STRESS CONCENTRATION AROUND CUT-OUTS IN PLATES AND CYLINDRICAL SHELLS

Rohit Chowdhury[1], M Saiteja Reddy[1], P. C. Jain[2] and P. Bangaru Babu[1]
[1]Department of Mechanical Engineering, National Institute of Technology Warangal, India
[2]Defense Research & Development Laboratory Hyderabad, India
E-Mail: rohitc.nitw@gmail.com

ABSTRACT

In this study, investigation is carried out to determine the effects on the stress concentration around a cut-out due to reinforcement around the cut-out in a flat plate and a cylindrical shell, subjected to axial and bending loads. The study is further extended, to a flat plate and a cylindrical shell, subjected to compressive load, containing two identical cut-outs along the length to determine the effect on the stress concentration factor, with the variation of the distance between the cut-outs. This study assumes the use of homogeneous, isotropic material. The investigation is carried out using the FEA software package, ANSYS APDL Mechanical. Parametric study is carried out to understand the variation of the stress concentration. Mathematical relations are also established that relate the diameter of the cut-outs to the distance between them at which they cease to influence one another, in a non-dimensional format.

Keywords: stress concentration, finite element analysis, circular cut-outs, reinforced cut-outs.

INTRODUCTION

The study of stress concentration is important for proper structural design of a machine components to assess the underestimation of the effects of applied loads. Amongst the various geometries encountered in mechanical and aerospace components and various other structures, cut-outs in a plates and shells are very common, especially in aerospace applications. Hence, extensive study of Stress concentration around cut-outs in shells and plates is essential for accurate design of structural components. Moreover, ways of reducing stress concentration are also very important to avoid failure of structures.

There has been extensive study in this field in the past. Exhaustive studies on stress concentrations for cut-out in two-dimensional bodies subjected to a wide variety of loading have been published by Pilkey [1] and Young [2]. Zheng Yang, *et al* [3] have studied three dimensional stress concentration in a plate containing a circular cut-out, subjected to out of plane bending. Chongmin She and Wanlin Guo [4] analysed the through-thickness variations of stress concentration factors along the wall of elliptic cut-outs in finite thickness plates of isotropic materials subjected to remote tensile stress using the finite element method. Hwai-Chung Wu and Bin Mu [5] have studied stress concentration in isotropic plates subjected to biaxial loading, and isotropic cylinders with uni-axial loading and internal pressure. They presented an empirical calculation method for the stress concentration in plates and cylinders with circular cut-outs. Rajaiah *et al.* [6] proposed cut-out shape optimization for stress mitigation in a finite plate using photo elasticity method by introducing auxiliary cut-outs around main cut-out. Mittal and Jain [7] proposed the optimization of design of square simply supported isotropic plate with central circular cut-out subjected to transverse static loading by Finite Element Method, by introducing auxiliary cut-outs and reported around 30% reduction in stress concentration factor.

Information about stress concentration in plates and cylindrical shells with a single cut-out, is available in literature for various loading conditions. However, for plates and cylindrical shells with reinforced cut-outs needs attention. Hence, the objective of this paper is to study the influence of reinforcement around a cut-out on the stress concentration. The analysis is further extended to plates and cylindrical shells containing two cut-outs to study the influence of the distance between the cut-outs on the stress concentration factor.

Throughout this analysis, the material used is aluminium and is assumed to be elastic, isotropic and homogenous. The Young's Modulus is assumed to be 70 GPa and the poisson's ratio to be 0.3. The following notation is used:

L- Length of the plate or Axial Length of the cylindrical shell
D- Outer Diameter of cylindrical shell
W- Width of the plate
T- Thickness of the plate or cylindrical shell
d- Diameter of cut-out
r- Radius of cut-out
w- Width of the reinforcement around the cut-out
t- Height of the reinforcement around the cut-out
l- Distance between two cut-outs
Y- distance between first cut-out and end of the plate or cylindrical shell
I- Areal moment of inertia of cross section area about neutral axis
A- cross section area
σ_{max} is defined as the maximum stress around the cut-out

PLATE WITH ONE REINFORCED CUT-OUT SUBJECTED TO TENSILE AND BENDING LOADS

A finite thickness flat plate with a reinforced cut-out is considered as shown in figure-1. For the analysis, the dimensions of the plate considered are as follows:

VOL. 11, NO. 18, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Width of plate, D=900 mm, Length of plate, L=1500 mm, Thickness of plate, T=5 mm such that the two ends lie at X=-750 and X=750. The height of reinforcement to thickness of plate ratio (t/T ratio) is varied from 1 to 6 in steps of 1. For each t/T ratio, the cut-out diameter, d and width of reinforcement, w, are parameters in our analyses, and varied such that cut-out diameter to plate width ratio, d/D varies from 0.04 to 0.24 in steps of 0.04 and the reinforcement width to cut-out radius ratio, w/r varies from 0.1 to 1 in steps of 0.1. The stress concentration factor is defined as

$$K = \frac{\sigma_{\max}}{\sigma_{avg}}$$

Where $\sigma_{avg} = \frac{F}{WT} + \frac{FT^2/4}{WT^3/12}$ for tensile load, where F is the tensile force

$\sigma_{avg} = \frac{FLT/2}{WT^3/12}$ for bending load, where FL is the bending moment

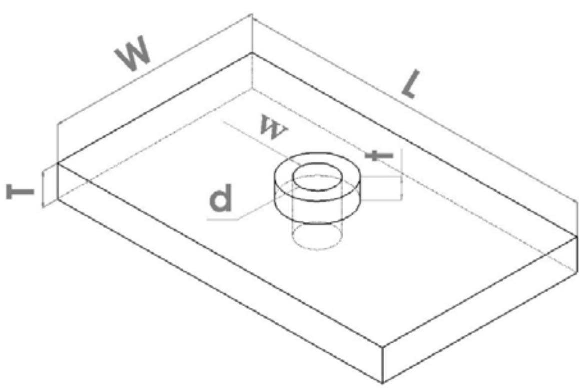

Figure-1. Geometrical model of reinforced cut-out in a plate.

