



# IS THERE AN ACCURACY ADVANTAGE TO USING TWO BASES IN A GNSS STATIC SURVEY UP TO 45 KM BASELINES?

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## ABSTRACT

This paper investigates the accuracy improvement of using two GNSS base receivers, instead of only one base receiver, in establishing GNSS static control points, up to baselines of 45 km. In this regards, a GNSS campaign was run to get the UTM coordinates of 15 new geodetic control points, using GNSS static technique by fixing one and two base receivers. The results showed that horizontal positional discrepancies  $P_{2d}$  between the two solutions have a mean value of 4.6 mm with 1.2 mm standard deviation, while the spatial positional discrepancies  $P_{3d}$  have a mean value of 5.6 mm with standard deviation 1.4 mm. These results can be considered insignificant from the accuracy point of view in establishing geodetic networks. On the other hand, from the economic and cost point of views, it is recommended to use only one dual-frequency GNSS receiver to save cost and time in establishing geodetic GNSS control points up to baselines of 45 km.

**Keywords:** GNSS, GNSS static technique, GNSS base receivers, GNSS baseline.

## 1. INTRODUCTION

The surveying and mapping community was one of the first to take advantage of GNSS because it dramatically increased productivity and resulted in more accurate and reliable data. Today, GNSS is a vital part of surveying and mapping activities around the world. Dual-frequency GNSS receivers can be used to position survey markers, buildings, bridges, and other large infrastructure. GNSS is widely used in mapping, including aerial mapping, and Geographic Information System (GIS) applications [1]. The applications of GNSS can be grouped into static and kinematic applications. Static applications of GNSS are used in geodetic surveying, photogrammetric surveying, land surveying, orthometric height determination, topographic mapping, monitoring structural deformations, and engineering surveys. One important static application of GNSS is to establishing geodetic control points for the daily topographic, cadastral, and engineering surveying works [2].

The objective of this paper is to investigate the improvement in coordinate's accuracy when using two base GNSS receivers, instead of one, on baselines up to 45 km in length. In this regard, review of GNSS observables and techniques of observation will be presented. The methodology of investigation, as well as the description of the field test will be described. Finally, the analysis of the obtained results supported with the statistical analysis will be presented, from which conclusions will be drawn.

## 2. REVIEW OF GNSS OBSERVABLES AND TECHNIQUES OF OBSERVATION

There are two types of GNSS observables, namely the code pseudoranges and carrier-phase observables. In general, the pseudorange observations are used for coarse navigation, whereas the carrier-phase observations are used in high-precision surveying applications. This is due to the fact that the accuracy of the carrier-phase observations is much higher than the accuracy of code observations [3]. Both types of the GNSS observables are affected by many systematic biases,

different in their source, nature, value, and the appropriate method of treatment. These errors are grouped into the satellite errors group, the receiver errors group, and the signal propagation group [4]. The mitigation of these errors can be done through modeling these errors mathematically; differencing the GNSS observations; linear combination between the GNSS observables; using precise products such as precise satellite ephemeris and satellite clock offsets from multinational GNSS agencies such as the International GNSS Service IGS.

The general form of code pseudorange observation equation is [5]:

$$P = \rho + c(dt - dT) + d^{ion} + d^{trop} + d^{orb} + \varepsilon^p \quad (1)$$

where  $P$  is the observed pseudorange,  $\rho$  the unknown geometric satellite to receiver range,  $c$  speed of light, which is equal to 299,792,458 m/s,  $dt$  and  $dT$  are satellite and receiver clock errors respectively,  $d^{ion}$ ,  $d^{trop}$ , the error due to ionospheric, tropospheric refraction, respectively,  $d^{orb}$  the orbital error, and  $\varepsilon^p$  the code measurement noise. The precision of a pseudorange derived from code measurement has been about 1% of the chip length. Consequently, a precision of about 3m, 0.3m is achieved with C/A-code and P-code pseudoranges, respectively [6]. The observation equation of the phase pseudorange is

$$\Phi = \rho + c(dt - dT) + \lambda N - d^{ion} + d^{trop} + d^{orb} + \varepsilon^p \quad (2)$$

where  $c$ ,  $dt$ ,  $dT$ ,  $d^{ion}$ ,  $d^{trop}$ , and  $d^{orb}$  are as previously defined, the measured phase is indicated in meters by  $\Phi$ ,  $\lambda$  the carrier wavelength,  $N$  the phase ambiguity, and  $\varepsilon^p$  the combined receiver and multipath noise.

