



# EFFECT OF STIFFNESS AND DAMPING COEFFICIENT OF SQUEEZE FILM CHARACTERISTICS OF MAGNETO-HYDRODYNAMICS NON-NEWTONIAN POROUS TRIANGULAR PLATES

Sundarammal Kesavan and Nisha

Department of Mathematics, SRM University, Kattankulathur, Tamilnadu, India

E-Mail: [sundarammal.k@ktr.srmuniv.ac.in](mailto:sundarammal.k@ktr.srmuniv.ac.in)

## ABSTRACT

A magneto hydrodynamic non-Newtonian porous triangular plate with couple stress fluid has been analyzed. A modified Reynolds's equation has been derived to account for the transverse magnetic field with lubricant as a couple stresses of porous triangular plates. The dynamic stiffness and dynamic coefficient are obtained and it shows a wide range of distribution when different externally magnetic fields are chosen.

**Keywords:** squeeze film, triangular plates, couple stress fluid, magnetic field, damping coefficient, stiffness coefficient.

## 1. INTRODUCTION

The objective of lubrication is to reduce friction, wear, and heating of machine parts that move relative to each other, lubricant is any substance that when inserted between the moving surfaces, accomplishes these purposes.

The study of magneto hydrodynamic (MHD) lubrication has attracted attention of several investigators in the field, because of its importance in many industrial applications. Traditionally, the analysis of porous squeeze film bearings was based on the Darcy's model, where the fluid flow in the porous matrix obeys Darcy's model, and the bearing/film interface the no-slip condition was assumed.

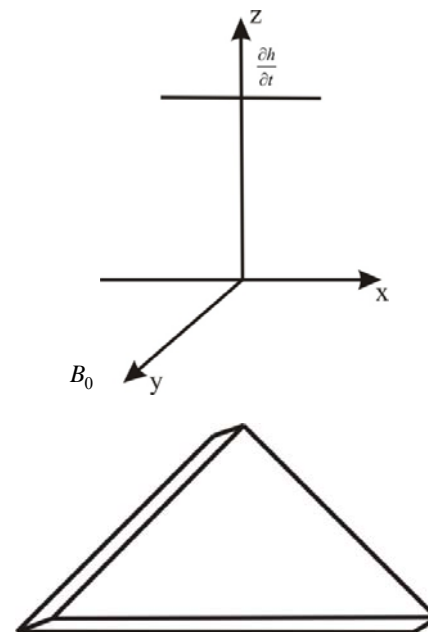
The MHD lubrication phenomenon has many industrial applications, because of increased use of liquid metal lubricants in high temperature. As most MHD bearings models have been developed from the view point of simplicity of mathematical analysis rather than their practicability hence the study of MHD lubrication is much more involved than that of hydro dynamic lubrication.

In 2014, Biradar Kasinath and B.N. Hanumagouda, have analyzed the combined effects of MHD and couple stress on the wide composite slider bearing, he found that the effect of couple stress is to increase the non-dimensional pressure, load carrying capacity and frictional force but decrease the coefficient of friction. In 2012, M. Rajashekar and Biradar Kashinath concluded that the presence of an externally applied transverse magnetic field provides an enhancement in the load carrying capacity. Stiffness is the rigidity of an object-the extent to which it resists deformation in response to an applied force. The complementary concept is flexibility or pliability; the more flexible an object is, the less stiffness it is. In 2010, Jaw-Ren Lin and Rong-Fang Lu has investigated theoretically on wide slider bearings with an exponential film profile and concluded that comparing with the Newtonian couple stress case, the MHD exponential shaped bearing provides an increase in the steady load and the dynamic characteristics of the bearing in which the improvements of dynamic damping

characteristics are emphasized for bearings designed at smaller profile parameters.

In 2015, Sundarammal Kesavan and Santhana Krishnan Narayanan concluded that comparing with the conventional Newtonian case, the MHNN curved circular porous squeeze film dynamic characteristics are proved by the use of an electrically conducting non-Newtonian fluid in the presence of external magnetic fields.

## 2. MATHEMATICAL FORMULATION



Here we are assuming that the lower plate is fixed while the upper plate moves normally towards the lower plate with velocity  $V = \frac{dh}{dt}$ , here we are using the lubricant as couple stress fluid.



