



A REVIEW ON ADVANCED OPTIMIZATION TECHNIQUES

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ABSTRACT

This paper mainly deals with the review on the various advanced optimization techniques. Optimization reveals significance advances in computing systems and it has become the most promising techniques for a variety of engineering applications. This paper highlights the various techniques such as evolutionary techniques, Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) to enhance the search process by improving the diversity, and the convergence toward the preferred solution have been analysed. A comparative study between the single and multi-objective based Optimization techniques including GA, PSO and Hybrid are presented. This comparison will be very helpful for industries to determine the optimal parameters and improve the process and quality of products. A variety of objective functions and its formulations are presented. Lagrangian relaxation is a tool to find upper bounds on a given arbitrary maximization problem. The main theme of this review is that the LD is naturally applied for a wide class of combinatorial algorithms which leads to get a significant solution. Among all the traditional optimization techniques, in recent years, heuristic algorithms are mostly applied to solve most of the combinatorial problems. Optimization algorithms can lead to appropriate solution for the real time applications.

Keywords: dynamic programming, lagrangian relaxation (LR), search algorithms, heuristic algorithms, multi stage decision process.

1. INTRODUCTION

Optimization techniques are one way to obtain operation (decision making), that will, to the extent possible, approach goals that have been set in response to a given problem. As a result, solutions such as design for minimal operating cost, optimal product quality, smallest device size and the like can be realized. The development of optimization techniques began during World War II, when they were used to optimize the trajectory of missiles. Subsequently, mathematical programming has been developed to realize optimization through the application of mathematical techniques. Additionally, the optimization technique of meta-heuristic, which imitates physical phenomena and the evolution of living organisms, has been developed since in the 1970s.

1.1 Dynamic programming

The powerful technique of dynamic programming was developed by Richard Bellman during the late 1940's and the early 1950's. The technique owes its current popularity also to him. Dynamic programming is a computational method for optimizing multi-stage (or sequential) decision processes. In fact, the phrase 'multi-stage decision process' can be associated with all the optimization problems that can be solved by this technique. In this method, we work in stages (sequences). This is achieved by decomposing a given problem into such subproblems or stages as can be handled more efficiently (from the computational viewpoint) that the given problem. The optimal solutions to these subproblems are then combined to lead an optimal solution, also known as the optimal policy, to the given problem. So far, there is no standard mathematical formulation of the dynamic programming problem. As such, some degree of ingenuity is needed to make out whether or not a problem can be solved by the dynamic programming technique. The ability to do this can be best

developed by examining a variety of problems. The basic concept of dynamic programming is contained in the principle of optimality enunciated by Bellman. He says, 'An optimal policy has the property that whatever the initial state and initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision'.

Dynamic programming solves those problems that satisfy this principle. According to this principle, given a initial state of a system, an optimal policy for the remaining stages does not depend on the policy adopted for the previous stages. In other words, the effect of the current decision on any of the decisions of the previous stages need not at all be considered. This is known as the Markovian property of the dynamic programming problems. The point (in time or space) or level at which we make a decision is referred to as a stage. A stage is a device to sequence the decisions i.e., it decomposes a problem into subproblems such that an optimal solutions to the subproblems. The successive stages of a problem can be separated by using the concept of stage. The state at a given stage represents the status of the process or system and enables us to make feasible decisions for the given stage without having to look back.

An oral review on global optimization during the period 1998 - 2008 has been taken by C.A. Floudas et al. (2008). The review covers various technique involves general twice differentiable Nonlinear Programming problems (NLP); mixed-integer nonlinear optimization problems Mixed Integer nonlinear programming problems (MINLP); bilevel nonlinear optimization problems; models with differential-algebraic constraints; semi-infinite programming problems and gray-box and nonfactorable models. Difficult combinatorial optimization problems are in practice frequently approached by means of Metaheuristics. This term has originally been introduced by Glover (1977) and



essentially refers to a broad class of problem-independent strategies for approximate optimization and problem solving, hence therefore to benefit from synergy. The large number of publications on Hybrid Metaheuristics and dedicated scientific events such as the ongoing series of Workshops on Hybrid Metaheuristics and Workshops on Metaheuristics document the popularity, success, and importance of this specific line of research. The special issue of Mathematical problems in engineering contains majority of optimization approaches for engineering applications which are classified into deterministic optimization, Heuristic Algorithms and Hybrid methods.