Finite element model

The finite element model is constructed with 4-node, 3D, shell element (SHELL 181). The section offset is set as 'bottom'. All degrees of freedom are fixed, for all nodes at X=0 (see Figure-2). In case of tensile load, a net tensile force, F= 37 KN is applied by applying a force on all nodes at X=1500. It resulted in σ_{avg} = 32.88 MPa. In case of bending load, a net transverse force of 185 N by applying forces on the nodes at X=1500 which results in σ_{avg} = 37 MPa

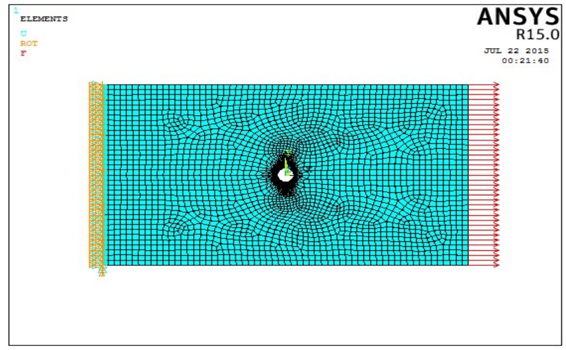

Figure-2. Meshed model of plate with reinforced cut-out subjected to tensile load.

Model verification

For verification, analysis was performed to determine the variation of the stress concentration factor around a single cut-out (without reinforcement) in a flat plate of finite thickness, subjected to tensile load. The results obtained were compared to that presented in Stress Concentration Factors by Walter and Deborah Pilkey [1]. It was observed that the two curves in Figure-3 are in good agreement with each other within an error band of 3%. In the analysis of plated with a reinforced cut-out described in this paper, the same method of formulation was used. This verifies the formulation of the presented analysis.

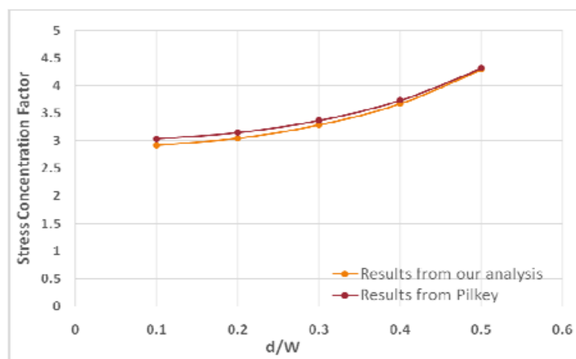

Figure-3. Variation of stress concentration factor with cut-out diameter in a flat finite plate.

Results

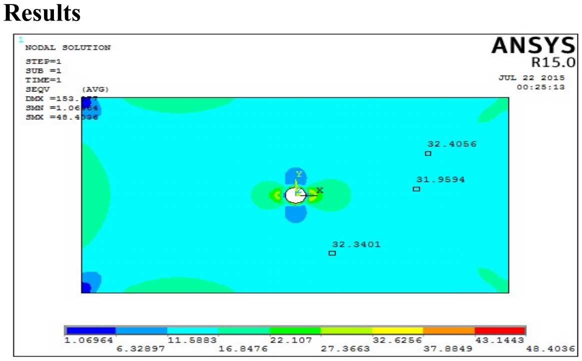

Figure-4. Von mises stress distribution in plate with reinforced cut-out subjected to tensile load.

ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

Figure-4 shows the von mises stress distribution in the plate with a reinforced cut-out. Figures 5-8 show the variation of stress concentration factor with w/r ratio with t/T as parameter. The following trends are observed from these figures.

In general, for both tensile and bending loads, for any cut-out radius, and for any height of reinforcement, the Stress Concentration Factor, K, decreases with increasing (w/r) ratio, i.e. K decreases with increasing width of reinforcement. This may be attributed to the addition of material, radial to the cut-out.

Keeping other parameters constant, K decreases with increasing height of reinforcement. This can be seen in the downward displacement of the K vs w/r curves, as t/T increases.

The slope of the K vs w/r curves are larger (magnitude-wise) for smaller values of w/r and the slope decreases (magnitude wise) with increasing w/r ratio.

As the d/D ratio increases, the curves appear to come closer together. This means, for given t/T and w/r ratios, the effect of cut-out diameter decreases as d/D ratio increases.

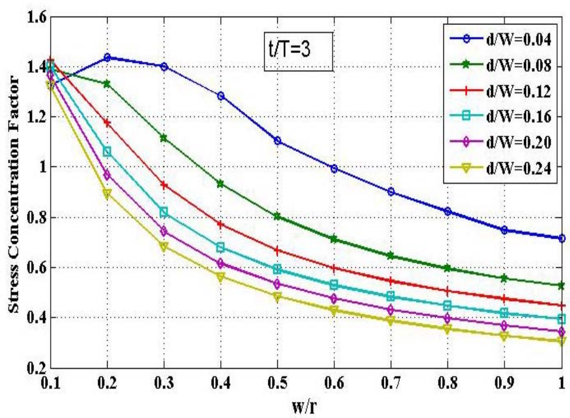

Figure-5. Stress concentration factor in plate with reinforced cut-out under bending load for t/T=3.

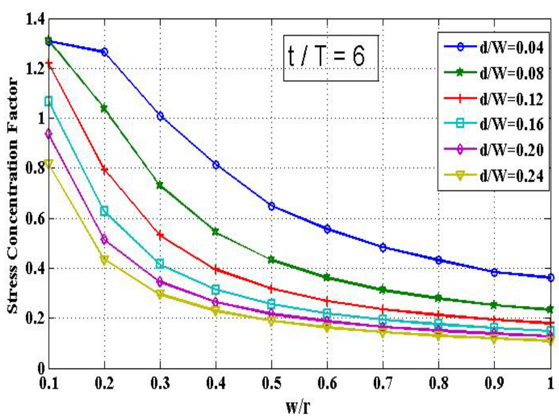

Figure-6. Stress concentration factor in plate with reinforced cut-out under bending load for t/T=6.

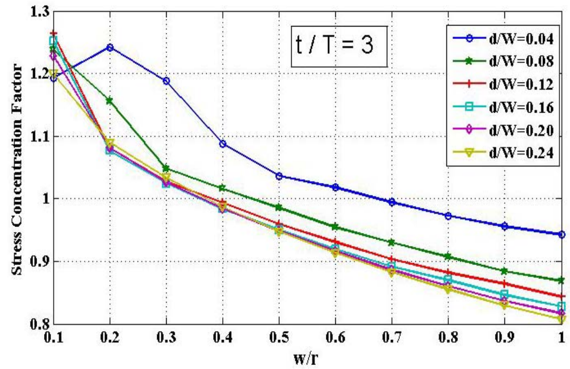

Figure-7. Stress concentration factor in plate with reinforced cut-out under tensile load for t/T=3.

