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## OPTIMIZATION OF ECONOMIC ENVIRONMENTAL DISPATCH (EED) USING RECENT MODIFIED ABC VARIANTS

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#### ABSTRACT

This paper presents a study on optimization of a complex real-world application; economic environmental dispatch (EED) using two recently proposed modified ABC variants, named as JA-ABC3 and JA-ABC4a. EED is known to be a problem that is difficult yet crucial to be solved. Thus, robust optimization algorithms are needed to solve this kind of problem. JA-ABC3 and JA-ABC4a are recently proposed ABC variants claimed to perform better than the standard ABC as well as other existing variants. Their performances in optimizing EED are compared with the standard ABC algorithm, the existing ABC variants as well as other optimization algorithms. Those observations and evaluations are done in order to identify which variants would give the best optimal solution. The results clearly depict the superior performances of JA-ABC3 and JA-ABC4a in comparison to others in solving 10-generator unit system EED problem.

Keywords: ABC variants, economic environmental dispatch, optimization.

#### INTRODUCTION

Economic environmental dispatch (EED) is known to be one of the complex power system problems to be solved. It is vital to be solved since it affects cost and the atmosphere. EED arose from the awareness of the emission of toxic gases during electricity generation (Breeze, 2005), (Po-Hung and Hong-Chan, 1995), (Sinha et al., 2003). Electricity generation from the combustion of fossil fuels has not only generated electricity, but other situation that create worst pollution to big cities. One of the effects is the emission of toxic gases. Toxic gases such as sulfur oxides (SO<sub>x</sub>), nitrogen oxides (NO<sub>x</sub>) and carbon dioxide (CO<sub>2</sub>), have affected the environment including the human beings (Breeze, 2005). Thus, it is necessary to efficiently minimize toxic gases emission to the atmosphere. In short, EED concerns minimizing toxic gasses and operating costs during electricity generation.

Many researchers attempted to solve the problem using various techniques as shown in the works of (Basu, 2011), (Kumar et al., 2008), (Nanda et al., 1988). Nevertheless, most of the techniques considered the problem as a single objective problem, thus requiring assumptions to be made (Farag et al., 1995). Meanwhile, there were also techniques that considered it as multiobjective problem. However, they were found to be computationally intensive and time consuming (Basu, 2011).

Thus, with the motivation from those problems, this research work is focusing on recently proposed modified ABC variants namely the JA-ABC3 (Sulaiman et al., 2015) and JA-ABC4a (Sulaiman et al., 2014a) to solve EED. ABC variants are developed with the aim to overcome the problem faced by the standard ABC algorithm; slow convergence speed and premature convergence. ABC algorithm is a swarm-intelligencebased algorithm that is inspired from the intelligence foraging behavior of honeybees. The effectiveness of ABC in solving numerous applications such as in the works of (Ayan and Kılıç, 2012), (Karaboga et al., 2012),

(Rodriguez et al., 2013) have driven the optimization researchers to carry out further research on ABC. Nonetheless, it actually suffers from slow convergence rate on unimodal functions (Gao and Liu, 2012), (Gao et al., 2012) and tendency for premature convergence on complex multimodal (Li et al., 2012). This is basically due to its mutation equation that is known to be good at exploration but poor at exploitation.

optimization Hence. researchers have implemented modifications to the standard ABC algorithm whether by introducing modified mutation equations, inserting new stages or both. The development of the ABC variants such as g-best guided ABC (GABC) (Zhu and Kwong, 2010), global best ABC (BABC1 and BABC2) (Gao et al., 2012), balanced ABC (BABC) (Jagdish Chand et al., 2013) are aimed to cater the problems. However, none of them are able to solve the problems simultaneously. Recently, JA-ABC variants have been proposed to overcome the problems (Sulaiman et al., 2015), (Sulaiman et al., 2014a) and (Sulaiman et al., 2014b). Among them are JA-ABC3 and JA-ABC4a. These variants are applied to solve EED problem since they have shown superior performance than the other variants (Sulaiman et al., 2015), (Sulaiman et al., 2014a).

This paper is organized into six sections. This section briefly introduced the root of EED, ABC and its variants. The next section provides the explanation regarding EED problem and its problem formulation, followed by ABC variants which are going to be used to solve EED problem. Then, the experimental setup will be explained before discussing the obtained results in section five. Lastly, conclusions will be made to summarize the whole findings.

### ECONOMIC ENVIRONMENTAL DISPATCH (EED)

Economic environmental dispatch (EED) aims to minimize the operating cost and toxic gases emission by determining the committed generating units to meet the demand and restricted to the system constraints. The

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constraints include power balance and active power lower and upper limit (Po-Hung and Hong-Chan, 1995), (Sinha et al., 2003). This makes EED a constrained-based multiobjective problem, which its objectives are contradicted to each other (Basu, 2011). Hence, EED is a complex and hard problem to be solved.