### 3. BASIC EQUATION

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u \quad (1)$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} - \sigma B_0^2 v \quad (2)$$

$$\frac{\partial p}{\partial z} = 0, \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z} \quad (4)$$

With geometry under consideration the relevant boundary conditions are

(i) At upper surface ( $z=h$ )

$$u=0, v=0, \frac{\partial^2 u}{\partial z^2} = 0, \frac{\partial^2 v}{\partial z^2} = 0,$$

$$w = \frac{dh}{dt} \quad (5(i))$$

(ii) At lower surface ( $z=0$ )

$$u=0, v=0, \frac{\partial^2 u}{\partial z^2} = 0, \frac{\partial^2 v}{\partial z^2} = 0, w = 0$$

$$u=0, v=0, \frac{\partial^2 u}{\partial z^2} = 0, \frac{\partial^2 v}{\partial z^2} = 0,$$

$$w = \frac{dh}{dt} \quad (5(ii))$$

$$u = \left\{ \frac{1}{\mu} \frac{\partial p}{\partial x} \left( \frac{h_0^2}{(C_1^2 - C_2^2) M_0^2} \left( \frac{C_2^2 \cosh \frac{C_1(2z-h)}{2l}}{\cosh \frac{C_1 h}{2l}} - \frac{C_1^2 \cosh \frac{C_2(2z-h)}{2l}}{\cosh \frac{C_2 h}{2l}} \right) + 1 \right) \right\} \quad (6)$$

similarly,

$$v = \left\{ \frac{1}{\mu} \frac{\partial p}{\partial y} \left( \frac{h_0^2}{(C_1^2 - C_2^2) M_0^2} \left( \frac{C_2^2 \cosh \frac{C_1(2z-h)}{2l}}{\cosh \frac{C_1 h}{2l}} - \frac{C_1^2 \cosh \frac{C_2(2z-h)}{2l}}{\cosh \frac{C_2 h}{2l}} \right) + 1 \right) \right\} \quad (7)$$

$$\text{Where } M_0^2 = B_0 h_0 \left( \frac{\sigma}{\mu} \right)^{1/2}$$

$$\begin{aligned} C_1 &= \left\{ \frac{1}{2} + \frac{1}{2} \left( 1 + 4l^2 \sigma B_0^2 / \mu \right)^{1/2} \right\}^{1/2} \\ C_2 &= \left\{ \frac{1}{2} + \frac{1}{2} \left( 1 - 4l^2 \sigma B_0^2 / \mu \right)^{1/2} \right\}^{1/2} \end{aligned} \quad (8)$$

Substituting (6) and (7) in the continuity equation (4) and integrating across the film thickness 'h' and applying the boundary condition 5(i) and 5(ii) give the modified Reynolds equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{dh}{dt} \frac{\mu M_0^2}{h_0^2 f_1(h, l, M_0)}$$

Where

$$f_1(h, l, M_0) = \frac{2l}{C_1^2 - C_2^2} \left( \frac{C_2^2}{C_1} \tanh \frac{C_1 h}{2l} - \frac{C_1^2}{C_2} \tanh \frac{C_2 h}{2l} \right) + h$$

The pressure boundary condition to be taken as  $p(x, y) = 0$

where

$$(x-a)(x+2a-\sqrt{3}y)(x+2a+\sqrt{3}y) = 0$$

Where 'a' is the length of the side of the equilateral triangle whose equation is given by

$$(x-a)(x+2a-\sqrt{3}y)(x+2a+\sqrt{3}y) = 0 \quad (10)$$

The origin is selected as the median of the triangle of (9) with the condition (10) gives the equation of the pressure in the form:

$$p = \frac{0.1792 M_0^2 \left[ \left( \frac{\sqrt{3}y^*}{2} \right)^2 - \left( 1 + \frac{x^*}{2} \right) (1-x^*) \right]}{F(h^*, l^*, M_0)} \quad (11)$$

$$\text{Where } x^* = \frac{x}{a}, y^* = \frac{y}{a}, h^* = \frac{h}{h_0}, l^* = \frac{2l}{h_0}$$

