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# ANALYTICAL SOLUTION OF THE PROBLEM OF COMBUSTION WAVE PROPAGATION IN A HOMOGENEOUS POROUS LAYER OF ORGANIC COMBUSTIBLE MATERIALS

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**ABSTRACT**

Paper contains model of combustion wave propagation in one-dimensional homogeneous layer of forest fuels taking into account the diffusion transfer of heat, release of energy due to combustion and dissipation of energy. Structure of combustion wave obtained. Conditions of equality of left and right limits of temperature and its spatial derivative in ignition point and ignition temperature used to obtain value of combustion wave propagation.

**Keywords:** combustion wave, the analytical solution, the diffusion of heat transfer.

## 1. INTRODUCTION

Modeling of extinguishing processes based on a full physical model is a complex computational problem [1, 2], which significantly complicates the analysis of the influence of parameters on the process characteristics. The problem of suppression the combustion wave is multi-parameter, thus investigation of key processes based on numeric methods represents a significant computational challenge. Such resource-intensive models are not suitable for operational forecasts. The simplest approach is to use cellular automata, which, as a rule, compare the state of the cell with the space temperature of corresponding region and condition of vegetation on it, such as in [3-5]. In order to use this approach more rigorous patterns are required. For this reason, in spite of the computer technology development, analytical solutions of combustion problems are still relevant.

Among the analytical solutions, obtaining burning speed, highlight set of papers based on the classical theory of combustion, which founders are the Frank-Kamenetskii and Zeldovich [6-7]. In these papers, the focus is on the investigating and forecasting of the combustion front velocity and the nature of its stability. The logical continuations of these works are [8-15].

Problems of the dynamics of forest fires are a arouse interest. Papers [16, 17] devoted to the determination of combustion wave velocity propagation at the ground and crown fires in the stationary case. One of the traditional approaches consists in assumption that the process propagates at predetermined velocity. It leads to the ordinary differential equation or equation system, where the velocity is a parameter [16-19]. So papers [16-17] consider the stationary propagation of the crown forest front of fire based on the simplified diffusion-thermal and thermal mathematical models. These papers give a solution of the problem of finding the velocity of propagation of the combustion wave and its configuration based on various assumptions. The author uses the assumption of the infinitely small thickness of combustion and pyrolysis front, which leads to a homogeneous differential equation with the weak discontinuity of temperature rupture in the pyrolysis and combustion fronts. In order to determine combustion wave propagation velocity, the author uses the condition of temperature equality of the fire front to the given value obtained numerically and experimentally. Such a solution is suitable only if the infinitely thin combustion front exists, that is a limitation focusing the assumption above. Moreover, the assumption of the instantaneous combustion reaction at a given temperature does not allow modeling the causal relationship in any point between the preset temperature and the process of ignition at this point because it would lead to an unlimited increase in the burning rate. That's why in the A.M. Grishins papers temperature at the point of ignition is introduced as a condition for the problem closing only.

In [8] the problem of flame propagation in a binary mixture is considered. This problem is formulated in the form of two equations (energy and concentration of one of the components). The reaction rate is determined by the Arrhenius law. The paper provides an expression of combustion wave velocity as an integral expression of the temperature, which is a useful result, but it does not allow obtain propagation wave velocity as function of independent input parameters. Paper [19] contains the relation between combustion wave configuration and its velocity, while the value of the combustion wave velocity is determined only numerically based on the numeric solution of the heat balance partial derivative equation. The paper considers one-dimensional and two-dimensional formulation of the combustion wave propagation problem. Two-dimensional formulation does not allow obtain a solution in the form of a stationary combustion wave. Even in the simplest case of the central symmetry in the polar coordinate system, the problem is reduced to one-dimensional equation with coefficients containing the radial coordinate. Therefore the solution approaches the stationary only if the values of the radial coordinate of the combustion wave are big, that is equivalent to line combustion front. That is, this approach may be used only if there is a stationary solution that is independent of some linear combination of spatial and time coordinates.

