TRANSFORMATION AND INTERFERENCE OF THE LASER RADIATION IN COMPOSITE CRYSTAL OPTICAL SYSTEMS

Abilhan Umbetovich Umbetov, Makhabbat Zhaksylykovna Umbetova, Gabit Mukhitovich Abildayev, Saule Svyazkhanovna Baizakova, Samal Arynovna Zhamalova, Arailym Boranbayevna Konussova and Kalamkul Kapenovna Dosmagulova

Arkalyk State Pedagogical Institute after I. Altynsarin, Auelbekova Street, Arkalyk, Republic of Kazakhstan
E-Mail: Umbetov.a@mail.ru

ABSTRACT

Currently, the use of crystal optical systems in the scientific and technological development has increased significantly. With their help it is possible to solve the control tasks of the amplitude, frequency, phase and polarization of the laser radiation and scanning by an optical beam in space. In addition, crystal optical systems are used for spatial encoding; creating controlled spatial filters in polarization interferometers, allowing investigating the quality of processing of optical parts, geometric parameters of the laser beams, the degree of coherence of the laser radiation to a high precision.

When the laser radiation is placed at the output of crystal optical systems, there appear the interference patterns, which are of optical parts, geometric parameters of the laser beams, the degree of coherence of the laser radiation to a high precision. With their help it is possible to solve the control tasks of the amplitude, frequency, phase and polarization of the laser radiation when transmitting through the CCOS; to study the interference mode of the laser radiation and to make assumptions of its use in laser polarized interferometers.

The calculations of the path of beams in the CCOS, known in the literature [8], are limited to the case of normal incidence of light in the input face of the system and held for each individual system separately. Shafranovsky and Fedorov studied the method of calculation of the beams’ path in refracting prisms of variable doubling angle [9]. The current paper considers the calculation of the path of electromagnetic waves in anisotropic uniaxial and biaxial crystals, the laws of reflection and refraction at the interface of two anisotropic media [10].

Therefore, the purpose of this research is to develop efficient techniques for calculation (transformation) of the path of laser radiation, to study the interference patterns when transmitting through the crystal optical system and the dispersion properties of the CCOS. Research objectives are as follows:

- Conducting a comprehensive study of the crystal optical system, the properties of transformation of the laser radiation when transmitting through the CCOS;
- Investigating the conditions for the occurrence of interference patterns at the output of the CCOS and their possible use in a variety of laser measuring devices;
- Studying the dispersion properties of the CCOS.

Keywords: interference, laser, radial vector composite system, path difference, dispersion vector, optical field.

1. INTRODUCTION

The investigation of composite crystal optical systems (CCOS) of anisotropic crystals is of interest for the construction of laser measuring devices. This raises the problem of the development of methods of calculation for such systems, which would describe more accurately their properties for the transmission and interference of electromagnetic waves (laser radiation) [1], [2]. This paper studies the distribution of the laser radiation through composite crystal optical systems made from uniaxial Iceland spar crystals. The research is based on newly developed techniques and examines the interference mode of the polarized laser radiation generated in the CCOS [3]. As shown in Figure-1, the CCOS consists of plano-concave (I) and plano-convex (II) uniaxial crystal lenses with different directions of optical axes in I and II, specified unit vectors \( \mathbf{a}_1 = (1,0,0) \) and \( \mathbf{a}_2 = (0,\sin \psi, \cos \psi) \), where the \( \psi \) angle is between \( \mathbf{a}_2 \) and the \( z \) axis, respectively.

A well-known covariant calculating method [4], [5] of the laser radiation in anisotropic media leads to composite general expressions, and its use for CCOSs is difficult. In general, the problem is not amenable to a rigorous analytical solution, where the main difficulty is the need to take into account the non-parallelism of the wave vector \( \mathbf{k} = 2\pi / \lambda \), describing the transfer of the wave phase, and the radial vector \( \hat{S} = [\hat{E}\hat{H}] \), describing the transfer of the wave energy (\( \lambda \) - the length of the wave, \( \hat{E}, \hat{H} \) - the intensity vectors of the electric and magnetic fields).