The selection of the observation technique in a GNSS survey depends upon the particular requirements of the project, and the desired accuracy. This is because the accuracy of code ranges is at the meter level, whereas the accuracy of the carrier phase is in the millimeter range [7]. However, the accuracy of the code ranges can be



improved by smoothing techniques. In addition, the code ranges are unambiguous unlike the carrier phase, which makes code ranges immune to cycle slip.

The GNSS observation techniques include precise point positioning (PPP) and relative positioning. PPP employs one GNSS receiver, while relative GNSS positioning employs two or more GNSS receivers, simultaneously tracking the same satellites. GNSS relative positioning includes static, rapid static, stop & go, kinematic, and real-time kinematic RTK techniques [8].

Surveying using GNSS has typically used relative positioning techniques due to the higher positioning accuracy obtained from the relative techniques compared to that of the PPP. A major disadvantage of GNSS relative technique is the dependency on the measurements or corrections from the base receiver. The largest difference between relative GNSS and PPP is the way that the satellite and receiver clock errors are handled. Instead of between-receiver differencing to remove the satellite clock errors, PPP uses highly precise satellite clock estimates. These satellite clock estimates are derived from a solution using data from a globally distributed network of GNSS receivers [9].

### 3. METHODOLOGY OF INVESTIGATION

As previously stated, the objective of this paper is to investigate the accuracy improvement on the UTM

coordinates when using two base GNSS receivers instead of one base for GNSS static baselines up to 45 km. The methodology will be based on the statistical analysis of the behavior of discrepancies in the UTM coordinates E, N, and h of 15 new GNSS geodetic control points established by static technique, where the length of the baselines is ranging from 2 to 45 km. The field test was conducted in Jeddah city, Saudi Arabia in January 2015.

The main GNSS campaign was started by set up two Leica Viva GNSS receivers at two municipality control points *A* and *B* (about 8-km apart). A third GNSS receiver of the same Leica model was setup at 15 new geodetic control points. Figure-1 shows the site vicinity, the general layout of the two municipality points, and the 15 control points. The observational operating parameters were the same for the three receivers, which were: static mode, elevation angle 15 degrees, and 15 seconds epoch rate. The observational duration for each baseline was 30 minutes for the baselines up to 5 km, with an increasing occupation time of 20 minutes for every 5 km of increasing in baseline length, i.e. 50 minutes for baselines up to 10 km, 70 minutes for baselines up to 15 km, and 190 minutes for baselines up to 45 km. The raw data of the GNSS campaign were downloaded and archived for processing using Leica Geo Office LGO software.



Figure-1. Site vicinity of the GNSS campaign.

A pre-test (3 days before the main GNSS campaign) for checking the internal consistency of the two municipality points *A* and *B* was made by occupying the two control points using the same GNSS receivers with a static mission having 2 hours of observations. The data were post processed using LGO software by fixing the

UTM coordinates of the first control point *A* and deriving the coordinates of the second control point *B*. The results from this pre-test showed that internal accuracy of the UTM coordinates for the two control points are within  $\pm 2$  mm in standard deviation.



The GNSS dual-frequency data of the main GNSS campaign were processed two times in the LGO software. The first run was by fixing the UTM coordinates of the control point *A*, where 15 GNSS static baselines were processed in this run. The second run was by fixing both municipality control points *A* and *B*, where 30 GNSS static baselines were processed; 15 baselines from control point *A*, and 15 baselines from control point *B*. After processing the 30 GNSS baselines in the second run, a network adjustment using least square technique for the 15 geodetic control points was done to get the adjusted coordinates for these 15 control points. The residuals of the least square adjustment were within  $\pm 4$  mm. The processing software parameters were as follow: broadcast ephemerides, 15-sec sampling rate, Hopfield tropospheric model, and Klobuchar ionospheric model. Finally, UTM coordinates referenced to WGS84 for every new control point produced using one or two municipality control points were archived for the statistical analysis.