Researchers implemented have various techniques to solve this problems, as shown in the works of (Nanda et al., 1988), (Basu, 2011) and many more. Nevertheless, most of the techniques treated it as a single objective problem, thus assumptions have to be made (Farag et al., 1995). There are techniques to solve EED as multi-objective problem as shown in the works of (Basu, 2011), (Das and Patvardhan, 1998). However, they have been detected to be lacked in efficiency since they are computationally intensive.

#### **Problem formulation**

The general mathematical formulation of a multiobjective problem can be written as follows:

 $\min(f(x))$ Such that

$$g(x) = 0 \text{ and}$$

$$h(x) \le 0$$
(1)

where f(x) is the objective function to be minimized, g(x)is the equality constraints and h(x) is the inequality constraints.

#### Objective function for cost

The mathematical formulation of the operating cost is as follows:

$$F_{i}(P_{i}) = \sum_{i=1}^{N} \left[ a_{i} P_{i} + b_{i} P_{i} + c_{i} \right]$$
 (2)

where  $P_i$  is the power generated by *i*-th generator unit,  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients of *i*-th generator unit and N is the total number of generator unit.

In this work, the valve point effect is being considered since it can cause higher order nonlinearity to the objective function (Coelho and Mariani, 2010). The cost objective function after considering the effect is given

$$F_i(P_i) = \sum_i [a_i P_i + b_i P_i + c_i + |d_i \sin\{e_i (P_i^{\min} - P_i)\}]$$
 (3)

where  $d_i$  and  $e_i$  are the fuel coefficients of *i*-th generator unit that show the valve point effect (Borckmans et al., 2014), (Coelho and Mariani, 2010). The rest of the parameters are the same as in (2).

#### Objective function for emission

The second objective of the problem is to minimize the emission of the hazardous gases into the atmosphere. The mathematical formulation of the emission is given by:

$$F_i(P_i) = \sum_{i=1}^{N} \left[ \gamma_i P_i^2 + \beta_i P_i + \alpha_i + \left| \eta_i \exp(\delta_i P_i) \right| \right]$$
 (4)

where  $P_i$  is the power generated by *i*-th generator unit,  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\delta_i$  and  $\eta_i$  are the emission coefficients of *i*-th generator unit and N is the total number of generator unit.

### **Equality constraint**

The control variable of EED problem; power generated by each of the committed generating units needs to satisfy the following equality constraints:

$$\sum_{i=1}^{N} P_i - P_D - P_L = 0 (5)$$

where  $P_i$  is the power generated by i-th generator unit,  $P_D$  is the active power demand and  $P_L$  is the transmission line losses. The rest of the parameters are the same as in (2).

Thus, the power generated by each of the committed generator unit has to satisfy this active power balance.

### **Inequality constraints**

The inequality constraint of EED is given by:

$$P_i^{\min} \le P_i \le P_i^{\max} \quad i = 1, \dots, N \tag{6}$$

where  $P_i^{min}$  is the lower limit of active power generated by *i*-th generator unit,  $P_i^{max}$  is the upper limit of active power generated by i-th generator unit and the rest of the parameters are similar to (2).

Hence, the power generated at each committed generating units are constrained within their upper and lower limits.

## **Penalty function**

Since EED is a multi-objective function with equality and inequality constraints, penalty term must be added in order to convert EED to a single objective function. The penalty function of EED problem, which includes the two objective functions and penalty term, is given by:

$$P(x) = \sum_{i=1}^{N} F_i(P_i) + \sum_{i=1}^{N} E_i(P_i) + K \times \left\{ \left( \sum_{i=1}^{N} P_i \right) - P_L - P_D \right\}^2$$
 (7)

where  $F_i$  is the cost function,  $E_i$  is the emission function,  $P_i$  is the active power generated at *i*-th generator unit, K is the penalty term of the penalty function,  $P_L$  is the power losses of the transmission lines and  $P_D$  is the power demand. This equation is the final equation that will be used as the objective function to be solved by the ABC variants.

More details of the problem formulation can be found in (Basu, 2011), (Coelho and Mariani, 2010).

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### ABC VARIANTS FOR SOLVING EED

As previously mentioned, ABC variants have been developed with the aims to overcome the limitations faced by ABC algorithm. However, most of the variants are unable to solve the problem simultaneously. The recently proposed variants have been claimed to perform better than the existing variants. In this work, JA-ABC3 (Sulaiman et al., 2015) and JA-ABC4a (Sulaiman et al., 2014a) were applied to solve the EED problem.