This paper solves the problem of extending the ordinary (0) and extraordinary (e) waves in an anisotropic dielectric medium with regard to the boundary conditions for the CCOS with the simplest and at the same time sufficiently general methods of geometrical optics in paraxial approximation, i.e. in supposing an infinitely narrow parallel beam, falling on different points of the input face of the anisotropic medium [6]. This calculating method of the laser radiation in anisotropic media allows to identify all the basic properties of the studied CCOS and to compare the results of theoretical calculations with experimental data. It also allows to describe in detail the interference mode of the laser radiation and to make assumptions of its use in laser polarized interferometers [7].
2. METHODS

This research studies the theory of distribution of electromagnetic waves (laser radiation) in anisotropic crystals, the laws of reflection and refraction at the interfaces of media [11]. The covariant method by F.I. Fedorov for calculating the distribution of electromagnetic waves in anisotropic media is known. However, a rigorous calculation of the beams’ path in crystal optical systems consisting of several anisotropic crystals results in cumbersome expressions, not suitable for engineering calculations and for investigating the general optical and interference properties of crystal optical systems. An effective calculating technique of the path of electromagnetic waves (laser radiation) in composite crystal optical systems and a description of the conditions for the occurrence of interference patterns at the output of the CCOS are required [12], [13].

This paper proposes a simple and fairly common technique of geometrical optics in paraxial approximation. For example, suppose that the narrow parallel beam of light with a circular polarization falls on the left face of the CCOS (on the XY plane) at an arbitrary point M (Figure 2) along the Z-axis. If we denote the coordinates of the point M on the XY plane: dcosφ, dsinφ, 0), where φ is the angle between the X-axis and the radius - vector d, drawn from the origin z=0 to the point M, then the assumption that d,<<R, where R is the radius of curvature of the spherical CCOS interface and the value \( \left( \frac{d}{R} \right)^2 \) will be negligible.

3. RESULTS AND DISCUSSIONS

The ways of transformation of the laser radiation through a composite crystal optical system are diverse, formed with the help of interference patterns. At the output of the CCOS, there are four beams with the same polarization states, but orthogonal between the pairs. However, a different distribution of the incident beam energy in the CCOS among four outgoing beams keeps the number of ongoing interference patterns under certain experimental conditions.

An exhaustive answer can be obtained by solving the Fresnel task to find the amplitudes of all the waves on three interfaces (Figure-1).

This problem cannot be completely solved by analytical methods. Nevertheless, sufficient information about the amplitudes of beams can be obtained from the following simple reasoning.

A flat (monochromatic) electromagnetic wave at the input face of the CCOS-1 is divided into two waves with orthogonal polarizations. The vibration directions of \( \vec{E}_0 \) and \( \vec{A}_0 \) vectors for the o(ordinary) wave coincide and are perpendicular to the main plane of the crystal. The unit vector of the electric vector (o) of the wave in the field I of the CCOS-1 can be written as follows:
The vibration directions of $\mathbf{E}_0'$ and $\mathbf{A}_0'$ and $\mathbf{E}_0'$ and $\mathbf{A}_0'$ vectors for the e (extraordinary) wave lie in the main plane and in general do not coincide. Given that the $\mathbf{A}_0'$ vector is perpendicular to the direction of the wave distribution and is related to the $\mathbf{E}$ vector by the ratio $\mathbf{A}_0' = e_0 e \mathbf{E}$, the unit vector of the electric vector can be written as follows:

$$\mathbf{e}_0^{(1)} = \frac{[b_2][a_2]}{[a_2][a_2]} = \frac{[b_2][a_2]}{\sqrt{1 - ([a_2][a_2])^2}} \quad (1)$$

As far as the o- wave is not refracted at the interface of the lens, then $k_0^{(3)} = k_0^{(2)}$ and for the CCOS-1 with mutually perpendicular optical axes in the fields I and II we have:

$$d_{00} = \frac{(k_0^{(1)} a_2^2) (a_2' a_2^2)}{(1 - ([a_2][a_2])^2)(1 - ([a_2'][a_2'])^2)} \quad (8)$$

Neglecting the reflection losses in the calculation of the coefficient of transformation of the (o) wave into the (e) wave, we have:

$$d_{0e} = 1 - d_{00} = \frac{1 - ([a_2][a_2'])^2 (a_2'[a_2'])^2}{(1 - ([a_2][a_2'])^2)(1 - ([a_2'][a_2'])^2)} \quad (9)$$

In the field I of the CCOS-1 the wave with polarization of the e - wave on the spherical surface of the interface will be split into o- and e- waves:

$$\mathbf{E}_{00}^{(1)} = A e_0^{(1)} (e_0^{(1)} e_0^{(1)}) ; \mathbf{E}_{ee}^{(1)} = A e_0^{(1)} (e_0^{(1)} e_0^{(1)}) \quad (10)$$

Neglecting the reflection losses, the transmission coefficient of the e- wave by intensity can be written as:

$$d_{ee} = \frac{([s_1][s_1])^2 |[s_1][s_1]|^2}{([s_1][s_1])^2 [([s_1][s_1])^2] \quad (11)$$