In [20] solved the problem of determination the velocity and extreme conditions of wildfires propagation. As part of the problem solution, the author emphasizes that practitioners are primarily interested in such characteristics of the process of burning of forest vegetation as the velocity of propagation, the maximum temperature, and the total radiative heat fluxes into the media. This paper uses a averaged by height one-dimensional model of the surface fire propagation on the vegetation layer in dimensionless variables associated with the position of the temperature profile of the maximum [16, 17, 20], which includes ordinary differential equations describing changes of temperature and concentration of the components of a binary mixture. This problem was solved by the diffusion approximation that allowed the author to exclude the kinetics of combustion processes. As a result of solution of the boundary value problem [20], the author found a relation between dimensionless form of the propagation velocity, the total thermal effect of combustion, heat and mass transfer coefficients and the maximum temperature [20].

Consideration of the heat balance differential equation in the partial, reducing it to an ordinary differential equation and its formal integration provides to set of solutions, depending on the propagation velocity; it is a useful result, but not complete. For a complete solution requires a physically reasonable algorithm for choosing solutions from this set to determine the propagation velocity.

Thus, the aim of this paper is determining the propagation velocity and structure the combustion wave based on the analytical solution taking into account conduction, heat dissipation and heat emission in combustion.

## 2. COMBUSTION WAVE FORMULATION

Let's consider the two-dimensional formulation of the problem of the stationary combustion wave propagation in the porous uniform layer of combustible materials. Analytical solution of this problem is possible, as a rule, in a substantially simplified formulation. Let's look at the equation of thermal balance, modeling propagation of the combustion wave.

$$\frac{\partial T}{\partial t} = -k\left(T - T_a\right) + K\Delta T - Q\frac{\partial \rho_v}{\partial t},$$

where $T$, K - the temperature in the layer of combustible materials; $k$, 1/s - heat transfer coefficient of the fuel layer; $T_a$, K – ambient temperature; $\Delta$ - Laplace operator; $K$, m$^2$/s - thermal conductivity within the layer of combustible materials; $Q$, K - thermal effect of flammable materials combustion divided by their specific heat; $\rho_v$ - the fraction of unburnt combustible materials; $t$, s - time.

This equation is a differential equation in partial derivatives. It can be solved numerically or analytically. It should be noted that the application of Fourier method for solving this equation represents a significant difficulty due to the presence in the equation $\rho_v$ that depends nonlinearly on the temperature.

According to the method, described in [9], it is assumed that the process propagates with a constant speed $c$. So transformation $T(x,t) = T\left(t - \frac{x}{c}\right)$ can be used. When the point temperature is more than ignition temperature, starts the combustion Thus, the heat balance equation with the boundary conditions takes the form

$$\begin{cases} \frac{K}{c^2}u'' - u' - ku = Q\rho_v' \\ u(-\infty) = u(+\infty) = 0 \end{cases}, \quad \rho_v = \begin{cases} 1, t \le 0 \\ e^{-\alpha t}, t > 0 \end{cases} \tag{1}$$

where $u = T - T_a$ .

## 3. NONDIMENSIONALIZATION OF COMBUSTION WAVE FORMULATION

Equation (1) has a discontinuous right-hand side, which complicates its solution by classical methods. Moreover, this equation contains parameter $c$, which was introduced during the transition from differential equations in partial derivatives, so the solution of (1) is not sufficient to determine speed of propagation of combustion and heat distribution.

Therefore, condition of equality ignition point temperature to temperature of ignition having form $u(0) = u_{th}$ should be used. If the temperature of ignition point is higher than $u_{th}$, than $c$ is lower bound of propagation velocity. If temperature of ignition point is lower than $u_{th}$, than $c$is upper bound of propagation velocity.

The problem (1), supplemented by the condition of ignition contains 6 parameters that complicates the analysis, therefore, for obtaining a solution that is easy to analyze, we should reduce equation (1) to dimensionless form by introducing a critical time and temperature of $u_{th}$ .

$$u = u_{th}\bar{u}, \; t = t_*\bar{t}, \; Q = u_{th}\bar{Q}, \tag{2}$$

$$\frac{Ku_{th}}{c^2 t_*^2}\bar{u}'' - \frac{u_{th}}{t_*}\bar{u}' - ku_{th}\bar{u} = \frac{u_{th}\bar{Q}}{t_*}\rho_v', \; \rho_v = \begin{cases} 1, \bar{t} \le 0 \\ e^{-\alpha t_* \bar{t}}, \bar{t} > 0 \end{cases}. \tag{3}$$