JA-ABC3 has been proposed by Sulaiman et al. in 2014 through three modifications to the standard ABC algorithm. The first modification is the insertion of proposed new stages between the initialization and employed-bees phases. The aim of these new stages is to enhance the average fitness of the population at the early phase of the algorithm. Thus, the algorithm is expected to converge faster since the solutions are being directed to the best food source. Second, the mutation equation of the employed-bees phase has been replaced with a modified mutation equation obtained from the work of (Gao and Liu, 2011):

$$z_{ij} = y_{kj} + \phi_{ij} (y_{ij} - y_{pj})$$
 (8)

where  $z_{ij}$  is the candidate solution of new food sources,  $y_{kj}$  and  $y_{pj}$  are the k-th and p-th food sources of j-th dimension, respectively. p and k are randomly chosen food sources and they are mutually exclusive while  $\emptyset_{ii}$  is a control parameter that represents random numbers within [-1, 1], inclusively.

The equation is known for its randomness, thus it can enhance the exploration process of the algorithm. This action will cater the problem that might be faced by the algorithm due to the first modification; premature convergence. By enhancing the exploration process of the algorithm, it should be able to avoid local minima traps. The third modification is in onlooker-bees phase. In this phase, onlooker-bees have been directed to update only few most-fit-selected food sources instead of updating all selected food sources. Thus, the algorithm is expected to converge faster since less number of food sources to be updated. More details on JA-ABC3 can be found in (Sulaiman et al., 2015).

JA-ABC4a has been developed by modifying the mutation equation of employed-bees phase of JA-ABC3. Instead of using the existing mutation equation, new mutation equation has been proposed. The proposed mutation equation is inspired from the work of (Gao and Liu, 2011) and (Abro and Mohamad-Saleh, 2012). Two mutation equations found in both works have been hybridized to produce modified mutation equation as given by:

$$z_{ij} = y_{best,j} + \phi_{ij} (y_{ij} - y_{r1j}) + \psi_{ij} (y_{r2j} - y_{best,j})$$
(9)

where  $z_{ij}$  is the candidate solution of food sources.  $y_{rl,j}$  and  $y_{r2,j}$  are rl-th and r2-nd possible solution with j-th dimension. Subscripts r1 and r2 are refer to the mutually exclusive possible solution,  $\psi_{ij}$  is a random number within

[0, T], where T is a user-defined number. The rest of the parameters are the same as (8). The first and third terms of the equation are inspired from (Abro and Mohamad-Saleh, 2012). They exploit the information of the best food source in the current population, and thus enhance the exploitation capability of the algorithm, resulting faster convergence rate. The second term of the equation is taken from (Gao and Liu, 2011). It is known for its randomness; hence the exploration process can be enhanced. The result is the ability of the algorithm to avoid premature convergence that might be caused by the first and third terms. The hybridization of the equations is expected to exhibit balanced exploration and exploitation capabilities of the algorithm. This way, it is expected to be able to faster while diligently obtaining global converge minimum.

Besides that, the existing sophisticated ABC variants have been used for the comparison purpose. They are global best ABC (BABC1) (Gao et al., 2012), improved ABC (IABC) (Gao and Liu, 2011) and enhanced ABC (EABC) (Abro, 2013). They have been chosen based on the experimental analysis that have been carried out.

BABC1 is recommended by Gao et al. in 2012 (Gao et al., 2012). They have introduced the new modified solution search equation into the algorithm besides applying chaotic systems and opposition-based learning method in initialization and scout-bee phase. The equation directs the candidate solution towards global best solution, which is aimed to enhance the exploitation capability of the algorithm.

IABC algorithm has been introduced by Gao and Liu in 2011 (Gao and Liu, 2011). Two improved mutation equations have been proposed; one is to enhance the convergence rate of the algorithm and the other is to randomly search for the candidate solution, thus avoid premature convergence. They emerged both equations using parameter p to ensure that the algorithm exhibit the advantages of both equations. Besides, they have also introduced chaotic systems and the opposition-based learning method during initialization.

EABC is proposed by Abro in 2013 (Abro, 2013). They have introduced new mutation equations during employed, onlooker and scout-bees phases. The equation aims to generate the candidate solution around gbest food source, and thus speed up the algorithm. Besides, to avoid premature convergence, they have introduced a novel Elite-update (EU) scheme in onlookerbees phase. Meanwhile, in scout-bee phase, intelligent scout-bee has been proposed with the aim to enhance the performance of the scout-bee phase.

#### **EXPERIMENTAL SETTINGS**

For the purpose of optimizing the EED problem, 10-generator unit system has been used. The dataset can be found in (Basu, 2011).

The performances of JA-ABC3 and JA-ABC4a at solving EED have been compared with the standard ABC (ABC) (Karaboga and Akay, 2009) and three sophisticated existing ABC variants available in the literature, i.e. BABC1 (Gao et al., 2012), IABC (Gao and Liu, 2011) and

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EABC (Abro, 2013). Besides, their performances have also been compared with the existing optimization algorithms; pareto differential evolution nondominating sorting genetic algorithm II (NSGA II) and strength pareto evolutionary algorithm 2 (SPEA2), available from the work of (Basu, 2011).