To consider the correlation (9) and (10), which can be given within the accuracy of $\delta^*$ as:

$$\langle \mathbf{k} \mathbf{s} \rangle = 1 \left( \mathbf{s} \mathbf{A} \right) = (\mathbf{k} \mathbf{A}) (1 + \delta - \delta(\mathbf{k} \mathbf{A})^2) \quad (12)$$

the equation (11) will take the form:

$$d_{ee} = \frac{([s_1][s_1])^2 |[s_1][s_1]|^2}{(1 - ([s_1][s_1])^2)(1 - ([s_1][s_1])^2)} \quad (13)$$

Without regard to reflection, the coefficient of transformation of the e- wave into the o- wave by intensity is equal to:

$$d_{eo} = 1 - \frac{([s_1][s_1])^2 |[s_1][s_1]|^2}{(1 - ([s_1][s_1])^2)(1 - ([s_1][s_1])^2)} \quad (14)$$

At the output of the CCOS-1 there is no mutual transformation of vibration modes (o and e). Therefore, within the accuracy of reflection losses, the formula (8), (9), (13) and (14) define the intensity of beams at the output of the CCOS-1.

The equation (8) shows that if the radiation is falling in the plane of optical axis, then $k_0^{(1)} = k_0^{(2)}$, $k_0^{(1)} = k_0^{(2)}$ and the equation of transformation of the o-wave is $d_{00} = 1$. In other words, the o-wave does not change its polarization when transmitting the interface in the CCOS-1 from the field I to the field II.

If the radiation is falling normally towards the optical axis plane, then $k_0^{(1)} = 0$, $k_0^{(2)} = 0$ and the coefficient of transformation is $d_{eo} = 1$. Thus, the o-wave
is transforming entirely into the e- wave when transmitting through the spherical surface. If the radiation is falling in the plane which is perpendicular to the plane of optical axes of the CCOS-1, and contains one of them, the transformation coefficient is set to unity at any angle of incidence of the wave on the input face.

With the \( \mathbf{K}_2 \) vector being presented in the form 
\[
\mathbf{K}_2 = \mathbf{K}_1 + \delta \mathbf{K}_2^{(1)}
\]
the expression (13) within the accuracy of \( \delta^2 \) can be written as follows:

\[
\mathbf{K}_1 = \{ \sin \alpha; 0; \cos \alpha \}
\]

With regard to the expression (4), from (19) we have:

\[
\mathbf{K}_1 = \left\{ \frac{n_o \sin \alpha}{\sqrt{n_o^2 n_e^2 - (n_o^2 - n_e^2) \sin^2 \alpha}}; 0; \frac{n_o \sqrt{n_o^2 - \sin^2 \alpha}}{\sqrt{n_o^2 n_e^2 - (n_o^2 - n_e^2) \sin^2 \alpha}} \right\}
\]

(20)

In the field II of the CCOS-1 for the wave having the polarization of the e- wave on both sides of the lens, the wave vector can be written as follows:

\[
\mathbf{K}_1 = \left\{ \sin \alpha; \eta_0 \log \theta + \text{decsin} \phi; \eta_0 \text{decsin} \phi; \sqrt{1 - \sin^2 \alpha} \right\}
\]

(21)

If the unit vectors of optical axes are set as:
\[
\mathbf{a}_1 = \{0, 0, 1\}; \mathbf{a}_2 = \{ \cos \varphi_0; \sin \varphi_0; 0 \}
\]

then from the expressions (7), (9), (13) and (14) we will get respectively:

\[
d_{00} = \frac{1 - \sin^2 \alpha}{n_o^2} \cos^2 \varphi_0
\]

(22)

\[
d_{0e} = \frac{\sin^2 \varphi_0}{1 - \sin^2 \alpha \cos^2 \varphi_0}
\]

(23)

\[
d_{ee} = \frac{\sin^2 \varphi_0}{1 - \sin^2 \alpha \cos^2 \varphi_0}
\]

(24)

\[
d_{eo} = \frac{\sin^2 \varphi_0}{\sin^2 \alpha \cos^2 \varphi_0}
\]

(25)
Thus, as can be seen from the expressions (22), (23), (24) and (25), at \( \varphi_0 = \frac{\pi}{2} \) there are four waves at the output of the CCOS-1 with approximately the same intensity (Figure 1a). At \( \varphi_0 = 0 \) we have \( d_{oo} = d_{ee} = 1 \) and regardless of the incidence angle of the radiation on the input face of the CCOS-1, the basic waves at the output of the BL-1 will be (oo) and (ee) waves (Figure 1b).