After dividing both hands of equations by $\frac{Ku_{th}}{c^2 t_*^2}$, and omitting the bar over dimensionless quantities we obtain

$$u'' - \frac{c^2 t_*^2}{Ku_{th}}\frac{u_{th}}{t_*}u' - \frac{c^2 t_*^2}{Ku_{th}}ku_{th}u = \frac{c^2 t_*}{K}Q\rho_v'. \tag{4}$$

After simple transformations we obtain

$$u''-\frac{c^2 t_*}{K}u'-\frac{c^2 t_*^2}{K}ku=\frac{c^2 t_*}{K}Q\rho_v'. \quad (5)$$

Let's define the relation between the reaction rate and the characteristic time as $t_*=\alpha^{-1}$, and multiply both sides of equation (5) by $\frac{K\alpha}{c^2}$. As a result of transformations we obtain

$$\frac{K\alpha}{c^2}u''-u'-\frac{k}{\alpha}u=Q\rho_v',\ \rho_v=\begin{cases}1, t\le 0\\ e^{-t}, t>0\end{cases} \quad (6)$$

In order to simplify the solution and its analysis, we introduce the following notation

$$c_{K\alpha}=c/\sqrt{K\alpha},\ k_\alpha=k/\alpha, \quad (7)$$

$$\frac{1}{c_{K\alpha}^2}u''-u'-k_\alpha u=Q\rho_v',\ \rho_v=\begin{cases}1, t\le 0\\ e^{-t}, t>0\end{cases} \quad (8)$$

## 4. BOUNDARY VALUE PROBLEM SOLUTION

The boundary value problem (8) has a solution which form depends on the $t$ sign. Cause of the function $\rho_v$ peculiarities we have a singular point $t=0$. Solution of equation (8) can be considered independently of each interval of continuity of right-hand side. Equation (8) is a linear ordinary differential equation of second order with constant coefficients and special right-hand side. The general solution without conditions of cross-linking and boundary conditions, in the absence of resonance can be written as:

$$u=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}+A_0e^{-t}, \quad (9)$$

where $\lambda_{1,2}$ - the roots of the characteristic equation, $A_0$ - function depending on the right-hand side of equation (8), $C_1, C_2$ - constants on each interval with continuous coefficients of the equation.

$$\lambda_{1,2}=\frac{c_{K\alpha}^2\left(1\mp\sqrt{1+4\frac{k_\alpha}{c_{K\alpha}^2}}\right)}{2},\ \lambda_1<0,\ \lambda_2>0, \quad (10)$$

$$A_0=\begin{cases}0, t\le 0\\ -A, t>0\end{cases}, A=\frac{c_{K\alpha}^2 Q}{1+c_{K\alpha}^2(1-k_\alpha)} \quad (11)$$

It should be noted that the roots of the characteristic equation are different considering the positive value of parameters. In the case where $\lambda_1$ equal to -1 resonance takes place. In this case denominator in (11) turns to zero. In case of resonance solution can be found as $u=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}+\frac{Q}{k_\alpha}te^{-t}$. However, resonance does not make differences in solutions.

As noted above, the right-hand side has a different structure for positive and negative values of the variable t, therefore in order to apply classical solutions it is necessary to determine the crosslinking condition for time $t=0$. To this end, we integrate equation (8) in a small neighborhood $\varepsilon>0$.

$$\int_{-\infty}^{t}\int_{t-\varepsilon}^{t+\varepsilon}\left(\frac{1}{c_{K\alpha}^2}u''-u'-k_\alpha u\right)dtdt=\int_{-\infty}^{t}\int_{t-\varepsilon}^{t+\varepsilon}(Q\rho_v')dtdt. \quad (12)$$

Let's integrate the expression (12) for the inner integral taking into account that the integration interval is infinitesimal, while $u$ and $\rho_v'$ are finite.

$$\int_{-\infty}^{t}\left[\left(\frac{1}{c_{K\alpha}^2}u'-u\right)\Bigg|_{t+\varepsilon}-\left(\frac{1}{c_{K\alpha}^2}u'-u\right)\Bigg|_{t-\varepsilon}\right]dt=0. \quad (13)$$

Taking into account the finiteness of the integral of $u$ over any segment in (13) and integrating, we obtain

$$\left\lfloor u\big|_{t+\varepsilon}-u\big|_{t-\varepsilon}\right\rfloor=0. \quad (14)$$