$$F(h^*, l^*, M_0) = h^* + \frac{l^*}{C_1^{*2} - C_2^{*2}} \left[ \frac{C_2^{*2}}{C_1^{*2}} \tanh \frac{C_1^{*2} h^*}{l^*} - \frac{C_1^{*2}}{C_2^{*2}} \tanh \frac{C_1^{*2} h^*}{l^*} \right] \quad (12)$$

where

$$C_1^* = \left[ \frac{1}{2} - \frac{(l^* M_0^2 - 1)^{1/2}}{2} \right]^{1/2} \quad (13(i))$$

$$C_2^* = \left[ \frac{1}{2} + \frac{(l^* M_0^2 - 1)^{1/2}}{2} \right]^{1/2} \quad (13(ii))$$

Now the force equation is given by

$$\begin{aligned} F^* &= \frac{F h^2 M_0}{\mu U L^2 B} \\ &= \int_{x^*=0}^{-1} p^* dx^* \end{aligned} \quad (14)$$

which gives,

$$F^* = -\frac{2.14}{9\sqrt{3}} M_0^2 \left[ \frac{1}{f_1(h^*, l^*, M_0) + h^*} \right] \quad (15)$$



#### 4. MHD STEADY AND DYNAMIC CHARACTERISTICS

The value of the stiffness coefficient under steady state is given by:

$$S_c^* = - \left( \frac{\partial F^*}{\partial h_m^*} \right) \quad (16)$$

$$= \frac{2.14}{9\sqrt{3}} M_0^2 \left( \frac{\partial}{\partial h_m^*} \left[ \frac{1}{f_1(h^*, l^*, M_0) + h^*} \right] \right) \quad (17)$$

$$= \frac{2.14}{9\sqrt{3}} M_0^2 \left[ h^* \frac{\partial f_1(h^*, l^*, M_0) - f_1(h^*, l^*, M_0)}{[f_1(h^*, l^*, M_0) + h^*]^2} \right] \quad (18)$$

The expression for the load carrying capacity is evaluated by:

$$W^* = (F)_0 = \left( \frac{\partial f_1}{\partial h_m} \right)_0$$

$$= \int_{x^*=-2a}^a \int_{-\frac{2a+x}{\sqrt{3}}}^{\frac{2a+x}{\sqrt{3}}} \frac{\sqrt{3} M_0^2}{60} \left\{ \frac{dy dx}{F(h^*, l^*, M_0) + h^*} \right\} \quad (19)$$

Where

$$f_1(h^*, l^*, M_0) = \frac{1}{C_1^{*2} - C_2^{*2}} \left( \frac{C_2^{*2}}{C_1^*} \tanh \frac{C_1^* h}{l^*} - \frac{C_1^{*2}}{C_2^*} \tanh \frac{C_2^* h}{l^*} \right)$$

$$\text{and } C_1^* = \left[ \frac{1}{2} - \frac{(l^* M_0^2 - 1)^{1/2}}{2} \right]^{1/2}, C_2^* = \left[ \frac{1}{2} + \frac{(l^* M_0^2 - 1)^{1/2}}{2} \right]^{1/2}$$

Similarly, under the steady state, the linear dynamics damping coefficient is given by:

$$D_c^* = - \left( \frac{\partial F^*}{\partial V^*} \right) \quad (20)$$

$$= \sqrt{6} M_0^2 \left( \frac{f_1(h^*, l^*, M_0) - \sqrt{7}}{12} \right) \quad (21)$$