Let us consider the interference properties of the lens of the CCOS-2 type. Start with the simplest case of a collimated laser beam that is normally falling on the input face of the CCOS-2 at \( z=0 \) (Figure 4). At the output of the CCOS-2 there are formed two main (oe) and (eo) waves. Partial (oe) and (eo) beams entering the CCOS-1 at the points \( M_0(d_0 \cos \varphi, d_0 \sin \varphi, 0) \) and \( M_0(d_e \cos \varphi, d_e \sin \varphi, 0) \), respectively, at the output of the CCOS-1 have the wave vectors derived from (12) and (14) at \( \varphi = \frac{\pi}{2} \).

\[
\mathbf{K}_{3ee} = \left\{ \frac{d_0}{R} (n_0 - n_e) \cos \varphi; \frac{d_0}{R} (n_0 - n_e) \sin \varphi; \sqrt{1 - \frac{d_0^2}{R^2} (n_0 - n_e)^2} \right\} \tag{26}
\]

and

\[
\mathbf{K}_{3eo} = \left\{ \frac{d_e}{R} (n_e - n_0) \cos \varphi; \frac{d_e}{R} (n_e - n_0) \sin \varphi; \sqrt{1 - \frac{d_e^2}{R^2} (n_e - n_0)^2} \right\} \tag{27}
\]
Figure-4. A scheme of polarized beams transmission through the CCOS-2 in the collimated laser beam

The formula (26) and (27) show that (oe) beams at the output of the CCOS-2 provide a divergent beam, and (eo) - convergent. Suppose the partial (oe) and (eo) beams intersect at the point M (x, y, z) at the screen (Figure-2). A direct calculation of transmission of the (oe) beam within the accuracy of $\left[\frac{d_{oe}}{R}\right]^2$ with regard to (26) allows to find its input parameter:

$$d_{oe} = \frac{R\rho}{(z-F_{oe})(n_0-n_{oe})},$$

(28)

where $\rho = \sqrt{x^2 + y^2}$

Similarly, for the (eo) beam, we have

$$d_{eo} = \frac{R\rho}{(z-F_{eo})(n_{eo}-n_0)}$$

(29)

As shown previously, the optical transmission difference between (eo) and (oe) beams within the accuracy of $\left[\frac{d_{eo}}{R}\right]^2$ is follows:

$$\Delta = (n_0 - n_{eo}) R \frac{d_{eo}^2}{2R^2}$$

(30)

The condition of interference maxima can be written as follows:

$$\Delta + (K_{eo}^o - K_{oe}^o)^2 = S\lambda$$

(31)

Hence, taking into account (28) and (29) and supposing that $\Delta_{L-F} << 1$, we get:

$$(x^2 + y^2) = \frac{S\lambda}{2} (L - F)$$

(32)

Thus, the collimated beam when transmitting through the CCOS-2 provides an interference pattern in the form of rings with radii:

$$R_S = \frac{S\lambda}{\sqrt{2}} (L - F)$$

(S=1, 2, 3)

(33)

at the distance between the adjacent rings (for S>> 1):

$$\Delta R = \frac{\lambda(L-F)}{4n_0}$$

(34)

This distance is proportional to the wavelength and inversely proportional to the radius of the S-th ring.

When the CCOS-2 is illuminated with the divergent laser beam, the interference pattern is getting complicated, and if the beams are moved in the transverse direction, i.e. in the xy plane, then the interference pattern is essentially dependent on the direction of transverse movement of the laser beam. All changes can be described by the method mentioned above. We will consider the "basic" (oe) and (eo) - beams.

a) the oe - wave

Suppose the axis of the divergent incident beam is determined by the equations $x=0; y=2d_0$ (Figure 5). Let us represent the field in the divergent beam in the form of a system of divergent plane waves defined by the unit wave vector:

$$\vec{K}_0 = \{\alpha \cos\phi; \alpha \sin\phi; 1\}$$

(35)