Equation (14) means that the function is continuous. In order to solve problem (8) it is necessary to find a condition similar to (14) for the derivative. Integrate equation boundary value problem (8)

$$\int_{t-\varepsilon}^{t+\varepsilon}\left(\frac{1}{c_{K\alpha}^2}u''-u'-k_\alpha u\right)dt=\int_{t-\varepsilon}^{t+\varepsilon}(Q\rho_v')dt. \quad (15)$$

Taking into account that the integration interval is infinitesimal and terms $u$ and $\rho_v'$ are finite, we integrate the expression (8)

$$\left(\frac{1}{c_{K\alpha}^2}u'-u\right)\Bigg|_{t-\varepsilon}^{t+\varepsilon}=0. \quad (16)$$

Taking into account the expression (14) we obtain the condition for the crosslinking of the derivative in form

$$\left\lfloor u'\big|_{t+\varepsilon}-u'\big|_{t-\varepsilon}\right\rfloor=0 \quad (17)$$

For obtaining solution weshould assume that at each interval of the members (1) continuity values $C_1, C_2$ are different. The solution of equation (8) has two

intervals continuity on which the integration constants are different, and therefore the solution looks like

$$u = \begin{cases} C_{1,1}e^{\lambda_1 t} + C_{2,1}e^{\lambda_2 t}, \ t \le 0 \\ C_{1,2}e^{\lambda_1 t} + C_{2,2}e^{\lambda_2 t} - Ae^{-t}, \ t > 0 \end{cases} \tag{18}$$

In order to determine integration constants we use crosslinking conditions for the function (14) and its derivative (17) and determine the left and right derivatives at the point $t = 0$ in the expression (18). Crosslinking conditions are supplemented by the boundary conditions at infinity (9), resulting in to the following expressions

$$\begin{cases} u(-\infty) = u(+\infty) = 0, \\ \left[u'|_{0+\varepsilon} - u'|_{0-\varepsilon}\right] = 0, \\ u(+0) - u(-0) = 0, \end{cases} \tag{19}$$

$$\begin{aligned} u'|_{0+\varepsilon} &= \lambda_1 C_{1,2}e^{\lambda_1 0} + \lambda_2 C_{2,2}e^{\lambda_2 0} + Ae^{-0}, \\ u'|_{0-\varepsilon} &= \lambda_1 C_{1,1}e^{\lambda_1 0} + \lambda_2 C_{2,1}e^{\lambda_2 0}. \end{aligned} \tag{20}$$

Since the value of $u$ in (18) should become zero according to the boundary conditions at the infinity, then according to (11) respecting signs of $\lambda_1$, $\lambda_2$ should be considered $C_{1,1} = C_{2,2} = 0$. As a result of simple transformations, we obtain a system of linear algebraic equations for the constants of integration in following form

$$\begin{cases} C_{2,1} = C_{1,2} - A \\ \left[\lambda_1 C_{1,2} + A - \lambda_2 C_{2,1}\right] = 0 \end{cases} \tag{21}$$

Thus, the values of the constants in the solution of (18) looks like

$$\begin{cases} C_{1,1} = 0, \\ C_{2,2} = 0, \\ C_{2,1} = -\dfrac{A(1+\lambda_1)}{\lambda_1 - \lambda_2}, \\ C_{1,2} = -\dfrac{A(1+\lambda_2)}{\lambda_1 - \lambda_2}. \end{cases} \tag{22}$$

Using the obtained values of the integration constants (22) into (18) we obtain the solution into the following form

$$u = \begin{cases} -\dfrac{A(1+\lambda_1)}{\lambda_1 - \lambda_2}e^{\lambda_2 t}, \ t \le 0 \\ -\dfrac{A(1+\lambda_2)}{\lambda_1 - \lambda_2}e^{\lambda_1 t} - Ae^{-t}, \ t > 0 \end{cases} \tag{23}$$