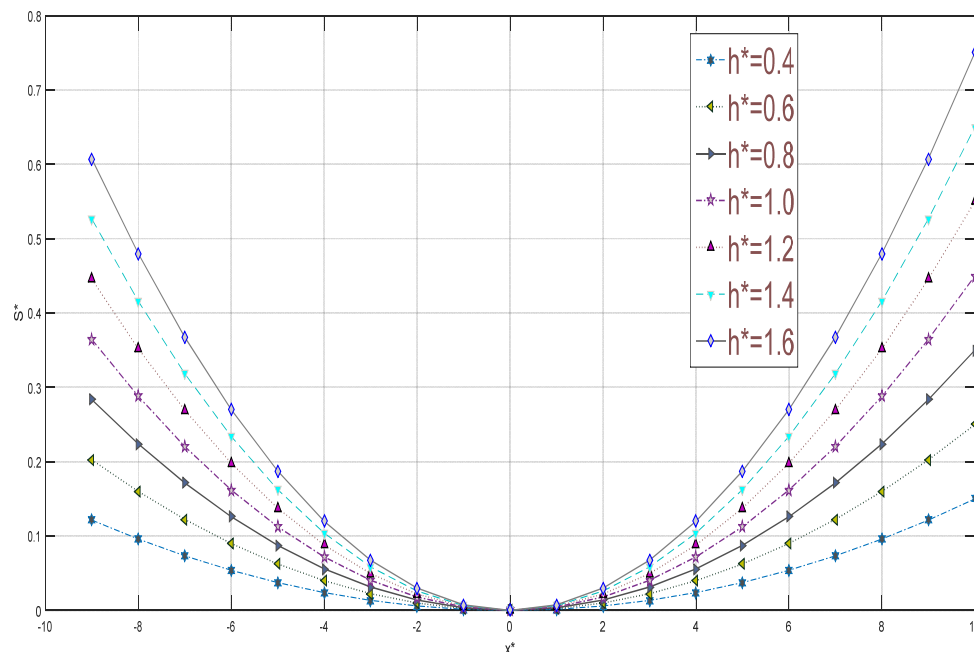
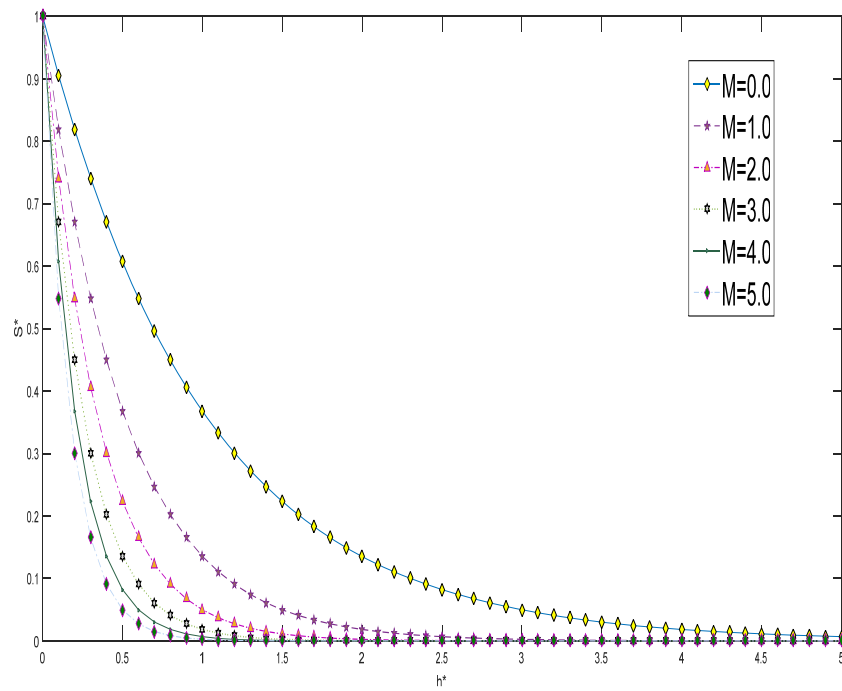
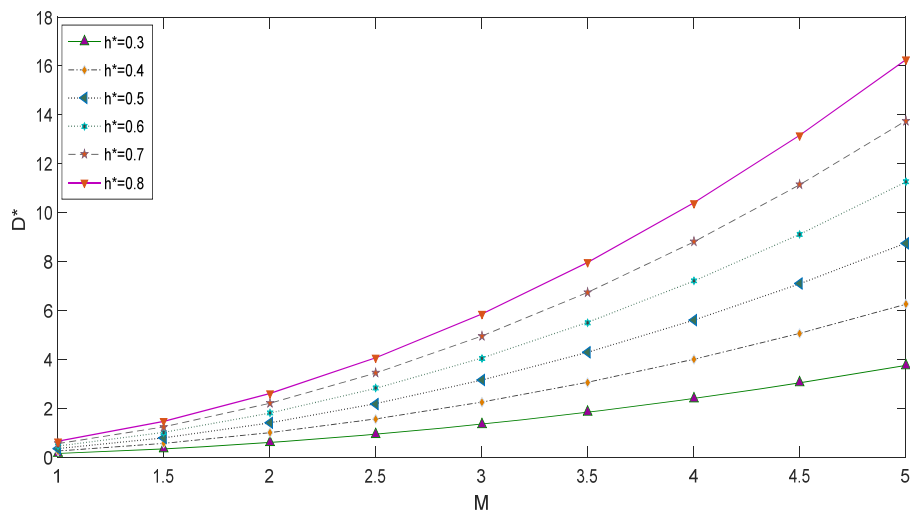