A consistent consideration of the refraction of waves (35) at the interface in the CCOS-1 gives the following equations for the unit wave and radial vectors in the fields I and II of the CCOS-2 and at the output of the CCOS-2:

$$\vec{K}_1 = \left\{\frac{\alpha \cos\phi}{n_0}; \frac{\alpha \sin\phi}{n_0}; 1\right\}$$
\[
\vec{K}_2^* = \left\{ \left[ 1 + \frac{\delta}{2} \left( \frac{z_0}{R} n_e + \frac{\sigma}{R} \right) \right] \frac{\cos \varphi}{n_0} - \frac{\delta d_0}{2 R} \cos \varphi_0; \left[ 1 + \frac{\delta}{2} \left( \frac{z_0}{R} n_e + \frac{\sigma}{R} \right) \right] \frac{\sin \varphi}{n_0} - - \frac{\delta d_0}{2 R} \sin \varphi_0; 1 \right\}
\]

\[
\vec{K}_2^* = \left\{ \left[ 1 + \frac{\delta}{2} \left( \frac{z_0}{R} n_e + \frac{\sigma}{R} - 1 \right) \right] \frac{\cos \varphi - \delta d_0}{2 R} n_e \cos \varphi_0; \left[ 1 + \frac{\delta}{2} \left( \frac{z_0}{R} n_e + \frac{\sigma}{R} - 1 \right) \right] \frac{\sin \varphi - \delta d_0}{2 R} n_e \sin \varphi_0; 1 \right\}
\]

\[
\vec{S}_2^* = \left\{ \left[ 1 + \frac{\delta}{2} \left( \frac{z_0}{R} n_e + \frac{\sigma}{R} \right) \right] \frac{\cos \varphi - \delta d_0}{2 R} \cos \varphi_0; \left[ 1 + \frac{\delta}{2} \left( \frac{z_0}{R} n_e + \frac{\sigma}{R} \right) \right] \frac{\sin \varphi - \delta d_0}{2 R} \sin \varphi_0; 1 \right\}
\]

\[x_1^0 = z_0 \cos \varphi; y_1^0 = z_0 \sin \varphi; z_1^0 = 0 \quad (37)\]

\[x_4^0 = z_0 + 2 \frac{\sigma - R}{n_0} + L_1 + \frac{\delta}{2} \left[ \frac{\sigma - R}{n_0} \left( \frac{z_0}{R} n_0 + \frac{\sigma}{R} \right) + L_1 \left( \frac{z_0}{R} n_0 + \frac{\sigma}{R} - 1 \right) \right] \frac{\cos \varphi - - \frac{\delta}{2} \left( n_e L_1 + \sigma - R \right) d_0}{R} \cos \varphi_0\]

\[y_4^0 = z_0 + 2 \frac{\sigma - R}{n_0} + L_1 + \frac{\delta}{2} \left[ \frac{\sigma - R}{n_0} \left( \frac{z_0}{R} n_0 + \frac{\sigma}{R} + 2 \right) + L_1 \left( \frac{z_0}{R} n_0 + \frac{\sigma}{R} - 1 \right) \right] \frac{\sin \varphi - - \frac{\delta}{2} \left( n_e L_1 + \sigma - R \right) d_0}{R} \sin \varphi_0\]

where \( \sigma \) is the distance from the origin of coordinates to the center of the spherical surface.

\[y_4^0 = L_1 + 2 \left( \sigma - R \right) \]

As in (36), they are the corresponding directing vectors for the segments of the beam’s trajectory (oe) as it passes through the CCOS-2. If we place the screen on which the form of the interference pattern is studied at \( Z = L_1 + 1 \), then the simple geometric calculation by (36) allows to find the intersection point \( M_4(x_4^0, y_4^0, L_1 + 1) \) of the selected partial beam with the screen, wherein:

\[b) \text{the eo - wave.}\]

Suppose that at the input of the CCOS-2 for the (eo) beam the partial wave vector forms the angles \( \alpha_4, \varphi_4 \), and \( \alpha_1, \varphi_1 \), that differ from \( \alpha \) and \( \varphi \) and

**Figure-5. a.** A scheme of polarized beams transmission through the CCOS-2 in the divergent laser beam displaced in the transverse / in the xy plane / direction.

**b.** Location of points and parameters in the XY plane.
\[ \alpha \text{ and } \phi \text{ for the (oe) beam. For the (eo) wave the radial and wave vectors are of the form} \]

\[ S_1 = \begin{cases} \frac{\alpha_1 \cos \phi_1 + \delta}{n_\ell}; & 1 \end{cases} \]

\[ K_1^\ell = \begin{cases} \frac{\alpha_1 \cos \phi_1}{n_\ell}; & 1 \end{cases} \]  

(39)

\[ z_0 \alpha_1 \cos \phi_0; y_0 \varepsilon \sin \phi_0; z_0^2 = 0 \]  

(40)

The coordinates of the \( M_1^\ell \) point of the input of the (eo) beam are as follows:

\[ x_1^\ell = z_0 + 2 \frac{\sigma - R}{n_e} \]

\[ y_1^\ell = \frac{\varepsilon}{\phi} \sin \phi_0 \]

\[ z_0^2 = 2(\sigma - R) + L_1; \]

As for the (oe) - beam by (39) and (40) after the solution of the corresponding geometric problem we find the coordinates of the intersection points of the partial beam with the screen:

\[ \Delta = \delta \frac{d_0}{R} \frac{\sigma - R}{n_e} (\alpha_1 + \alpha_2) \cos (\phi_0 - \phi_1) \] - the path difference of the (eo) and (oe) beams at the output of the CCOS-2.