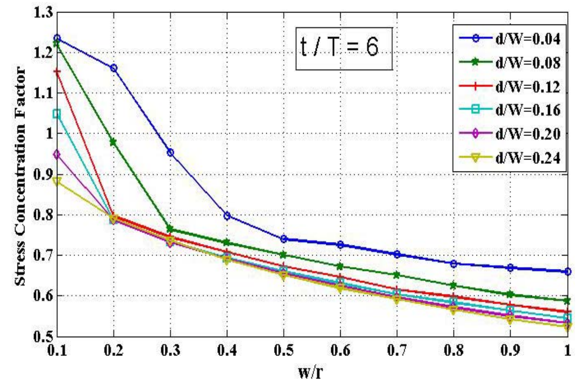

Figure-8. Stress concentration factor in plate with reinforced cut-out under tensile load for t/T=6.

For a given height of reinforcement, K=1 is obtained at a smaller w/r ratio for a cut-out with larger d/D ratio. This statement need to be interpreted properly. For example, Table-1 shows what volume of reinforcement per unit height of reinforcement, K=1, for various d/D ratios, at t/T=4. Here, for a given d/D ratio, K=1 occurs at $(w/r)=(w/r)_0$.

Table-1. Volume of reinforcement per unit height for different d/D ratios at K=1.

d/W	W (mm)	r (mm)	w_0/r	w_0 (mm)	Volume/t $=\pi[(w_0+r)^2-r^2]$
0.04	1500	30	0.46	13.8	3197.90
0.08	1500	60	0.28	16.8	7216.47
0.12	1500	90	0.20	18	11190.96
0.16	1500	120	0.155	18.6	15108.27
0.20	1500	150	0.14	21	21166.74
0.24	1500	180	0.125	22.5	27023.63

It is clear from the above table, that a cut-out with a larger diameter requires more volume of reinforcement (or in other words, more material).

VOL. 11, NO. 18, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

PLATE WITH TWO CUT-OUTS

The study is extended by introducing one more cut-out in the flat plate, but this time, none of them are reinforced (as shown in Figure-9) The end of the plates are at X=0 and X=L. The two cutouts, of diameter, d are placed at distances Y and Y+l from X=0 such that Y≥1.5d. This analysis is carried out with various values of L and W such that the L/W ratio varies from 2 to 5 in steps of 1. The d/W ratio is varied from 0.1 to 0.3. Plate thickness, T=5mm. The stress concentration factor, K is

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

Where $\sigma_{avg} = \frac{F}{WT}$, where F = total axial compressive force

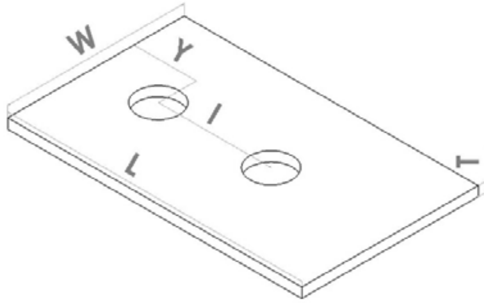

Figure-9. Geometrical model of a plate with two cut-outs.

Finite element model

The finite element model is constructed with 4-node, 3D, shell element (SHELL 181). The section offset is set as 'mid'. The model is freely meshed with mesh size 10. The nodes lying on the cut-out are refined to obtain a fine mesh at the edge of the cut-out. All degrees of freedom are fixed, for all nodes at X=0, i.e. one end of the plate.

A line pressure of 300000 N/m is applied on the edge of the plate at X=L units, i.e. on the other end of the plate as shown in Figure-9. As a result σ_{avg}= 60 MPa

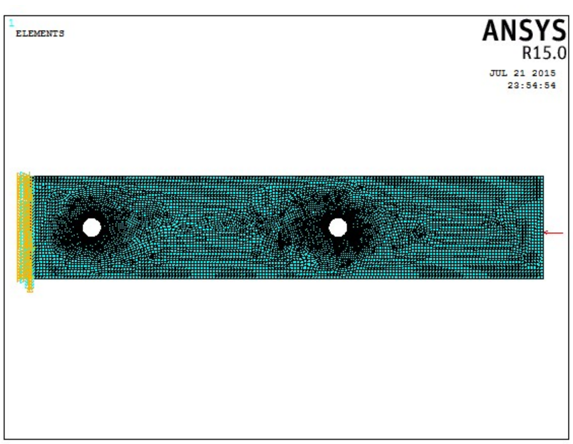

Figure-10. Meshed model of plate with two cut-out.

Model verification

Verification of the model is shown in section 2.2

Results

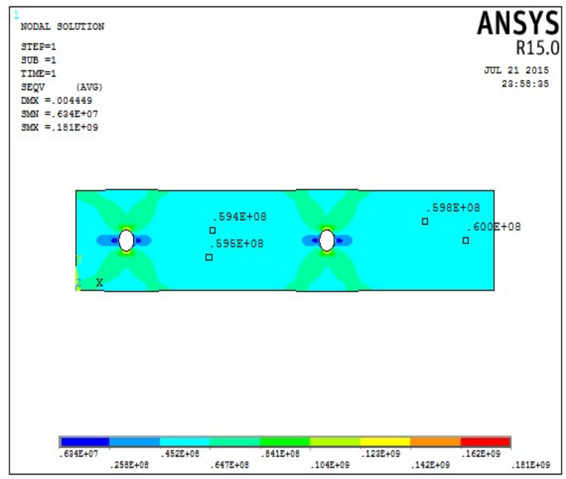

Figure-11. Von mises stress distribution in plate with two cut-outs subjected to compressive load.