2. SEARCH ALGORITHMS

Multi-objective formulations are combinatorial models for many decision making problems. Search techniques are demonstrated to determine feasible solution to these problems. In many real world problems the objectives under consideration conflict with each other, and optimizing a particular solution with respect to a single objective can result in unacceptable results with respect to the other objectives. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution. Figure-1 illustrates the popular search algorithms such as Uninformed Search, Evolutionary algorithms and Heuristic Search etc. This review provides the comparison, analysis of these algorithms for different problems.

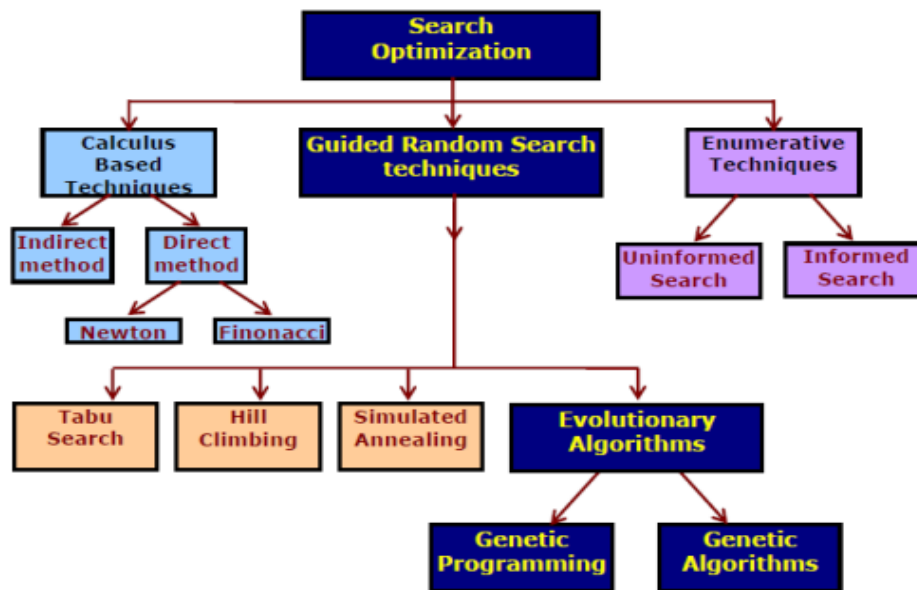


Figure-1. Classification of different Search Techniques.

2.1 Genetic algorithm (GA)

Genetic Algorithm is a heuristic search technique based on the process of natural evolution. This technique normally used to generate useful solutions for optimization and search problem. Modeling natural selection is the base of GA which does not need any secondary functions such as derivatives computation. Some positive characteristics

of GA which make it more usable in optimization problems are as follows: (a) probability of local minimum trapping is decreased (b) computations of going from one state to another is declined and (c) evaluation of the fitness of each string guides the search. An advantage of the GA techniques is that they lead, in most of the cases, to the global optimal Pareto frontier.

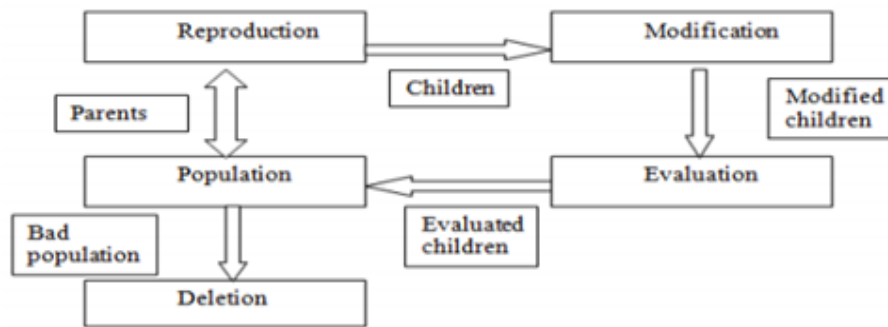


Figure-2. Process of GA.