The connection between the input angles \( \alpha, \phi, \alpha_1, \phi_1 \), and the coordinates of the observation point can be obtained from the condition that the points \( M_1^\ell \) and \( M_2^\ell \) are identical. Using (38) and (42), we obtain the condition for the maxima:

\[ \Delta = \delta \frac{d_0}{R} \frac{\sigma - R}{n_e} (\alpha_1 + \alpha_2) \cos (\phi_0 - \phi_1) \]

(41)

If we assume that the observation plane (p) is located close to the output face of the CCOS-2, i.e. \( L_1 \) is very small, then from (43) we have:

\[ x_0^2 \left[ z_0^2 + \frac{1}{n_e} z_0 + \frac{R + i^2}{4n_e^2} \right] + y_0^2 \left[ z_0^2 + \frac{1}{n_e} z_0 - \frac{R - i^2}{4n_e^2} \right] = \frac{d_0}{R} \left[ z_0^2 + \frac{1}{n_e} z_0 - \frac{R - i^2}{4n_e^2} \right] \]

(44)

For the parameters used in the experiments with the CCOS-2

\[ (l = 7.4 \text{ mm}, R = 50 \text{ mm}; n_e = 1.49, \lambda = 0.6328 \text{ mm}, \delta = 0.2) \]

(44)

we get the following equation of the second order curve:

\[ x^2 (z_0^2 + 5z_0 + 90) + y^2 (z_0^2 + 5z_0 - 78) - \frac{d_0}{R} (x \cos \phi_0 + y \sin \phi_0) (z_0^2 + 5z_0 - 78) \]

\[ 5.5 \times 10^{-2} \times (1.49z_0 + 7.4) \]

(45)

The shape of this curve is shown in Figures 6 and 7.
Figure-6. Interference patterns generated by the CCOS-2 with the longitudinal displacement of $\Delta z$ along the z axis of the focused laser beam at the input. Pictures $\alpha$-$\omega$ are for $\Delta z = +22, +12, +11, +10, +0, -5, -5.5, -6.5, -18\text{ mm}$, respectively. $\Delta z = 0$ corresponds to the front face of the CCOS-2. $-\Delta z$ corresponds to the displacement of the focused laser beam to the front face of the CCOS-2.

Figure-7. Interference patterns generated by the CCOS-2 with the displacement of $\Delta z$ of the z axis of the focused laser beam displaced at a distance $d=4\text{ mm}$ from the center of the CCOS-2 in the XY plane. Displacement occurs along the x-axis at $\phi_0 = 0^\circ$. Pictures $\alpha$-$\omega$ are for $\Delta z = 30, 3, 0, -3.5, -3, -25\text{ mm}$, respectively. $\Delta z = 0$ corresponds to the front face of the CCOS-2. $-\Delta z$ corresponds to the displacement of the focused laser beam to the front face of the CCOS-2.

Figure-8. Interference patterns generated by the CCOS-2 with the displacement of $\Delta z$ of the z axis of the focused laser beam displaced at a distance $d=4\text{ mm}$ from the center of the CCOS-2 in the XY plane. Displacement occurs in the direction of the composing angle $45^\circ$ with the axes X and Y at $\phi_0 = 45^\circ$. Pictures $\alpha$-$\omega$ are for $\Delta z = 30, 5, 1.5, 0, -4, -25\text{ mm}$, respectively. $\Delta z = 0$ corresponds to the front face of the CCOS-2. $-\Delta z$ corresponds to the displacement of the focused laser beam to the front face of the CCOS-2.

between the (o) and (e) waves at the output of the CCOS-2 there is:

$$\Delta = \frac{1}{4} \frac{\delta}{n_e} (\alpha^2 + \alpha_i^2)$$  \hspace{1cm} (48)
Then the equation of the geometric place of points for the maxima of the interference pattern is written in the following form:

$$\left[ z_0^2 + z_0 \left( \frac{l}{n_0} + 1 - 2n_0^2 \right) + \frac{e^R}{2n_0^2} + \frac{R^2}{4n_0^4} \right] x^2 + \left[ z_0^2 + z_0 \left( \frac{l}{n_e} + 1 - 2n_e^2 \right) - \frac{e^R}{2n_e^2} + \frac{R^2}{4n_e^4} \right] y^2 = \frac{R^2}{8n_e^4} \left[ (z_0 + 1)n_e + 1 \right]$$