Figure-11 shows the distribution von mises stress in a plate with two cut-outs. Figure-12 shows variation of K with G for various L/W ratios, where $G = \left(\frac{d}{W}\right) * \left(\frac{l}{L}\right)^{0.625}$

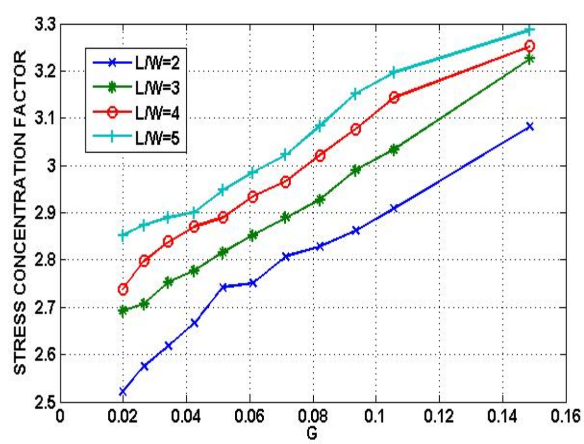

Figure-12. Variation of stress concentration factor with G.

For a given plate even for different cases of d/W and l/L the stress concentration is same when $(d/W)x(l/L)^{0.625}$ is almost same with an error of ±5%, with respect to the first value. This fact is demonstrated in tables 2 and 3.

Table-2. Error between stress concentration factor obtained by FEA analysis and that by proposed formula for L/W=5.

d	l	G	S_{max}	K	Error %
200	1000	0.07314	181.2	3.02	0
220	858.56	0.07314	181.56	3.026	1.98
240	746.98	0.07314	181.59	3.0266	2.17
260	657.18	0.07314	180.41	3.0068	-4.37

Table-1. Error between stress concentration factor obtained by FEA analysis and that by proposed formula for L/W=3.

d	l	G	S_{max}	k	Error %
100	800	0.04377	174.085	2.901	0
120	597.58	0.04377	173.565	2.892	-2.98
140	466.96	0.04377	171.414	2.851	-1.53

For a constant l/L ratio, with increase in the d/D ratio the stress concentration factor increases. For a constant d/D ratio, with increase in the l/L ratio the stress concentration increases. This may be explained as follows: when the cut-outs are closer to each other, the force flow lines are smoothened out more. However as the distance between the cut-outs increases, the smoothening effect decreases too.

For a given G, with increase in the value of L/D the stress concentration factor increases. It is observed that diameter of the cut-outs has more effect on the stress concentration than the distance between cut-out centers. Using this plot one can find the maximum distance between the centers of the two cut-outs, such that there will not be failure due to compressive load.

Distance at which cut-outs cease to affect each other

The stress concentration factor around the first cut-out first increases and then it becomes constant after a certain distance between cut-out centers. The stress concentration factor around the second cut-out also tends to a constant value as the distance between the cut-out centers increases, but increases sharply as the second cut-out approaches the edge of the plate. A similar behavior is observed in plates with different thickness and L/W ratio. Figure-13 shows variation of the distance with cut out radius after which the second cut-out has no effect on the first cut-out for L/W=3.75

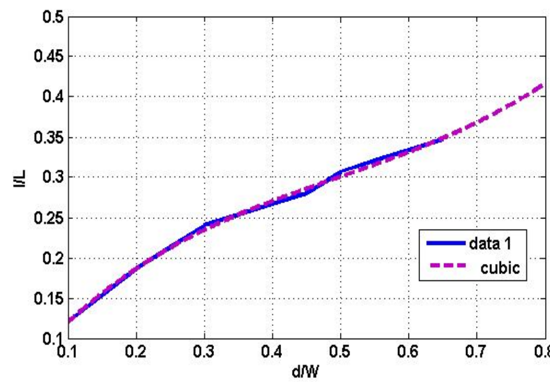

Figure-13. l/L ratio at which the cut-outs cease to influence one another for a d/W ratio.

In Figure-13, the solid line is plotted using data from ANSYS.

The dotted line is plotted by using basic curve fitting tool in matlab. The following equation obtained from polynomial curve fitting, shows the relationship between the radius of the cut-outs, and the center to center distance at which they cease to affect each other: $y = 0.99703x^3 - 1.4917x^2 + 1.0382x + 0.03082$

Where, y represents l/L ratio, and x represents the d/W ratio of the cut-out.

Using the above graph one can find out the minimum distance after which the first cut-out is not affected by the second cut-out. This graph and equation can be used for other plates with various L/W ratios as well.

CYLINDRICAL SHELL WITH ONEREINFORCED CUT-OUT SUBJECTED TO COMPRESSIVE AND BENDING LOAD

Next, a finite thickness hollow cylindrical shell is considered that contains a reinforced cut-out as shown in Figure-14. For the purpose of analysis, D=900 mm, L=1500 mm, T=5 mm such that the two ends lie at X=0 and X=1500. For a given t/T ratio, d and w, are parameters in our analyses, and varied such that d/D varies from 0.04 to 0.24 in steps of 0.04. w/r is varied from 0.0625 to 1 in steps of 0.0625 for compressive loads and from 0.1 to1 in steps of 0.1 for bending loads. t/T ratio is varied from 1 to 6 in steps of 1.

The stress concentration factor is defined as

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

Where $\sigma_{avg} = \frac{4F}{\pi\left(D^2 - (D-2T)^2\right)}$, for compressive load, where F is the compressive force

$\sigma_{avg} = \frac{32M}{\pi\left(D^2 - (D-2T)^2\right)}$, for bending load, where M is the bending moment

VOL. 11, NO. 18, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

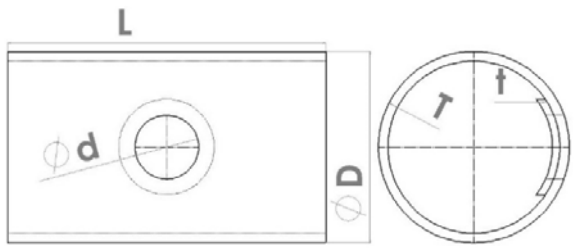

Figure-14. Geometrical model of cylindrical shell with reinforced cut-out.

Finite element model

The finite element model is constructed with 4-node, 3D, shell element (SHELL 181). The section offset is set as 'top'. The model is freely meshed with mesh size 25. All degrees of freedom fixed, for all nodes at X=0 (Figure-15). In case of compressive load, a net compressive force, F= 257.6 kN is applied by applying a force on all nodes at X=1500. As a result σ_{avg}= 18.32 MPa. In case of bending load, a constant bending moment M_z=1.0 x10^8 through linearly forces on the nodes at X=1500. As a result σ_{avg}= 31.97 MPa

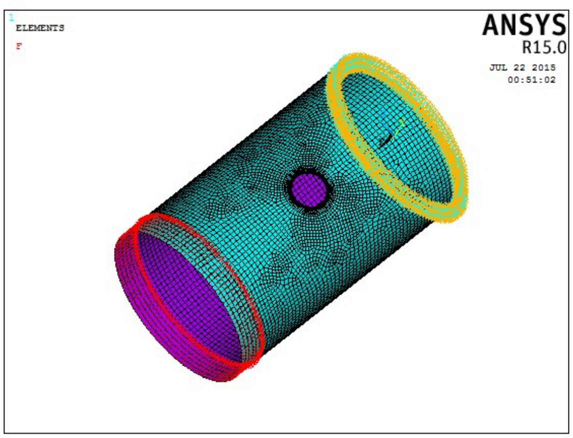

Figure-15. Meshed model cylindrical shell with reinforced cut-out under compressive load.