Figure-1 reveals that flow start from the generation of population. In next step parents are selected from the pool and reproduce to give children, this is known as crossover. Then these newly generated children are modified by the process known as mutation. They are evaluated against fitness criterion. Those children are selected that have high fitness function value and become a part of population others that do not pass the fitness test are bad population and are deleted.

Pareto-optimal solutions obtained by a non-dominated sorting genetic-based algorithm which is a combination of graph theoretical concept and GA was adapted to reach the minimum number of inputs. Milosevic *et al.* (2003) considered two competing objectives including minimizing the number of Phasor measurement units (PMU) and maximizing the measurement redundancy in the OPP solution. Unlike most of the applied procedures, the entire Pareto optimal solutions exist for the Optimal PMU Placement (OPP) problem, instead of a single point solution. Important steps including crossover, mutation, and population are mentioned to be problem-dependent, where crossover values are in high probable value; regardless of mutation probability, crossover would be a good choice for non-dominated sorting genetic based algorithm parameters. Different crossover and mutation probabilities are applied to reach several and common Pareto-optimal fronts.

Repairing infeasible solutions which confront computation analysis with difficulty narrows the application of this method in plenty of optimization problems. In formulation of the OPP problem was taken by a topologybased algorithm and GA was used as a solution for this problem. This method considered zero-injection buses in a power network for solving the OPP problem. A comparison of the results between utilized method and earlier applied solutions was also made. Achieving completely observable power system utilizing GA algorithm was presented, which could successfully provide a solution for the OPP problem considering two important objectives including (i) one PMU/branch outage and (ii) maximum redundancy in the system observability. Crossover and mutation were applied as two operators of GA method to cause the accurate number of PMUs for solving the OPP problem. Observing maximized redundancy in the number of buses was the result of

optimum location of PMU determination. There was an increase in the number of PMUs which was needed to make the system observable when it had branch/PMU failure. A solution for the OPP problem using genetic algorithm-based procedure was presented to make the system observable for utilizing in linear state estimation. A new generation with fitness evaluation for a new population, started by opting crossover and mutation of individuals from the old population.

The various steps involved in GA is composed by,

- Step 1:** Random initialization for the population. (Consists of initialization for individual population. The designer can define size of the population, the coding of the individuals and can have some influence on the algorithm convergence (speed)).
- Step 2:** Evaluation of the population. (Based on the fitness of the population, corresponding weight can be assigned)
- Step 3:** Generate offspring. (Generate new individuals by applying crossover with parents).
- Step 4:** Apply mutation to offspring. (Evaluating the population). (Repeat Steps (iii - v) until the convergence criterion is reached).
- Step 5:** The size of the populated is constant.

2.2 Particle swarm optimization (PSO)

Particle Swarm Optimization (PSO) has a biologically inspired computational search and Optimization method by Eberhart et al (1995) based on the social behaviors of birds flocking or fish schooling. It is similar method to GA (population-based search technique) in which a population of random solutions is initially given to the system is PSO which have no operators of "mutation", "recombination", and no notion of the "survival of the fittest". Particles remark the individuals that are flown through the multi-dimensional space. The best position for each particle is obtained by the appropriate solution (fitness) faced by itself and its neighbors. As mentioned, the process of this algorithm starts with an initial position and velocity for each particle,



in which the velocities are bounded due to not flying in unusable fields and also overflowing forbiddance. A new concept for solving the OPP problem and reaching a completely observable network which satisfies the constraints of PMU loss or a transmission line outage was presented by Hajan et al (2011), which was marked utilizing a modified binary particle swarm optimization (BPSO) method. BPSO algorithm is a discrete binary version of PSO in which variables can only take 0 and 1 values. The rules presented in topological observability in the majority of papers have been completed in the

presented paper by developing the new rule based on analyzing the observability of a group of zero-injection buses to reach maximum usage of the existing data. As mentioned in the formulation of the OPP problem in the present paper, this rule ensures the observability of zero-injection buses whose adjacent buses had known values. Results of the presented method and different algorithms were compared in different situations including normal condition and a PMU/branch outage. Recently, there are many variants of PSO, and it may always grow rapidly. Figure-3 describes the variants of particle swarm.