#### 4. ANALYSIS OF RESULTS

The UTM coordinates *E*, *N*, and *h* for the 15 new geodetic control points, resulted from processing the GNSS campaign using one and two base control points, were compared based on the following equations:

$$\Delta E = E_{two} - E_{one}; \Delta N = N_{two} - N_{one}$$

$$\Delta h = h_{two} - h_{one} \quad (3)$$

where  $\Delta E$ ,  $\Delta N$ , and  $\Delta h$  are the *E*, *N*, and *h* discrepancies between one and two reference control points, respectively,  $E_{two}$ ,  $N_{two}$ , and  $h_{two}$  are the adjusted *E*, *N*, and *h* coordinates using two base receivers,  $E_{one}$ ,  $N_{one}$ , and  $h_{one}$  are the *E*, *N*, and *h* coordinates using only one base receiver.

The 2-D and 3-D positional discrepancy between the two solutions  $\Delta P_{2d}$ ,  $\Delta P_{3d}$ , as well as the standard deviation, can be calculated using [10]:

$$\Delta P_{2d} = \sqrt{(\Delta E)^2 + (\Delta N)^2}$$

$$\Delta P_{3d} = \sqrt{(\Delta E)^2 + (\Delta N)^2 + (\Delta h)^2} \quad (4)$$

$$\sigma_{\Delta P_{2d}}^2 = \sigma_{\Delta E}^2 + \sigma_{\Delta N}^2 \quad \sigma_{\Delta P_{3d}}^2 = \sigma_{\Delta E}^2 + \sigma_{\Delta N}^2 + \sigma_{\Delta h}^2 \quad (5)$$

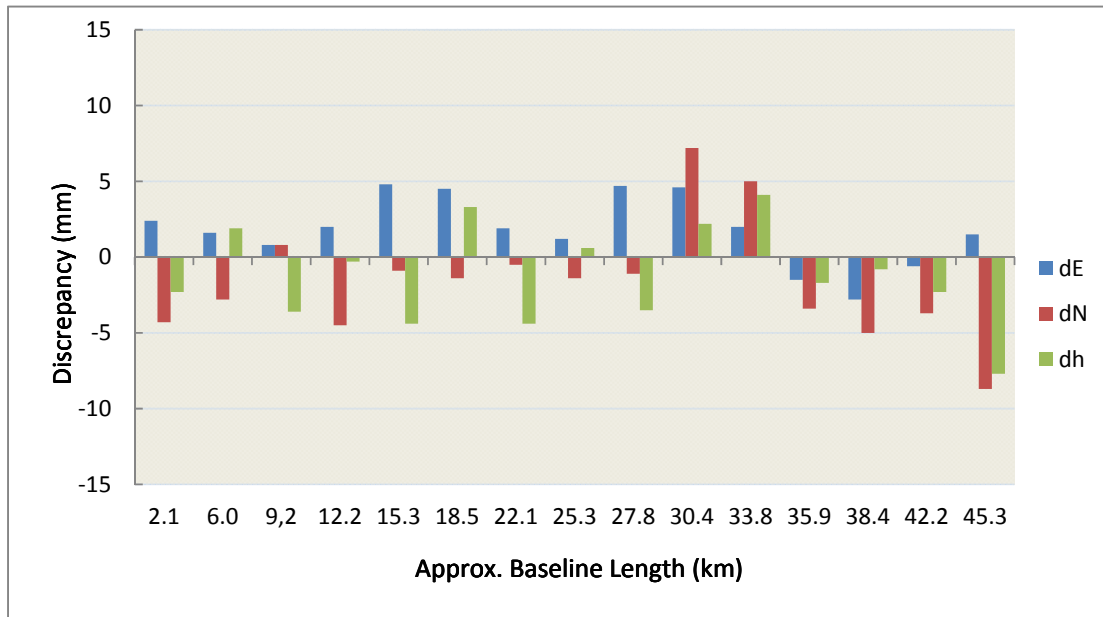
The discrepancies in *E*, *N*, *h*, 2-D and 3-D position *P*, between processing the GNSS data using one or two municipality control points are shown in Table-1.