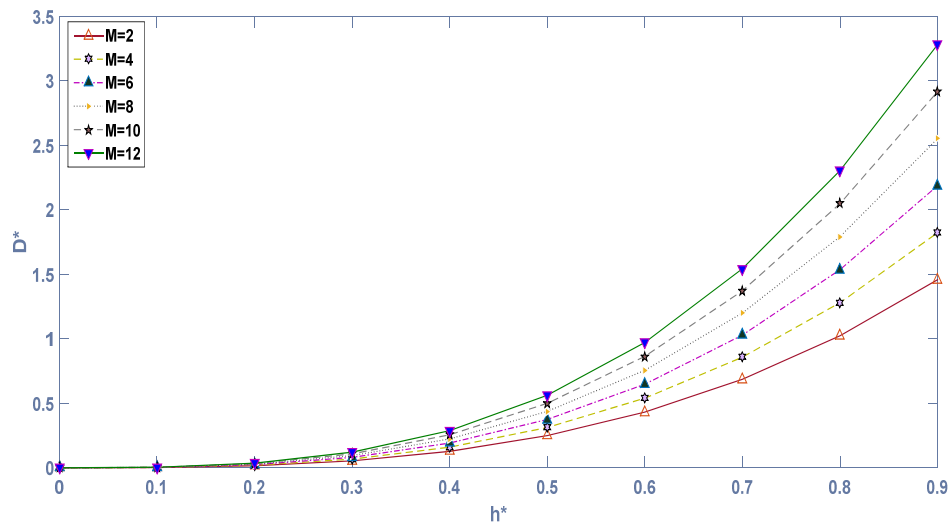
Figure-1. MHD dynamic stiffness coefficient as a function of M for different value of film thickness.



**Figure-2.** MHD dynamic stiffness coefficient as a function of film thickness for different values of Hartmann number.



**Figure-3.** MHD dynamic coefficient as a function of  $M$  for different value of film thickness.



**Figure-4.** MHD dynamic stiffness coefficient as a function of film thickness for different values of Hartmann number.

## 5. RESULTS AND DISCUSSIONS

According to the present studies, Figure-1 represents  $M \rightarrow 0, N \rightarrow 0$

i.e., a Newtonian case of triangular squeeze film lubrication.

Figure-2, shows a smooth and uniform distribution when  $x^* \rightarrow 0$

Between the coordinate and stiffness coefficient.

Figure-3, represents when  $M \rightarrow 0$  the stiffness coefficient has a decrease in effect over film thickness of triangular plates.

Figure-4 shows the equal and uniform distribution of dynamic coefficient over Hartman number  $M$ .

From Figure-1, it is found that as the film thickness of a profile increases from 0.4 to 1.6, the rigidity of the film profile in response of an applied force increasing, i.e., the flexibility or pliability decreases. when the non dimensional coordinate  $x^* \rightarrow 0$  then the stiffness coefficient reverses, i.e, the tortional stiffness came into existence, which allows less energy to be dissipated through heat and friction.

Figure-2, by the definition, the Hartmann number  $M$  characterizes the influence of an externally applied magnetic field on the MHD characteristics of bearings in the presence of an electrically conducting fluid.

According to Lin and Hung, when the value of Hartmann number is zero, the dynamic MHD equation reduces to the Newtonian non conducting lubricant case.

From the fig, the stiffness decreases or the flexibility increases and couple stress fluid will give time to bearings to overcome its inefficiency.