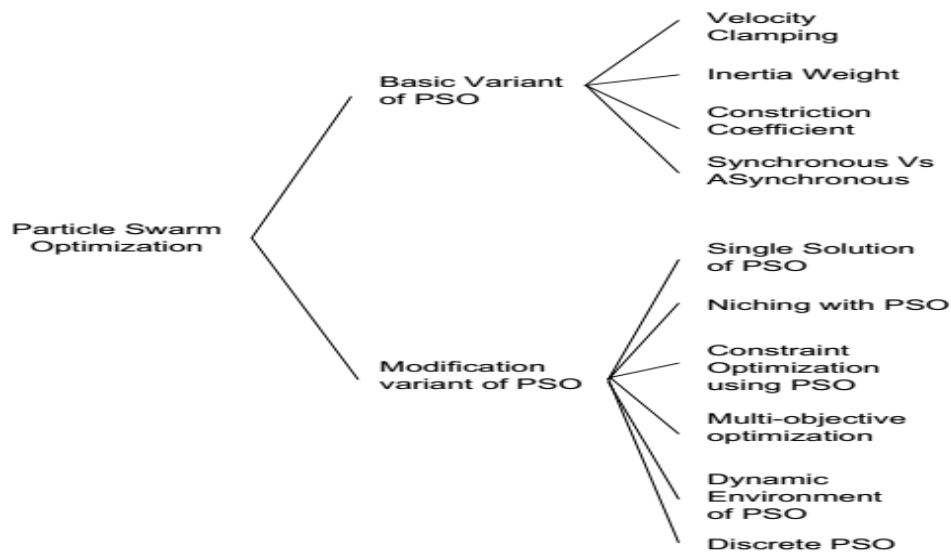


Figure-3. Variant of particle swarm optimization.

The main advantages of this technique is,

- Insensitive to scaling of design variables
- Very few parameters
- Implementation is simple
- Fitness functions are derivative free
- It is very suitable for large scale optimization

The disadvantages are,

- Slow convergence in refined search stage (Weak local search ability) in multidimensional problems.

2.3 Simulated annealing (SA)

Simulated annealing (SA) is a procedure for solving complicated combinatorial optimization in which the current solution is randomly altered. The new solution is the worse alteration with the probability that is reduced as the computation proceeds. An optimal solution for a large combinatorial optimization problem needs a fit perturbation mechanism, cost function, solution space, and cooling schedule to be solved by SA. Sufficiency of SA can be found by searching a large-scale system and obtaining good speed in terms of finding an optimal or near-optimal solution. Local search techniques, such as steepest descend method, are very good in finding local

optima. However, difficulties arise when the global optima is different from the local optima. Since all the immediate neighboring points around a local optima is worse than it in the performance value, local search cannot proceed once trapped in a local optima point. We need some mechanism that can help us escape the trap of local optima. And the simulated annealing is one of such methods.

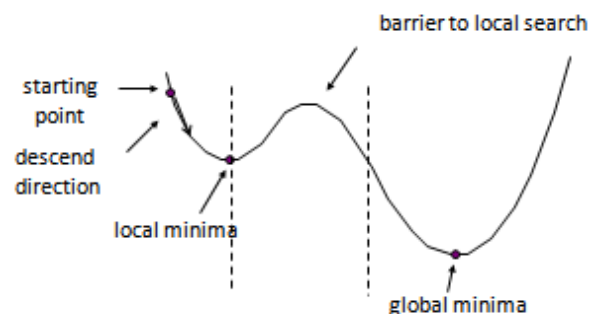


Figure-4. Difficulty in searching global optima.

A solution for pragmatic communication-constrained PMU placement problem is SA method which