(49)

One of the well-known materials for the construction of the CCOS is an Iceland spar (CaCO₃), that has a high value of $\Delta n = n_0 - n_e = 0.17$. The dispersion of the Iceland spar was presented in Figure 9, for $\lambda = 0.4 + 2.3 \mu m$. In the visible field of the spectrum ($\lambda = 0.4 + 0.7 \mu m$) the change of the refraction index CaCO₃ for the $o$-beam is $\Delta n_0 = 0.032$, and for the $e$- beam is $\Delta n_e = 0.016$.

As seen from the dispersion curve of the Iceland spar, the values of $\Delta n$ increase in the ultraviolet field of the spectrum and in the infrared field - decrease, remaining almost constant in the visible field. These small changes in the refraction indices of the Iceland spar, depending on the wavelength, give a significant effect of the change of focal distances of the CCOS.

Figure-9. The dispersion curve of the Iceland spar (CaCO₃).

The experiment was conducted to measure the changes of the focal length of the CCOS, depending on the wavelength of radiation. Two waves were used: $\lambda = 0$, 6328 $\mu m$ and $\lambda = 1.15 \mu m$ of the coherent radiation (LG-126).

The recording of the light intensity in focuses was carried out with the help of an infrared vidicon (LI-438) of PTU-32 installation. The spectra in the visible field were obtained from the source of a "white" light using different polarization filters. In this case, the focal points of different wavelength were recorded by a microscope (OSM-2). Experimental results and theoretical calculation are presented in Figures 10 and 11.

As can be seen from Figures 10 and 11, with the decreasing of the wavelength, the focal distances $(F_{oo} + F_{ee})$ decrease, and the effect of increasing of the difference of focal distances $\Delta F = F_{oo} - F_{ee}$ is observed, and with the increasing of the wavelength there is an increase in the focal distances and a decrease in $\Delta F$. The change of the difference of focal distances $\Delta F$ with the change of the wavelength of radiation becomes clear if to take into account that the trajectory of the $e$ - beam in the field II of the CCOS-1 is given by the direction of the radial vector.

Changes of $\Delta F$ depending on the wavelength can be used for the selection of focal distances when using incoherent light sources.

It is necessary to note one more important feature of the CCOS-1 which consists in the ability to form an interference pattern using a "white" light source. When the CCOS-1 is illuminated by a convergent beam of "white" light, at the output of the CCOS-1 the interference pattern is formed in the shape of a light and dark cross with multicolored circles.

A variety of interference patterns in case of using "white" light is shown in Figure-12.

Figure-10. Dependence of the focal distance $F_{oo}$ and $F_{ee}$ on the wavelength. The calculated dependence of focal distances $F_{oo}$ and $F_{ee}$, respectively, $x$, the experimental dependence of the focal distance $F_{ee} = \square$. 
Figure-11. Dependence of the difference of focal distances $\Delta F = F_{oe} - F_{ee}$ on the wavelength.

Figure-12. Interference patterns formed by the CCOS-I in the "white" light.

To get a full picture of the structure of the optical image, it is necessary to know the light distribution not only in the focal plane, but also near the focus, in the accurate restructuring of the system, having a large depth of focus. The exact definition of the location of the focal plane of the system plays an important role in holography.

The diffraction patterns of the optical field near the focuses and the dependence of the light intensity distribution in the focal plane on various primary aberrations are discussed by many researchers.

Here are some interesting observation results of the structure of the optical field near the focal plane of the CCOS-1 and CCOS-2.

A functional scheme of the installation is shown in Figure-13. The laser beam from the source 1, after its expansion by a collimator, including a micro-lens (2), a diaphragm (3) and a collimating lens (4), falls on the CCOS 5. The structure of the field near the focus of the CCOS is observed by a microscope OSM-2 of the sixth type, the accuracy of which by the internal scale of Archimedes spiral is 10 µm.

Figure-13. An installation scheme for monitoring the interference structure of the optical field in the CCOS focuses. Note. 1 - a gas laser, 2 - a microlens, 3 - a diaphragm, 4 - a collimating lens, 5 - CCOS-1, 6 - a microscope.