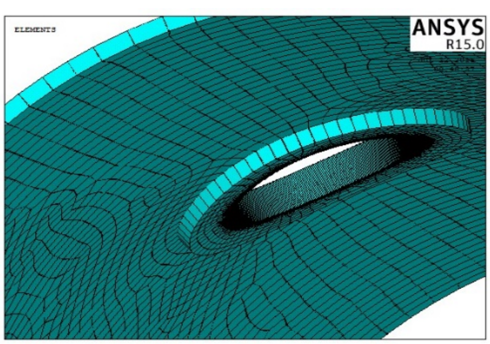

Figure-16. Meshed reinforcement around cut-out.

Model verification

For verification, analysis was performed to determine the variation of the stress concentration factor around a single cut-out (without reinforcement) in a cylindrical shell of finite thickness, subjected to bending load. The results obtained were compared to that presented in Stress Concentration Factors by Walter and Deborah Pilkey [1]. It was observed that the two curves, as shown in figure-17 are in good agreement with each other within an error band of 5%. In the analysis of the cylindrical shell with a reinforced cut-out described in this paper, the same method of formulation was used. This verifies the formulation of the presented analysis.

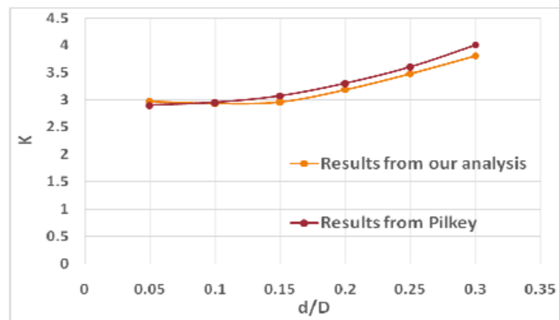

Figure-17. Variation of stress concentration factor with the cut-out diameter.

Results

Figure-18 shows the von mises stress distribution in a cylindrical shell with a reinforced cut-out, subjected to compressive load. Figures-19-22 show the variation of stress concentration factor with w/r ratio. The following trends are observed from these figures.

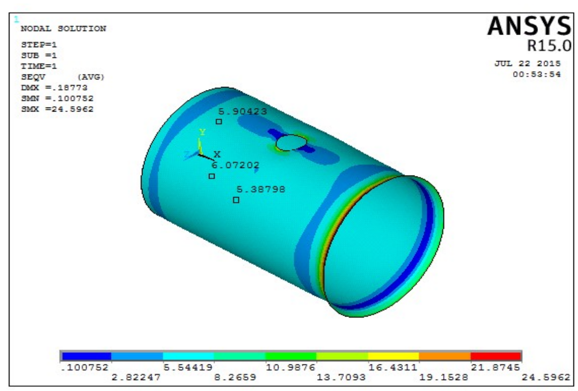

Figure-18. Von mises stress distribution in cylindrical shell with reinforced cut-out subjected to compressive load.

In general, for any cut-out radius, and for any height of reinforcement, the Stress Concentration Factor, K, decreases with increasing (w/r) ratio. In other words, K decreases with increasing width of reinforcement as evident from studies on the plate.

VOL. 11, NO. 18, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

Keeping other parameters constant, K decreases with increasing height of reinforcement. This can be seen in the downward displacement of the K vs w/r curves, as t/T increases as shown in Figures 19 and 20. This can also be attributed to the additional material, adding to the height of reinforcement.

The slope of the K vs w/r curve increases (magnitude-wise) with increasing d/D ratio. This means that K decreases more rapidly with increasing w/r ratio for a larger cut-out, than in the case of a smaller cut-out. This can be explained through the following example:

Let there be two curves corresponding to $(d/D) = (d/D)_1$ (curve 1) & $(d/D)=(d/D)_2$ (curve 2), such that

$(d/D)_2 > (d/D)_1$

Then, for constant D,

$d_2 > d_1$ or $r_2 > r_1$

Therefore, at a given (w/r) = c (say)

⇨ $w_2/r_2 = c$ for curve 2 and $w_1/r_1= c$ for curve 1

⇨ $w_2=cr_2$ and $w_1= cr_1$

But $cr_2 > cr_1$ (since $r_2> r_1$)

⇨ $w_2>w_1$

Hence, at any given w/r ratio, the curve having larger d/D ratio will have a larger width of reinforcement, w, and hence more material in the reinforcement. So with increasing w/r ratio, more material is added around a larger cut-out than in the case of a smaller. This is the reason why a curve with larger d/D ratio has larger slope (magnitude-wise). For a given t/T ratio, all curves appear to meet at a common point or a common region. This point can be referred to as the point of concurrence.

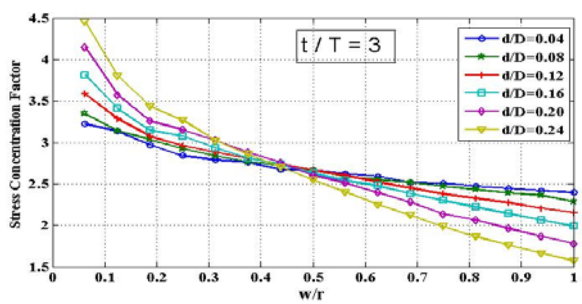

Figure-19. Stress concentration factor in cylindrical shell with reinforced cut-out under compressive load for t/T=3.