## 5. OBTAINING COMBUSTION WAVE PROPAGATION VELOCITY

Expression (23) is a solution of (8), but it implicitly includes an arbitrary parameter - the dimensionless combustion wave velocity $c_{k\alpha}$, introduced in progress of solving the problem, so to determine the dynamics of the combustion wave propagation additional analysis is necessary. We introduce the function, depending on the parameter $c_{K\alpha}$ specified as $u_0(c_{K\alpha}) = u(0)$. As shown previously, the function $u(t)$ is continuous, that is $u(0) = u(-0) = u(+0)$, therefore, value at t=0 in (23) can be chosen at any interval. As a result of simple transformations (23), we have

$$u_0(c_{K\alpha}) = -\frac{A(1+\lambda_1)}{\lambda_1 - \lambda_2} \tag{24}$$

The process of propagation the combustion wave is only possible in case of the condition $u_0(c_{K\alpha}) = 1$, taking into account (4)

$$\frac{Q\left(-c_{K\alpha}^2\sqrt{\dfrac{4k_\alpha + c_{K\alpha}^2}{c_{K\alpha}^2}} + 2 + c_{K\alpha}^2\right)}{2\left(1 + c_{K\alpha}^2(1-k_\alpha)\right)\sqrt{\dfrac{4k_\alpha + c_{K\alpha}^2}{c_{K\alpha}^2}}} = 1. \tag{25}$$

In order to solve the equation (25) square root term should be moved to the left-hand side, and all the other terms in the right-hand side of equation

$$\sqrt{\frac{4k_\alpha + c_{K\alpha}^2}{c_{K\alpha}^2}} = \frac{Q\left(2 + c_{K\alpha}^2\right)}{c_{K\alpha}^2(Q - 2k_\alpha + 2) + 2}. \tag{26}$$

The solution (26) leads to the following set of roots

$$\begin{aligned} c_{K\alpha,1,2} &= \sqrt{\frac{Q_1{}^2 - 4k_\alpha - 1 \pm \sqrt{\left((Q_1-1)^2 - 8k_\alpha\right)(Q_1+1)^2}}{2(Q_1 + k_\alpha + 1)}}, \\ c_{K\alpha,3,4} &= -\sqrt{\frac{Q_1{}^2 - 4k_\alpha - 1 \pm \sqrt{\left((Q_1-1)^2 - 8k_\alpha\right)(Q_1+1)^2}}{2(Q_1 + k_\alpha + 1)}} \end{aligned} \tag{27}$$

where $Q_1 = Q - 2k_\alpha$.

Necessary and sufficient conditions for real solutions existence are radical expression in (27) and the left-hand side of equation (26). They are non-negative and radical expression under external root is positive. Those conditions are correspond to the system of inequalities

$$\begin{cases} \left((Q_1-1)^2-8k_\alpha\right)(Q_1+1)^2 \ge 0 \\ c_{K\alpha}^2(Q_1+2)+2>0 \\ \left(Q_1{}^2-4k_\alpha-1\right)^2 \ge \left((Q_1-1)^2-8k_\alpha\right)(Q_1+1)^2 \end{cases} \quad (28)$$

This condition is equivalent to the system

$$\begin{cases} \left[\begin{array}{l}(Q_1-1)^2 \ge 8k_\alpha \\ Q_1=-1\end{array}\right. \\ c_{K\alpha}^2(Q_1+2)+2>0 \\ Q_1{}^4+(-8k_\alpha-2)Q_1{}^2+(-4k_\alpha-1)^2 \ge Q_1{}^4+(-8k_\alpha-2)Q_1{}^2-16k_\alpha Q_1-8k^{\alpha}+1 \end{cases} \quad (29)$$

After substituting expression $Q_1=-1$ in the solution (27), we obtain

$$c_{K\alpha\,1,2}=\sqrt{\frac{(-1)^2-4k_\alpha-1}{2(k_\alpha-1+1)}}=\sqrt{\frac{-4k_\alpha}{2k_\alpha}}=\sqrt{-2}\,,$$

which leads to an imaginary solution. Therefore, the value should be excluded from consideration. After some simple transformations we obtain

$$\begin{cases} \left[\begin{array}{l}Q_1 \le -\sqrt{8k_\alpha}+1 \\ Q_1 \ge \sqrt{8k_\alpha}+1\end{array}\right. \\ c_{K\alpha}^2(Q_1+2)+2>0 \\ 8k_\alpha(2k+1) \ge 8k_\alpha(-2Q_1-1) \end{cases}$$

(30)