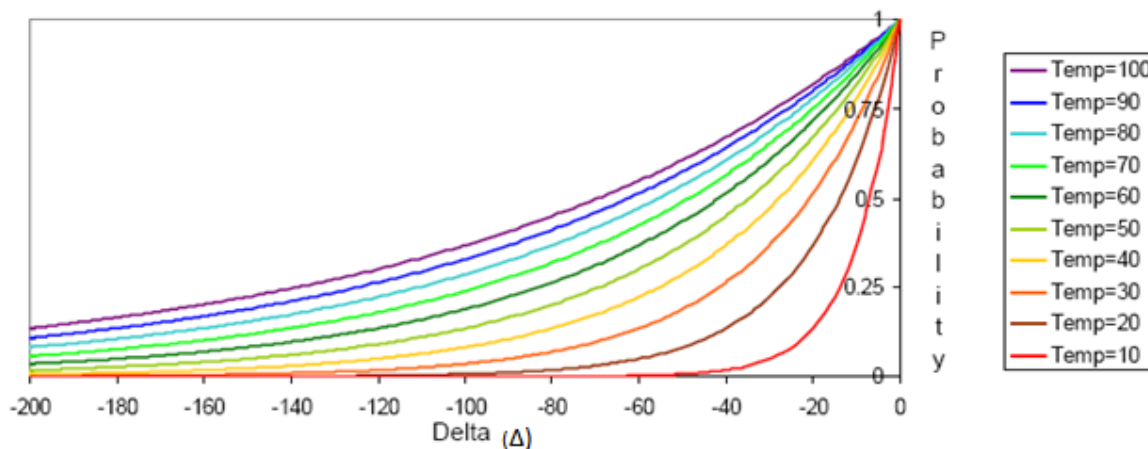
is utilized to solve the OPP problem based on incomplete observability by Nuqui *et al.* (2005). Optimal locations of PMUs based on a desired depth of unobservability and impact of depth of unobservability on the number of PMUs were presented in this paper. There was a relationship between the certitude of state estimation of unobserved buses and a given unobservability depth. Lower depths of unobservability caused a particulate state estimation. This method provided optimal PMU installation for estimation with available communication facilities and certified that the unobserved buses were far from the observed buses. To detect bad data in a power network which turns the measurement to critical measurement (CM), utilizing PMU installation is considered. Any bad data detection incidentally needs a critical measurement free. To identify critical measurement, several methods have been proposed; this paper used residual analysis to identify critical measurement. The absence of critical measurement means a power system that loses single measurement. A solution for the OPP problem to make a power system topologically observable, considering bad data revelation using SA, was presented by Tai *et al.* (2013). A hybrid genetic algorithm and simulated annealing (HGS) was used as a solution for the OPP problem and a comparison organ with the results of SSA method. Difference between a system in its normal situation and with critical measurement free is observable, when for the second system, losing any single branch does not impact the observability of the power system. Better usage of specific PMU measurement values and accessing highly sensitive system data were considered to optimally install PMUs for making power network completely observable. Reaching

initial PMU configuration to have a system with full observability was analyzed by an observability topology algorithm based on incidence matrix. Sensitivity constrained OPP detection and completely observable power system were solved by applying SA method. Dynamic character of a network could be better defined by the data with high sensitivity. The various steps involved in SA as follows:

- Step 1:** Initialize - Compute randomly next position.
- Step 2:** Calculate different Δ (determine the difference between two consecutive positions)
- Step 3:** If $\Delta < 0$, assign next position to the current position
- Step 4:** If $\Delta > 0$, compute the probability(score) of accepting the next position
- Step 5:** Choose - If the probability is $< e^{(-\Delta / \text{temperature})}$, then assign the next position to current position. (Depending on the change in score, accept or reject the move. The probability of acceptance depending on the current temperature)
- Step 6:** Update and Repeat - Loop to step 1 until "Freezing Point" is reached.

Figure-5 reveals that the Acceptance criterion and cooling schedule.

if ($\Delta > 0$) accept
else if ($e^{-\Delta / \text{temperature}}$) accept, else reject / * $0 \leq \text{random} < 1$ */



Initially temperature is very high (most bad moves accepted)
Temperature slowly goes to 0, with multiple moves attempted at each temperature
Final runs with temp = 0 (always reject bad moves)

Figure-5. Acceptance criterion and cooling schedule.

