\right]^{0.4} \quad (A.56)$$

$$h_y=\left[\frac{1}{21.2484L_w^{0.85}h_x^{0.2}h_z^{0.25}}\left(\frac{k_y t_a(P)_{el-ss5i}}{\phi}\right)^{1.5}\right]^{0.4} \quad (A.57)$$

$$h_y=\left[\frac{1}{12.425137}\left(\frac{t_a(P)_{Ell-ss5i}}{\phi}\right)^{1.36}\frac{k_y^{1.5}}{L_w^{0.57}h_z^{0.25}h_x^{0.2}k_x^{0.14}}\right]^{0.4} \quad (A.58)$$

$$h_y=\left[\frac{1}{1.22316906}\frac{t_a(P)_{pr-ss5i}}{\phi}\frac{L_w^{0.15}k_y^{1.5}}{h_x^{0.2}h_z^{0.25}k_x^{0.5}}\right]^{0.4} \quad (A.59)$$

$$h_y=\left[\frac{1}{18}\left(\frac{k_y t_a(P)_{ll-ss5i}}{\phi}\right)^{1.5}\frac{L_w^{0.15}}{h_x^{1.2}h_z^{0.25}}\right]^{0.4} \quad (A.60)$$

$$A=\frac{L_w}{11.1965}\left(\frac{k_y t_a(P)_{el-pssi}}{\phi}\right)^{0.5} \quad (A.61)$$

$$(t_{Da})_{Ell-pssi}^{0.64}=\frac{1}{18.07}\frac{A}{L_w^{0.72}r_w^{1.28}}\left(\frac{k_y}{k_x}\right)^{0.14} \quad (A.62)$$

$$A=\frac{1}{109.735}\frac{t_a(P)_{pr-pssi}}{\phi}(k_y k_x)^{0.5} \quad (A.63)$$

$$A=\frac{h_x}{7.454}\left(\frac{k_y t_a(P)_{ll-pssi}}{\phi}\right)^{0.5} \quad (A.64)$$

$$A=\frac{h_x}{24.13074}\left(\frac{k_y t_a(P)_{hl-pssi}}{\phi}\right)^{0.5} \quad (A.65)$$

A.2. Skin factors

$$(s_z+s_m')=\left[\frac{1}{29.4434h_z}\sqrt{\left(\frac{k_z t_a(P)_{el}}{\phi}\right)}\right]\left[\frac{m(1/q)_{el}}{[t*m(1/q)']_{el}}-2\right] \quad (A.66)$$

$$s_{Ell}=\left[\frac{1}{20.3163}\frac{k_y^{0.5}L_w^{0.28}}{k_x^{0.14}h_z}\left(\frac{t_a(P)_{Ell}}{\phi}\right)^{0.36}\right]\left[\frac{m(1/q)_{Ell}}{[t*m(1/q)']_{Ell}}-2.777\right] \quad (A.67)$$

$$(s_m'+s_z)=\frac{1}{2}\frac{L_w}{h_z}\sqrt{\frac{k_z}{k_y}}\left[\frac{m(1/q)_{pr}}{[t*m(1/q)']_{pr}}-\ln\left(\frac{k_x t_a(P)_{pr}}{L_w^{\,2}\phi}\right)+4.659\right] \quad (A.68)$$

$$(s_x+s_z+s_m')=\frac{1}{29.4434}\left(\frac{L_w}{h_x h_z}\right)\sqrt{\left(\frac{k_z t_a(P)_{ll}}{\phi}\right)}\left[\frac{m(1/q)_{ll}}{[t*m(1/q)']_{ll}}-2\right] \quad (A.69)$$

$$(s_x+s_z+s_m'+s_{hl})=\frac{L_w}{9.0951h_z h_x}\sqrt{\left(\frac{k_z t_a(P)_{hl}}{\phi}\right)}\left[\frac{m(1/q)_{hl}}{[t*m(1/q)']_{hl}}-2\right] \quad (A.70)$$

$$(s_x+s_z+s_m'+s_{PB})=\left[\frac{297.0224L_w^{0.62}b_y^{2.3}}{k_y h_z^{0.62}h_y^{1.05}r_w^{0.25}}\sqrt{\frac{k_z\phi}{t_a(P)_{PB}}}\right] \quad (A.71)$$

$$\left[\frac{m(1/q)_{PB}}{[t*m(1/q)']_{PB}}+2\right]$$