Figure-3, represent the dynamic damping coefficient as a function of coordinate for different values of Damping coefficient is increasing with increase in values of Hartmann number. As the squeeze film height decreases, the value of the damping characteristics

enhances which helps the bearing to work in an unexpected condition. We can found that the increment in values of the damping coefficient is more applicable for MHD triangular plates with smaller squeezing film heights and larger hartmann number.

Figure-4, shows a uniform distribution when we increase the values of hartmann number with respect to film thickness, it merges till the squeezing film is of 0.2 and then it shows a different distribution which indicate that it improves the bearing characteristics after certain stage.

## 6. APPLICATIONS

This paper is about the triangular shape bearing which reduces the fiction and it helps the engineer to improve the bearing characteristics. The triangular shape bearings needs time to improve its efficiency, it absorbs the lubricant and gap indicates the time taken to come to its normal function. According to these characteristics, the engineers can design the bearings for its purpose and can use it in a different field.

### Nomenclature

$a$  – the length of the side of the equal triangle

$B_0$  - Applied magnetic field

$h$  – Film thickness

$h^*$  - Dimensionless film thickness ( $h/h_0$ )

$h_0$  - Minimum film thickness

$l$  - Couple stress parameter

$s^*$  - Dimensionless couple stress parameter ( $2l/h_0$ )

$M_0$  - Hartmann number ( $= B_0 h_0 (\sigma^* / \mu)^{1/2}$ )

$P$  - Pressure in the film region

$P$  - Dimensionless film pressure



$$\left( = -ph_0^3 / \mu(dh/dt)3\sqrt{3}a^2 \right)$$

$x, y, z$  - Coordinates (in Cartesian)

$\eta$  - Material constant responsible for couple stress

$V$  - Squeezing velocity ( $= dh/dt$ )

$\omega$  - Work done (load carrying capacity)

$\mu$  - Viscosity of the fluid

$\sigma$  - Electrical conductivity

$$S_c, S_c^* - \text{Stiffness coefficient, } S_c^* = \frac{S_c h^2 m_0}{\mu U L^2 B_0}$$

$$D_c, D_c^* - \text{Damping coefficient, } D_c^* = \frac{D_c h^3 m_0}{\mu L^3 B}$$

Hamza E.A. 1998. The magneto hydrodynamic squeeze film, J. Tribology. 10: 375.

Huges W.F and Elc0o R.A. 1962. Magneto hydrodynamic lubrication flow between parallel rotating disks, J. Fluid Mech. 13(1): 21.

## REFERENCES

Malik M. and Singh D.V 1980. Analysis of finite MHD Journal bearings, wear. 64: 273-280.

Hamrock B.J. 1994. Fundamentals of Fluid Film lubrication, Mc Graw-Hill. pp. 286-287.

Stokes V.K. 1966. Couple stresses in fluid. Physics of Fluids. 9: 1709-1715.

Jaw-Ren Lin and Rong-Fang Lu. 2010. Dynamic characteristics for magneto-hydrodynamic wide slider bearings with an exponential film profile. Journal of Marine Science and Technology. 18: 268-276.

Sundarammal K. Ali. J. Chamkha and Santhana Krishnan N. 2014. MHD squeeze film characteristics between porous parallel rectangular plates with surface roughness. International Journal of Numerical methods for heat and fluid flow. 24(7): 1595-1609.

Santhanakrishnan N and Sundarammal Kesavan. 2016. Stiffness and Damping coefficients of magneto-Hydrodynamic squeeze film characteristics for Non-Newtonian porous curved circular plates. ARPN Journal of Engineering and Applied Sciences. 11(5).

Santhanakrishnan N and Sundarammal Kesavan. 2015. Magneto Hydrodynamics Non-Newtonian Squeeze Film Characteristics of Porous Circular Plates. ARPN Journal of Engineering and Applied Sciences. 10(14).

Shukla J.B. 1965. Hydro magnetic theory for squeeze films. Journal of basic engineering. 87: 144-147.

Oliver D.R. 1988. Load enhancement effects due to polymer thickening in a short model journal bearing. Journal of Non-Newtonian fluid mechanics. 30: 185-189.

Lin J.R 2120. Steady and dynamic characteristics for wide tapered land slider bearings. Tribology international. 43: 2378-2383.