The main feature of SA is to provide local optima by allowing hill climbing moves (i.e., moves which worsen the objective function value). As the temperature parameter is decreased to zero, hill climbing moves occur

less frequently, and the solution distribution associated with the in homogeneous Markov chain that models the behavior of the algorithm converges to a form in which all



the probability is concentrated on the set of globally optimal solutions

2.4 Differential evolution (DE)

Differential evolution (DE) concept employs N-dimensional element vectors to minimize ongoing space functions. Mutation, crossover, and selection are the principle operators utilized to carry out the global optimization. This heuristic method could be widely used in different cost function problems such as nondifferentiable, non-linear and, multi-modal functions. Parallel computations, easy usage, and good convergence properties are other benefits of this approach. Peng et al. (2010) presented multi-objective OPP using a non-dominated sorting differential evolution (NSDE) algorithm which is an organic integration of Pareto non-dominated sorting operation and differential evolution algorithm (NSDE). In addition to solving the OPP problem this concept considered maximum measurement redundancy and voluntary PMU failure to reach a completely observable network. Usage of DE algorithm obtained from GA led to proposing NSDE algorithm procedure.

Achieving particular and complete Pareto front and finding many Pareto-optimal solutions were mentioned as the betterment of this procedure. Reaching the minimum number of PMUs required for system observability was discussed by utilizing integer linear programming (ILP), which provided an optimal solution by DE method. Three operators containing mutation, recombination, and selection process were functioned in this concept until the stopping criteria was accessed. Finally, solutions for the OPP problem considering fault observability were given considering the system with and without zero-injection. Utilizing conventional measurements moreover than PMU usage in the system, is to reach lower cost and also, get more accurate state estimation. This presented algorithm optimally provided a global solution in test systems which were benchmarked by state estimation. Also, the best solution was opted using the formulated procedure. The presented DE procedure provided a method for minimal PMUs and their configuration in the power system to analyze the observability of the system. Results of the proposed model were compared with other methods to show the minimal reached number of PMUs compared to others.

2.5 Tabu search (TS)

Tabu search (TS) is an adaptive algorithm that utilizes many other methods such as linear programming algorithms and heuristic concepts. This procedure is

presented to solve the combinational optimization problems in scheduling and covering. Tabu list which is one of the main elements of TS consists of the number of recently visited states plus a number of unwanted states. Other main elements of TS are aspiration, diversification, and definition of a state and the surrounding area. There is a reset in TS when it is not converging. A Solution for the OPP problem solution in terms of reaching a completely observable power system and enough redundancy using TS based on the linear state estimator model of a system was presented. This fast method of topologically observability analysis needed loss computation function based on incidence matrix for solving the OPP problem and was highly robust. Comfortableness and high speed of accessing an observable power system by the manipulation of integer numbers is also concerned by this method. Most of the observability analysis techniques utilize topological method, while a combination of a numerical method with TS, called Recursive Tabu Search (RTS), was presented to reach a completely observable network with maximum redundancy (Koutsoukis et al. 2013). This optimization method found the best solution of the executions where the initial solution always obtained from greedy algorithm was utilized as executed recursively. This procedure considered the methods including Modified Tabu Search (MTS) approach to reach the minimum number of PMUs as a solution for the OPP problem.

New parallel TS for solving the OPP problem providing a shorter process time was presented, which introduced four parallel spaces created by state division. Each of the newly obtained spaces was analyzed by Tabu list. In this method, a graph theory concept was utilized to reach an initial configuration for power system. Considering a constraint called lack of communication in substations was another phase of this paper. Two other methods called "Step Elite Solution" and "PMU Site Selector" were operated to calculate the functioned energy. Finally, an optimal solution for the OPP problem was obtained for the system considering the system with and without constraint. The objective is to solve the OPP problem using different methods to reach the minimal PMUs while analyzing complete observability of a power system by considering of observability constraints. Moreover than TS, Power System Analysis Toolbox (PSAT) software was utilized to compare different results of each method. Different algorithms have been applied in PSAT to analyze the system observability and obtain the minimum number of PMUs, which have illustrated the efficiency of the proposed method. Fig - 6 reveals that the standard TS algorithm,

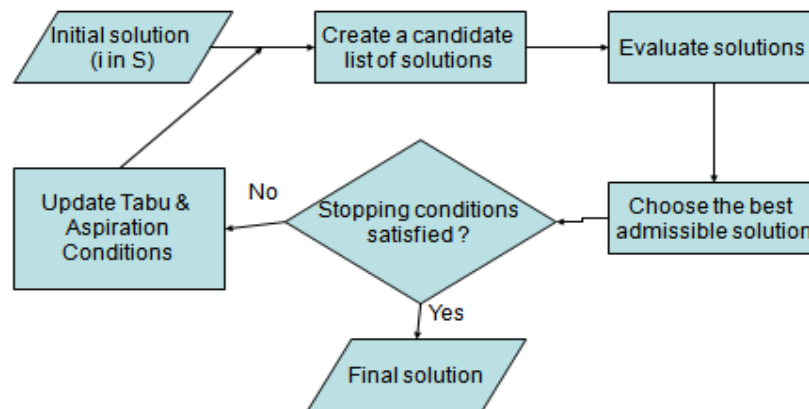


Figure-6. Standard TS algorithm.