**Table-1.** The discrepancies in *E*, *N*, *h*, horizontal and spatial position *P*.

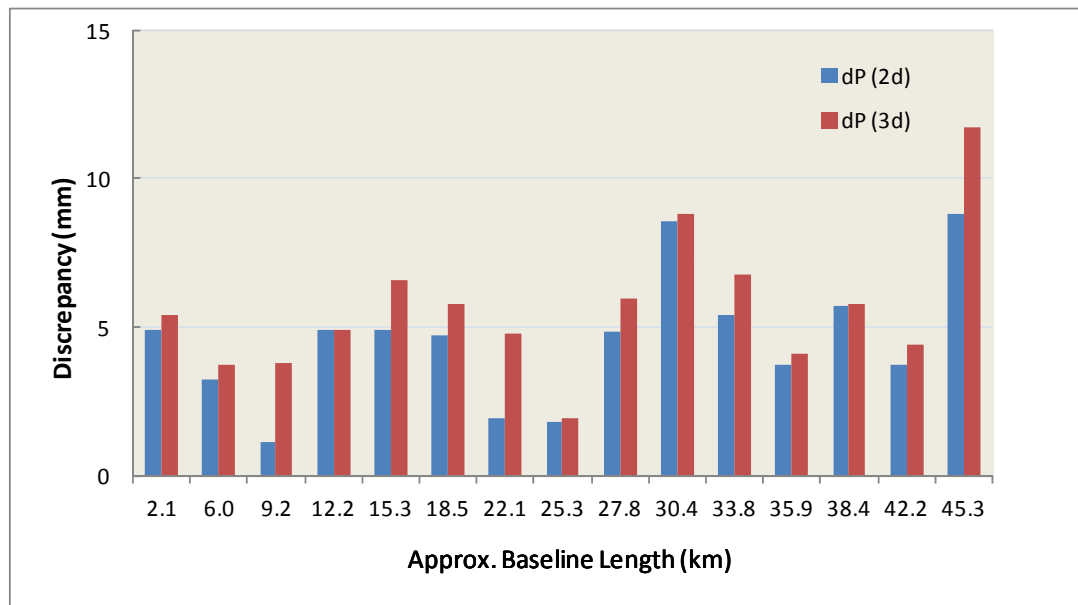
Control point	Baseline (km)	$\Delta E$ (mm)	$\Delta N$ (mm)	$\Delta h$ (mm)	$\Delta P_{2d}$ (mm)	$\Delta P_{3d}$ (mm)
1	2.1	2.4	-4.3	-2.3	4.9	5.4
2	6.0	1.6	-2.8	1.9	3.2	3.7
3	9.2	0.8	0.8	-3.6	1.1	3.8
4	12.2	2.0	-4.5	-0.3	4.9	4.9
5	15.3	4.8	-0.9	-4.4	4.9	6.6
6	18.5	4.5	-1.4	3.3	4.7	5.8
7	22.1	1.9	-0.5	-4.4	2.0	4.8
8	25.3	1.2	-1.4	0.6	1.8	1.9
9	27.8	-4.7	-1.1	-3.5	4.8	6.0
10	30.4	4.6	7.2	2.2	8.5	8.8
11	33.8	2.0	5.0	4.1	5.4	6.8
12	35.9	-1.5	-3.4	-1.7	3.7	4.1
13	38.4	-2.8	-5.0	-0.8	5.7	5.8
14	42.2	-0.6	-3.7	-2.3	3.7	4.4
15	45.3	1.5	-8.7	-7.7	8.8	11.7

Figures 2 and 3 show the *E*, *N*, and *h* coordinate discrepancies for the 15 GNSS geodetic control points,

and the 2-D and 3-D positional discrepancies *P* for the same new control points.



**Figure-2.** Variation of the E, N, and h coordinate discrepancies.



**Figure-3.** Variation of the horizontal and spatial positional discrepancies.

The descriptive statistics of the above findings are summarized in Table-2. For instance, the *E*-coordinate discrepancies have a mean value 1.2 mm and standard deviation (SD)  $\pm 2.7$  mm for single determination. The *N*-coordinate discrepancies have a mean value of  $-1.6$  mm and  $\pm 3.9$  mm SD for single determination. The *h*-

coordinate discrepancies have 1.3 mm mean value and  $\pm 3.3$  mm SD for single determination. On the other hand, the horizontal positional discrepancy 2-D and the spatial positional discrepancy 3-D have 4.6 mm and 5.6 mm mean values, respectively, with standard deviations of  $\pm 4.5$  mm and  $\pm 5.6$  mm, respectively.