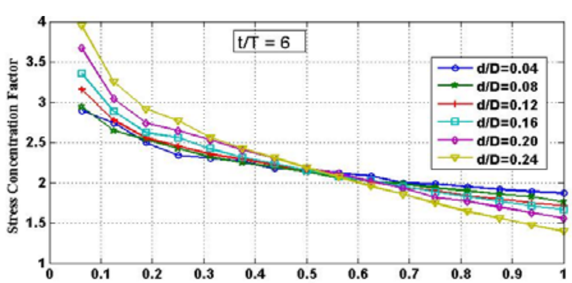

Figure-20. Stress concentration factor in cylindrical shell with reinforced cut-out under compressive load for t/T=6.

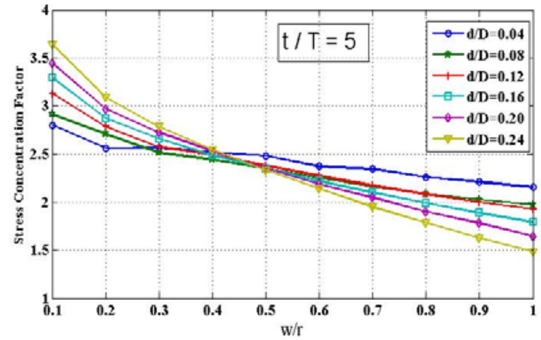

Figure-21. Stress concentration factor in cylindrical shell under bending load for t/T=5.

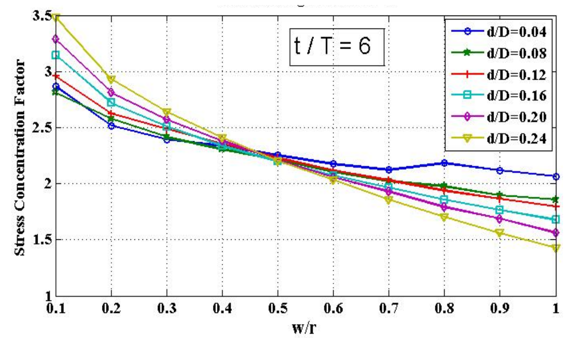

Figure-22. Stress concentration factor in cylindrical shell under bending load for t/T=6.

Hence, for a given t/T ratio, K is approximately constant for all d/D ratios at a particular (w/r) ratio. Let this particular w/r ratio be called the critical width ratio, denoted by w/r = α, and the corresponding constant K value be Kα. With increasing t/T ratio, the point of concurrence shifts rightwards and downwards. In other words, α increases and Kα decreases with increasing t/T ratio.

For practical purposes, the height and width of cut-out reinforcement, the stress concentration factor (SCF) never comes down to K=1. At best, i.e., with thickness of reinforcement= 6 times the shell thickness, cut-out radius of 0.24 times shell radius and width of reinforcement = radius of cut-out, SCF comes down to K=1.37 for compressive loads and K=1.48 for bending loads.

CYLINDRICAL SHELL WITH TWO CUT-OUTS

Now, one more cut-out is introduced in the cylindrical shell, but this time, none of them are reinforced (as shown in Figure-23. The ends of the cylinder are at X=0 and X=L. The two cutouts, of diameter, d are placed at distances Y and Y+l from X=0 such that Y≥1.5d. This analysis is carried out with various values of L and W such that the L/W ratio varies from 2 to 5 in steps of 1. The d/D ratio is varied from 0.1 to 0.3. Thickness of cylindrical shell, T=5mm.

VOL. 11, NO. 18, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

The stress concentration factor, K is

$$K = \frac{\sigma_{\max}}{\sigma_{avg}}$$

Where $\sigma_{avg} = \frac{4F}{\pi\left(D^2 - (D-2T)^2\right)}$

Where, F = total axial compressive force

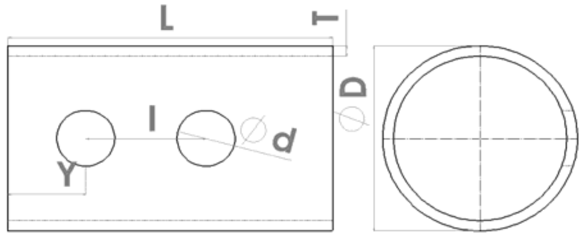

Figure-23. Geometrical model of cylindrical shell with two cut-outs.

Finite element model

The finite element model is constructed with 4-node, 3D, shell element (SHELL 181). The section offset is set as 'mid'. The nodes lying on the cut-out are refined to obtain a fine mesh at the edge of the cut-out as shown in the figures 24 and 25 All degrees of freedom are fixed, for all nodes at X=0, i.e. one end of the cylindrical shell.

A line pressure of 200000 N/m is applied on the circumference of the cylinder at Z= -L. As a result σ_{avg}= 40 MPa.

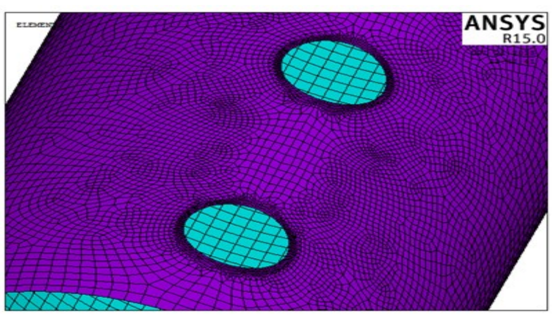

Figure-24. Refined mesh around the cut-outs.

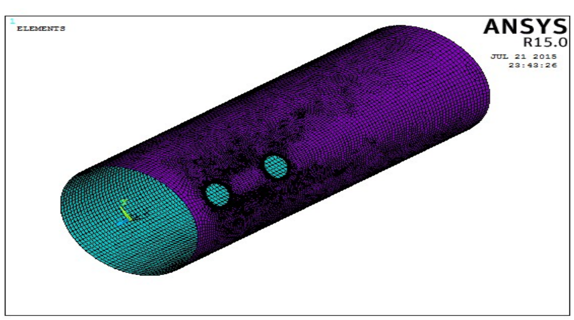

Figure-25. Meshed model of cylindrical shell with two cut-outs.

Model verification

Verification of the model is shown in section 4.2.