The condition $Q_1 \le -\sqrt{8k_\alpha}+1$ is not physical, since according to it reduction of heat of combustion contributes to combustion wave propagation. Perform also simple transformations of the last condition of (31). In view of the foregoing, the system (31) takes the form

$$\begin{cases} Q_1 \ge \sqrt{8k_\alpha}+1 \\ Q_1 \ge -\dfrac{2}{c_{K\alpha}^2}-2 \\ 8k_\alpha(2k+1) \ge 8k_\alpha(-2Q_1-1) \end{cases} . \quad (32)$$

Obviously, all solutions of second inequality are also solutions of first inequality. Thus, the necessary condition for the existence of solutions of (25) has the form $Q_1 \ge \sqrt{8k_\alpha}+1$. In case of fulfilling this condition, the right-hand side of the third inequality is non-positive, while the left-hand side - non-negative for any non-negative values of the variables satisfying first inequality. Then the third inequality should be excluded from consideration too. It should also be noted that if $Q_1 \ge \sqrt{8k_\alpha}+1$ then denominator in (27) does not become zero.

In order to determine velocity of combustion wave propagation only positive solutions should be considered, namely $c_{K\alpha,1,2}$ . If there are two positive solutions of (27) then $u_0(c_{K\alpha})>1$ when $c_{K\alpha,1}<c_{K\alpha}<c_{K\alpha,2}$. Thus, a steady combustion wave regime corresponds to the velocity $c_{K\alpha,2}$. Condition $Q_1=\sqrt{8k_\alpha}+1$ corresponds to the single solution $c_{K\alpha,1}=c_{K\alpha}=c_{K\alpha,2}$.

Thus, the velocity of combustion wave propagation has the following form

$$c_{K\alpha}=\begin{cases} \sqrt{\dfrac{Q_1{}^2-4k_\alpha-1+\sqrt{\left((Q_1-1)^2-8k_\alpha\right)(Q_1+1)^2}}{2(Q_1+k_\alpha+1)}} & ,Q_1 \ge \sqrt{8k_\alpha}+1 \\ 0 & ,Q_1<\sqrt{8k_\alpha}+1 \end{cases} \quad (33)$$

where $Q_1=Q-2k_\alpha$ .

Dimensionless temperature distribution in the combustion wave is determined by (1.23), with the substitution of (1.10), (1.11) and (1.33).

For illustration, let's present a numerical solution of the original problem and find its stationary solution in the one-dimensional case. Statement of the problem is as follows [19]

$$\frac{\partial T}{\partial t}=-k(T-T_a)+K\frac{\partial^2 T}{\partial x^2}-Q\frac{\partial \rho_v}{\partial t}\,,\ \frac{\partial \rho_v}{\partial t}=-\omega\alpha\rho_v \quad (34)$$

$$\omega(x,t)=\begin{cases} 1, \max\limits_t(T(x,t)) \ge T_{th} \\ 0, \max\limits_t(T(x,t)) < T_{th} \end{cases} \quad (35)$$

**Boundaries**

$$\rho_v(x=-\infty)=1, \rho_v(x=+\infty)=0, T(x=\pm\infty)=T_\alpha\,,$$
$$\omega(x=-\infty)=0, \omega(x=+\infty)=1, \quad (36)$$

where $\omega$ equals 1 in points, where ignition have occurred and 0 in rest points.

Initial conditions are

$$\rho_v(t=0)=(1-sign(x)) \quad (37)$$

Initial temperature conditions are chosen in a way that they should be a upper bound for the analytical solutions. This choice ensures convergence to a stationary regime, subject to existence. The following parameters were used for solving the problem: $k=0.071\,s^{-1}$, $K=3.1\cdot 10^{-5}\,m^2 s^{-1}$, $Q=1200\,C$, $\alpha=0{,}19 s^{-1}$, $T_\alpha=29\,C$, $T_{th}=300\,C$ .