As can be seen from Figure 14, the distribution of light intensity in the focus area by the cross section of the beam is diverse. There are diffraction structures in the sections of the beam; the field near the focuses is "blurred", and the inexplicitly expressed focus is being formed.

When the CCOS-1 is rotated around the y axis (Figure-12), the field structures are changing. The peculiarities of change of the optical field structure near the focuses at the rotation of the CCOS-1 around the "y" axis at different angles are shown in Figure 15.

Variability of the distribution of light intensity in the focus area Fee at different values of $\Delta Z = 0 - 3; -5$ mm is shown in Figures 16, 17 and 18. In this case, as compared to Figure-13, the CCOS-1 is rotated around the y-axis at 180°.

As noted in the experiment with the use of an ideal lens MIR-1, the diffraction structure of the field in the cross sections of the beam near the focus decreases. However, at a large turn angle of the lens MIR-1 around the y-axis, the structure of the field in the form of the caustic is observed.

For comparison with the obtained pictures, the optical field structure near the focus for a variety of lenses was studied. However, its mathematical description is a complex task; therefore, it is not considered in this article.

Our purpose was to present some interesting experimental data of the characteristics of light intensity distribution near the focuses.
Figure-14. Section type of the laser beam focused with the help of the CCOS in different distance $\Delta Z$ from the focuses $F_{oe}$ and $F_{ee}$. Pictures /a/ - /w/ are for $\Delta Z = -17, -14, -9, -7, -4, -2, -1, 0, +1$ mm, respectively. The value of one large division of the ocular-micrometer is 0.1 mm. $\Delta Z = 0$ corresponds to the conditional focus of the CCOS. / - / $\Delta Z$ / corresponds to the retraction of the microscope from the CCOS, i.e. in the course of the laser beam up to the conditional focus.

Figure-15. The interference field structure in the focal area of the CCOS-1 for the focuses $F_{oe}$ and $F_{ee}$ at $\Delta Z = -5$ mm in the laser radiation $\lambda = 0.6328 \mu m$. Pictures /a/ - /e/ show the individual stages of observation of the caustic along the axis when the CCOS is rotated around the y-axis at an angle $\phi = 0^\circ, 5^\circ, 10^\circ, 12^\circ$, respectively. The pattern scale: the distance between the adjacent lines of the spiral of the ocular-micrometer is equal to 0.1 mm.

Figure-16. Section type of the laser beam $\lambda = 0.6328 \mu m$ focused with the help of the CCOS-1, depending on the distance of $Z$ from the focus $F_{ee}$. Pictures /a/ - /e/ are for $\Delta Z = -11, -6, -5, -0, 5, 0, +1.5$ mm, respectively. Photos are obtained when the CCOS-1 is turned at 1800 around the axis, as compared with Figure 13. Astigmatism can be noticed, as compared with Figure-14.

Figure-17. The interference field structure in the focal area $F_{ee}$ at $\Delta Z = -3$ mm. Pictures /a/ - /e/ show the individual stages of observation of the caustic along the $Z$-axis when the CCOS-1 is rotated around the y-axis at an angle $\phi = 0^\circ, 7^\circ, 9^\circ, 11^\circ, 13^\circ, 16^\circ$, respectively. The pattern scale: the distance between the adjacent lines of the spiral of the ocular-micrometer is equal to 0.1 mm.
4. CONCLUSIONS

This paper studied the functioning of composite crystal optical systems of two types: CCOS-1 and CCOS-2 in the mode of interference of polarized beams.

The peculiarity of interference in the system of CCOS-1 type was theoretically described and experimentally justified. The interference of polarized beams was performed without analyzers when the optical axis was located in the input field of the CCOS-1 along the propagation direction of the laser beam.

There were obtained the expressions describing the dependence of the type of interference patterns formed in the CCOS-2 on the angle of radiation divergence, falling on the front face of the CCOS, on the longitudinal displacement (along the Z-axis) of the focused laser beam, and on the transverse displacement (in the xy plane) of the focused laser beam.

The interference structure of the optical field and its spatial transformation in the focal field during the laser irradiation of the CCOS was experimentally studied.

The experiments confirming the regularities of interference patterns allow recommending the use of the CCOS-1 and CCOS-2 in laser measuring devices.

The dispersion properties of the CCOS were considered. The focal lens distances of CCOS-1 and CCOS-2 types were measured depending on the length, and the calculations were made in the range of 0, 4 ≤ λ ≤ 2,3 µm, which are in good agreement with each other.

The interference structure of the optical field in the focal area during the laser radiation of the CCOS was experimentally studied.

REFERENCES