Results

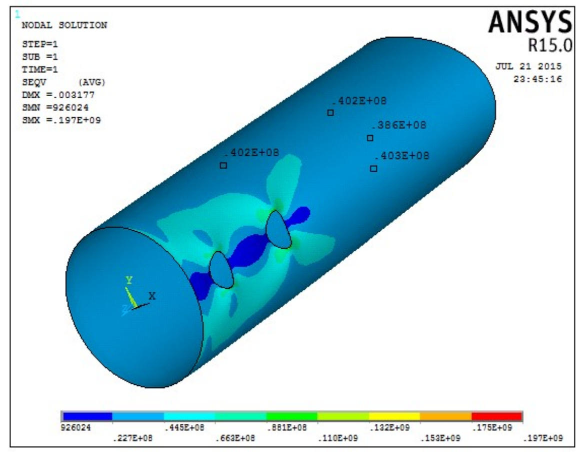

Figure-26. Von mises stress distribution in cylindrical shell with two cut-outs subjected to compressive load

Figure-27 shows variation of stress concentration factor, K with G, for various L/D ratios, where $G = \left(\frac{d}{D}\right) * \left(\frac{l}{L}\right)^{0.3458}$

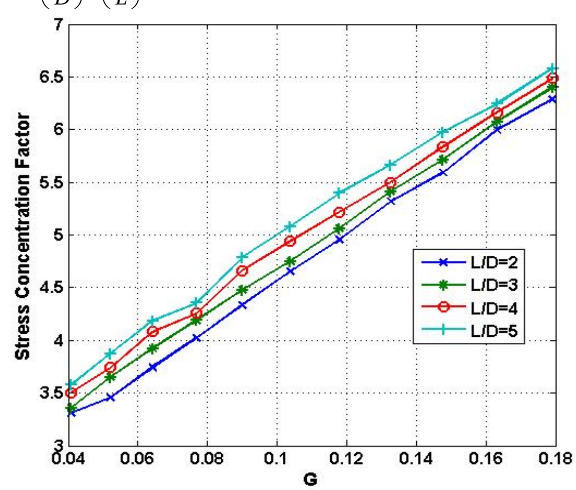

Figure-27. Variation of stress concentration factor with G.

For a given cylinder, even for different cases of d/D and l/L, the stress concentration is same when $(d/D)x(l/L)^{0.3458}$ is almost same. In other words, even on changing the cutout diameter and the distance between the cut-outs, as long as the value $(d/D)x(l/L)^{0.3458}$, the stress concentration factor is also constant, within an error of 5%, with respect to the first value. The tables 4 and 5 demonstrates this observation, through the examples of two cylinders:

VOL. 11, NO. 18, SEPTEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Table-2. Error between stress concentration factor obtained by FEA analysis and that by proposed formula for L/D=3.

d	l	G	S_{max}	K	Error %
140	1100	0.098958	182.055	4.551	0
160	747.633	0.098958	186	4.65	2.175
180	531.818	0.098958	186.604	4.66	2.395

Table-5. Error between stress concentration factor obtained by FEA analysis and that by proposed formula for L/D=5.

d	l	G	S_{max}	K	Error %
220	330	0.08594	184.039	4.6009	0
180	589.57	0.08594	187.112	4.6778	1.69
160	904.174	0.08594	187.768	4.6942	2.04
140	1219.46	0.08594	181.24	4.531	-1.5

For a constant l/L ratio, with increase in the d/D ratio the stress concentration factor increases. This is similar to corresponding observation in plates. The stress concentration factor increases as the cut-out diameter increases in a finite plate. For a constant d/D ratio, with increase in the l/L ratio the stress concentration increases. This may be explained as follows: when the cut-outs are closer to each other, the force flow lines are smoothened out more. However as the distance between the cut-outs increases, the smoothening effect decreases too.

For a given G, with increase in the value of L/D the stress concentration factor increases. It is observed that diameter of the cut-outs has more effect on the stress concentration than the distance between cut-out centers.

Distance at which cut-outs cease to affect each other

The stress concentration factor around the first cut-out first increases and then it becomes constant after a certain distance between cut-out centers. This means that after a certain distance, the cut-outs cease to act as a stress reliever for one another. The stress concentration factor around the second cut-out increases continuously as the distance between the cut-out centers increases. A similar behavior is observed in cylinders with different thickness and L/D ratios.

Figure-28 shows variation of the distance with cut out radius after which the second cut-out has no effect on the first cut-out for L/D=1.67

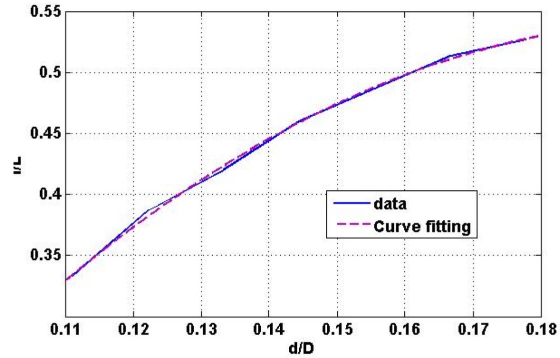

Figure-28. l/L ratio at which the cut-outs cease to influence one another for a d/D ratio.

In Figure-28, the solid line is plotted using data from ANSYS.

The dotted line is plotted by using basic curve fitting tool in matlab. The following equation obtained from polynomial curve fitting, shows the relationship between the radius of the cut-outs, and the center to center distance at which they cease to affect each other. $y = 7.84 * 10^{-6} x^3 - 25.07 x^2 + 10.136 x - 0.482$

Where, y represents l/L ratio at which the cut-outs cease to influence one another, and x represents the d/D ratio of the cut-out. This graph and equation can be used for other cylinders with various L/D ratios as well.

Check for buckling: Although the analyses up to this point is informative about the Stress Concentration Factor, it gives no indication, as to whether the structure would fail due to buckling under compressive loads, before it may fail due to yielding of the material. The plots below shows, are for a thickness of 5mm, a given L/D ratio and applied load 628.318 KN, how the buckling load factor varies with the l/L ratio for different d/D ratios. The Buckling Load Factor is defined as follows:

Buckling Load Factor = Load at which the structure buckles (in our analyses, for 1st mode) / Applied Load

The plots below show the variation of buckling load factors in a cylinder with two identical cut-outs with l/L ratio for various d/D ratios when L/D=3, L/D=5.

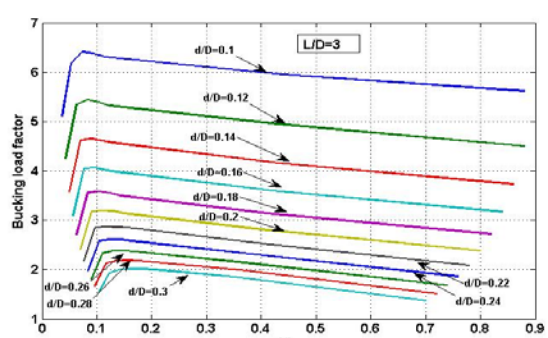

Figure-29. Variation of buckling load factor with the distance between the two cut-outs.