In order to solve problem (34)-(37) following numeric scheme was used:

$$G_i^n = \begin{cases} 1, T_i^n \geq T_{th}, \\ 0, T_i^n < T_{th}. \end{cases} \quad (38)$$

$$\omega_i^n = \max\left(\omega_i^n, G_i^n\right) \quad (39)$$

$$T_i^{n+1} = \Delta t\left(-k\left(T_i^n - T_a\right) + K\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} + Q\alpha\left(\rho_v\right)_i^n \omega_i^n\right) + T_i^n \quad (40)$$

$$\left(\rho_v\right)_i^{n+1} = \left(\rho_v\right)_i^n - \alpha\omega_i^n\left(\rho_v\right)_i^n \Delta t \quad (41)$$

Values of spatial and temporal steps were $\Delta x = 3{,}5\cdot 10^{-4} m$, $\Delta t = \frac{\Delta x^2}{4K}$. Number of spatial steps was enough to make temperature near edges of computation domain would differs from ambient less than $10^{-10}$C according to analytic solution. Number of time steps was 400000. Combustion wave propagation velocity measured after 200000 time steps in numeric solution. It helped to eliminate effects of convergence process and mitigate numeric fluctuations at velocity measurement.

In order to provide effective solutions of infinite boundary conditions problem, algorithm of boundary motions was implemented. According to it, when ignition starts at the point with zero coordinates, values of all variables shifts by one space step to right and value in most left cell obtains in according to boundary conditions. Quantity of such shifts was a base of combustion wave propagation calculation.

Figure-1 contains numeric and analytic solutions of combustion wave propagation problem comparison. The ordinate axis represents temperature of the media and abscissae axis coordinate connected with the combustion wave.

As shown by numerical calculations, total relative error between numerical and analytical solution equals 0.3% according to the formula below:

$$\varepsilon = \frac{\int_{-\infty}^{+\infty}\left|T_{analytic} - T_{numeric}\right|dx}{\int_{-\infty}^{+\infty}\left|T_{analytic}\right|dx}$$

The relative error of the combustion wave propagation velocity was 0.24%. Using implicit scheme and more time steps would reduce the error, but this is not required in this work. Figure-1 (a) shows the numerical and analytical solution of this problem. Cause of small error the graphs are virtually indistinguishable, and therefore the structure of error $\Delta T = \left|T_{analytic} - T_{numeric}\right|$ is shown at the Figure-1(b). As can be seen at the Figure-1 (b), maximum error occurs at the point of ignition, which is associated with the error of its determination using numerical solution.

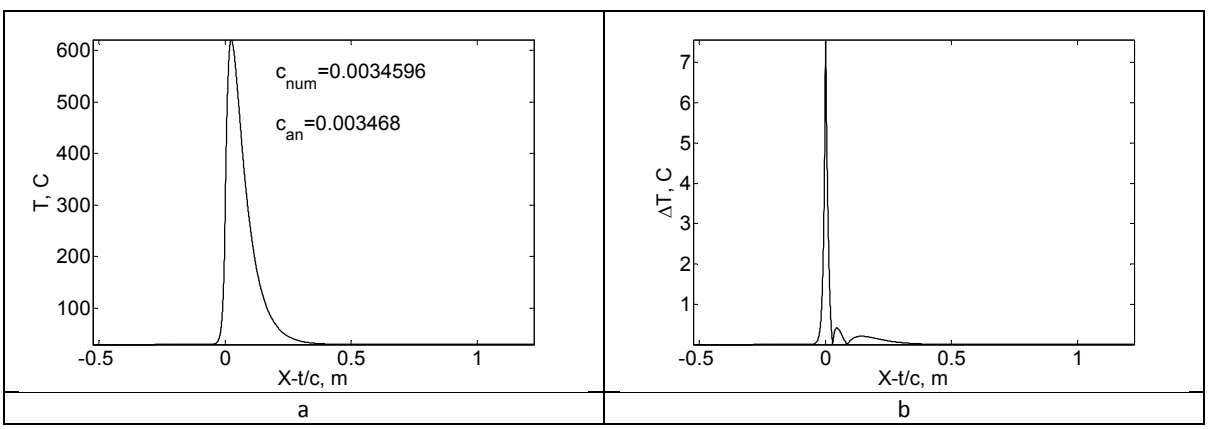


**Figure-1.** Numerical and analytical solutions of the combustion wave propagation problem and structure of the error.

## 6. CONCLUSIONS

In this paper a formula for determining the velocity of the combustion wave propagation in a homogeneous layer is obtained. In [19] condition of equality the temperature in zero point and the ignition temperature is used to solve the boundary value problem, in this article temperature and its derivative continuity used for it. Due to this, the temperature in the ignition point is used to find the velocity of the combustion wave, whereas in [9] it done numerically. The formulas obtained in this paper can be used to determine the patterns of propagating real fires.

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