**Table-2.** Descriptive statistics of the discrepancies (mm).

Disc.	Max.	Min.	Range	Mean	S. D <sub>single</sub>	S. D <sub>mean</sub>
ΔE	4.8	-4.7	9.5	1.2	2.7	0.7
ΔN	7.2	-8.7	15.9	-1.6	3.9	1.0
Δh	4.1	-7.7	11.8	-1.3	3.3	0.8
ΔP <sub>2d</sub>	8.8	1.1	7.7	4.6	4.5	1.2
ΔP <sub>3d</sub>	11.7	1.9	9.8	5.6	5.6	1.4

The above discrepancies were tested for any systematic errors in the sample. In this regards, the sample mean was examined according to its deviation from the mean of the population which is theoretically is zero [10]. The assumption for this sample is to be normally distributed and the sample data were collected from a population of unknown true mean  $\mu$  equal to zero. The following confidence interval can be used:

$$\mu - T.SD_{mean} < m < \mu + T.SD_{mean} \quad (6)$$

where  $\mu$  is the population mean ( $\mu = 0$ ),  $SD_{mean}$  is the sample standard deviation,  $T$  is critical  $t$  value from Student's  $t$ -distribution corresponding to sample size  $n$  and degree of freedom  $df$  equal to  $n - 1$ , using a confidence level of 95%. Table (3) summarizes the results, which shows that no significant systematic errors are found in the discrepancies between using one or two municipality control points to establish new geodetic control points.

**Table-3.** Testing the deviation of the sample mean.

Disc.	df	T	SD <sub>mean</sub>	Lower limit	Mean	Upper limit	Test result
ΔE	14	2.14	0.7 mm	-1.5 mm	+1.2 mm	+1.5 mm	pass
ΔN	14	2.14	1.0 mm	-2.1 mm	-1.6 mm	+2.1 mm	pass
Δh	14	2.14	0.8 mm	-1.7 mm	-1.3 mm	+1.7 mm	pass

## 5. CONCLUSIONS

The objective of this paper is to study the accuracy improvement in case of using two GNSS reference receivers as bases instead of using one base receiver, to establish geodetic GNSS control points using static technique on baselines up to 45 km in length. In this context, a field GNSS campaign was run in Jeddah city of Saudi Arabia to establish 15 new geodetic control points. These 15 control points were measured using GNSS static technique using either one or two reference control points. The office work involved processing of GNSS static baseline ranging from 2 to 45 km based on one reference municipality control point; and processing of 30 GNSS static baselines for the same 15 control points based on two reference municipally control points, and applying the least square adjustment technique to get the most probable values for the 15 new control points.

The discrepancies in E, N, and h coordinates for the 15 control points between processing the GNSS baselines using one or two reference control point (s) were statistically analyzed. The results showed that a mean discrepancy in the E-coordinate of 1.2 mm with a standard deviation of  $\pm 0.7$  mm. The mean discrepancy in N-coordinate was -1.6 mm with a standard deviation of  $\pm 1.0$  mm. The mean discrepancy for the h-coordinate was -1.3 mm with a standard deviation of  $\pm 0.8$  mm. On the other hand, the horizontal positional discrepancy (P<sub>2d</sub>) between the two solutions had a mean value of 4.6 mm with a

standard deviation of  $\pm 1.2$  mm, while the mean spatial positional discrepancy (P<sub>3d</sub>) had a mean value of 5.6 mm with standard deviation of  $\pm 1.4$  mm.

The above results showed that using two GNSS dual-frequency base receivers for processing static GNSS data when establishing new geodetic control points is enhancing the accuracy of the UTM coordinates for these control points by about 5.6 mm, for GNSS static baselines up to 45 km. This result is about 0.12 ppm, which can be considered insignificant from the accuracy point of view in establishing geodetic networks. Thus from an economic and cost point of views, cost of equipment operations, and maintenance, it is recommended to use only one dual-frequency GNSS base receiver to save the cost and time of establishing geodetic GNSS control points up to baselines lengths of 45 km.

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