ARPN Journal of Engineering and Applied Sciences

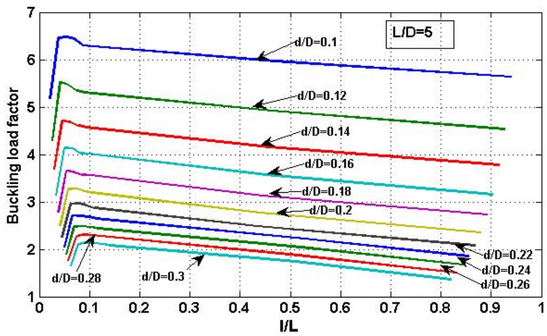

Figure-30. Variation of buckling load factor with the distance between the two cut-outs.

When the cut-outs are very close the buckling load factor is less and it increases with increase in the distance between cut-out centers and attains a maximum and starts decreasing after that, this is because the second cut-out approaches the edge where load is applied. With increase in the L/D ratio for same l/L and d/D ratio the buckling load factor increases.

Using Figure-27 and 28 for compression and Figures 29 and 30 for buckling one can find out the minimum and maximum distance that can be between two identical cut-outs on a cylindrical shell for a given load.

CONCLUSIONS

For designing engineering structures with circular cut-outs, a dependable estimation of stress concentration factors is important. The plots featured in our study can be used for preliminary design of cylindrical shells and plates with reinforced cut-outs. The following useful conclusions can be drawn from our study.

Reinforcement of cut-outs in cylindrical shell and plate:

- In both cylinders and plates, under axial and bending load, the stress concentration factor decreases, with the increase in the width of reinforcement.
- In both cylinders and plates, under axial and bending load, the stress concentration factor decreases, with the increase in the height of reinforcement.
- In cylinders, for a given t/T ratio all curves appear to meet at a common point called the point of concurrence. Hence, for a given t/T ratio, the stress concentration factor is approximately constant for all d/D ratios at a particular w/r ratio. With increasing t/T ratio, the point of concurrence shifts rightwards and downwards. In other words, α increases and K_α decreases, with increasing t/T ratio.
- In plates under tensile load and bending load, the slope of the K vs w/r curves are larger (magnitude-wise) for smaller values of w/r and the slope decreases (magnitude wise) with increasing w/r ratio.
- In plates under tensile load and bending load, as the d/D ratio increases, the curves appear to come closer together. This means, for given t/T and w/r ratios, the effect of cut-out diameter on the stress concentration factor decreases as d/D ratio increases.
- Reinforcement of a cut-out in a plate leads to better reduction of stress concentration compared to that in a cylindrical shell. While SCF could be brought down to a minimum of K=1.37 in case of cylinders, K=1 is easily achieved in plates for all practical cut-out diameters at $t/T \geq 3$.

Two identical cut-outs in cylindrical shell and plate:

- In both, plate and cylindrical shell, it is observed that one cut-out acts as a stress reliever for the other and the stress concentration increases with increase in the distance between the cut-outs. The reason one cut-out acts as a stress reliever for the other may attributed to the smoothening of the force flow lines around the cut-outs.
- For a constant l/L ratio, with increase in the d/D ratio the stress concentration factor increases. This is similar to corresponding observation in plates. The stress concentration factor increases as the cut-out diameter increases
- For a cylindrical shell with a given length, diameter and thickness, even for different cases of d/D and l/L, the stress concentration is same when $(d/D)x(l/L)^{0.3458}$ is almost same. In other words, even on changing the cut-out diameter and the distance between the cut-outs, as long as the value $(d/D)x(l/L)^{0.3458}$ is same, the stress concentration factor value is also constant, within an error of 5%. For a given value of $(d/D)x(l/L)^{0.3458}$, with increase in the value of L/D, the stress concentration factor increases
- The following equation is obtained in case of cylindrical shells:
- $y = 7.84*10^{-6}x^3 - 25.07x^2 + 10.136x - 0.482$, where y represents l/L ratio at which the cut-outs cease to influence one another, and x represents the d/D ratio of the cut-out.
- For a given plate even for different cases of d/W and l/L the stress concentration is same when $(d/W)x(l/L)^{0.625}$ is almost same with an error of ±5%. For a given value of $(d/W)x(l/L)^{0.625}$ with increase in the value of L/D the stress concentration factor increases
- The following equation is obtained for flat plates:
- $y = 0.99703x^3 - 1.4917x^2 + 1.0382x + 0.03082$, where y represents I/L ratio at which the cut-outs cease to influence one another, and x represents the d/W ratio of the cut-out.

ACKNOWLEDGEMENTS

The authors would like to thank DRDL, Hyderabad and NIT Warangal for facilitating to carry out their study

REFERENCES

[1] Pilkey WD. Peterson's stress concentration factors, 2nd ed. John Wiley and Sons: New York; 1997.

[2] Young WC, Budynas RG.Roark's formulas for stress and strain, 7th ed. McGraw-Hill: NewYork; 2002.

[3] The stress and strain concentrations of out-of-plane bending plate containing a circular cut-out by Zheng Yang, Chang-Boo Kim, Hyeon Gyu Beom, Chongdu Cho, International Journal of Mechanical Sciences 52 (2010) 836–846

[4] Chongmin She, Wanlin Guo, "Three-dimensional stress concentrations at elliptic cut-outsin elastic isotropic plates subjected to tensile stress." International Journal of Fatigue 29 (2007) 330–335

[5] Hwai-Chung Wu, Bin Mu, "On stress concentrations for isotropic/orthotropic plates and cylinders with a circular cut-out", Composites: Part B 34 (2003) 127–134

[6] N. D. Mittal and N. K. Jain, The optimize design of a square simply supported isotropic plate with central circular cut-out for reduction of stress concentration subjected to transverse static loading, Proc. Of ICTACEM, 2007.

[7] K. Rajaiah and A. J. Durelli, "Optimum cut-out shapes in finite plates under uni-axial load", Applied Mechanics, vol. 46(3), pp. 691-695, 1979.