For all compared algorithms, the dimension of the benchmark function has been set to 30, the population size has been set to 50, number of generations has been limited to 1000 and the parameter *limit* has been set as  $D \times SN$ , where D represents the dimension of the search space and SN is the number of food sources. The T-value for EABC has been set to 0.5 (Abro, 2013) and the p-value of IABC has been set to 0.25 (Gao and Liu, 2011). For global solution validation, each of the compared algorithms has been simulated for 30 times in solving the EED problem (Abro and Mohamad-Saleh, 2012). All these values follow those used and recommended in the literature (Abro, 2013), (Gao et al., 2012), (Gao and Liu, 2011), (Karaboga and Akay, 2009), (Sulaiman et al., 2015), (Sulaiman et al., 2014a).

The simulation and testing phases have been done using Matlab 2010a programming language on Intel Core i7 CPU 2.80 GHz computer.

### RESULTS AND DISCUSSIONS

The results of JA-ABC3 and JA-ABC4a in solving EED problem is illustrated by Table-1. The table shows the mean values of the operating cost and toxic gases emission together with the optimized values of the variables. The results depict that JA-ABC3 has outperformed JA-ABC4a in obtaining the least values of operating cost and toxic gases emission. However, the difference is actually small. Thus, it can be said that they are actually performing better at solving EED problem.

Their performances then are being compared to the standard ABC algorithm, the existing ABC variants and other optimization algorithms in order to further evaluate their effectiveness. The results are tabulated in Table-2. The acquired results in Table-2 clearly portray the superiority of JA-ABC3 and JA-ABC4a at solving EED problem.

The results show that JA-ABC3 has the least values of operating cost and toxic gases emission in comparison to others. This is followed by JA-ABC4a. Meanwhile, the compared optimization algorithms obtained from the work of (Basu, 2011) have the worst performance among all. Thus, the results vividly show that the ABC algorithm and its variants perform better than other optimization algorithms.

**Table-1.** The performance results of JA-ABC3 and JA-ABC4a at solving EED on 10-generator unit system.

Variables	Algorithms	
	JA-ABC3	JA-ABC4a
$P_{G1}$	47.1022	48.6635
$P_{G2}$	72.2871	72.9381

$P_{G3}$	103.6006	98.2306
$P_{G4}$	104.1198	99.3552
$P_{G5}$	115.5461	116.7651
$P_{G6}$	115.5704	127.1116
$P_{G7}$	288.0000	280.1056
$P_{G8}$	330.2652	324.1790
$P_{G9}$	453.6176	461.6643
$P_{G10}$	454.5126	455.6653
EED (10 <sup>5</sup> )(\$(lb))	1.1671	1.1676

**Table-2.** The performance results of the optimization algorithms at solving EED on 10-generator unit system.

Algorithms	EED (10 <sup>5</sup> )(\$(lb))
PDE	1.1762
MODE	1.1760
NSGA-II	1.1767
SPEA2	1.1763
ABC	1.1737
BABC1	1.1686
IABC	1.1693
EABC	1.1692
JA-ABC3	1.1671
JA-ABC4a	1.1676

Among the ABC variants, the standard ABC algorithm has the worst performance. Thus, it can be concluded that ABC variants perform better and have the ability to solve complex real-world problems such as EED, particularly the recently proposed ABC variants; JA-ABC3 and JA-ABC4a. Both modified ABC variants exhibit the best results among all, due to the balanced exploration and exploitation capabilities that are achieved by them.

### **CONCLUSIONS**

This work optimizes economic environmental dispatch (EED) problem using two recently proposed ABC variants, JA-ABC3 and JA-ABC4a. The variants have been developed by modifying the phases and mutation equations of the standard ABC algorithm. The aim of these modifications is to balance out the exploration and exploitation capabilities of the algorithm since a robust optimization algorithm should have balanced exploration and exploitation capabilities.

A 10-generator unit system has been used as the test system. The performances of the variants in optimizing EED have been compared with the standard ABC algorithm and the existing ABC variants available in the literature. A few optimization algorithms have also been included.

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The obtained results clearly showed the effectiveness of JA-ABC3 and JA-ABC4a in obtaining the least amount of operating cost and toxic gas emission in comparison to others. Between the two, JA-ABC3 has the best performance. However, since the difference is quite small, it can be concluded both variants have the ability to efficiently solve the problem by obtaining minimal solution.

To further evaluate the effectiveness of the ABC variants at optimizing the problem, various test system should be used. Besides, other recently proposed ABC variants should also be included prior to analyzing the effectiveness of each of the proposed variants.

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