Various step involved in TS are:

Step 1: Generate initial solution for X

Step 2: Generate Tabu List

Step 3: Find the solution for the candidate X

If the solution set X^* is not complete,
Generate solution x^* (the neighbors of X which are not in the Tabu List) for candidates from current solution x

Add x^* to X^* if x^* is not Tabu (minimum one aspiration criterion is satisfied)

Step (iv): Among all the solutions choose best one (x^* in X)

Step (iv): If the $F(x^*) > F(x)$ then set $x = x^*(F(x))$ denotes the fitness of x

Step (v): Modify the Tabu List (aspiration criteria).

Step (vi): If the termination condition reaches then stop, otherwise loop to the *step (iii)*.

2.6 Ant colony optimization (ACO)

Another concept utilized for presenting a solution to optimization problems is ant colony optimization (ACO) which initially uses a population of ants. Role of the colony of ants is to move through adjacent states of the problem by applying a stochastic local decision optimal controller (policy), which results in the solution for the optimization problem. Pheromone trail evaporation and daemon actions are other processes in ACO. Computational problems can be reduced using ACO to find good paths through graph. Most of the papers presented in Asia-Pacific power and energy engineering conference, reveals the optimal PMU placement problem for obtaining an observable power system with the minimum number of PMUs and considering maximum measurement redundancy was solved by utilizing an improved ACO. Depth first search as a graph theoretic method was applied to build a measurement tree so that the network observability could be analyzed.

Efficient calculation and equivalency between the exploration of new solution and that of aggregated problem learnt were mentioned as characteristics of ant colony system (ACS). Development of ACS by an adaptive stochastic perturbing ACS (ASPACS) was proposed in this paper to adaptively conduct the pheromone trail persistence coefficient (PTPC) and stochastic perturbing progress (SPP). Providing the OPP solutions containing approximate solutions and global solutions considering maximum measurement redundancy in power systems. Feasible solutions scope was propagated by proposing a function which simplified an extended scheme access for engineers. Finally, a comparison was made between results of this method and adaptive GA and SGA. The following are the merits of ACO,

- Efficient method for Travelling Salesmen Problem (TSP) with minimum number of nodes
- The greedy heuristic helps to find the acceptable solution in early stages of search process.
- For TSP it performs better solution compared to other global optimization techniques (Neural net, genetic algorithms, simulated annealing)
- Derivative free algorithm (good choice for constrained discrete problems)

The following are demerits of ACO,

- For theoretical analysis is difficult, such as sequences of random decisions (not independent); Probability distribution changes by iteration; Research is experimental rather than theoretical; Convergence is guaranteed, but time to convergence uncertain
- Difficult to solve TSP with large number of nodes (more than 75 cities)

3. LAGRANGIAN BASED SOLUTION ALGORITHM

3.1 The method of lagrange multipliers



Instead of replacing the constraint into the objective function, the method of Lagrange multipliers introduces one or more variable λ , into the problem. This variable is known as Lagrange multiplier and has an important economic interpretation which lies between zero and one. The method of Lagrange relies on maximizing an associated function, called Lagrangian. The Lagrangian function can be formed by adding λ times the constraint to the objective function and maximizing over the control variables and also the Lagrangian Multiplier.

Suppose the given problem is of the form

$$\begin{aligned} & \text{Max } f(x, y) \\ & \text{Subject to} \\ & g(x, y) \leq m \end{aligned}$$

Then the Lagrangian function is of the form,
 $L[x, \lambda] = \text{Max } f(x, y) + \lambda (m - g(x, y))$

In order to maximize the Lagrangian, we take partial derivatives with respect to the three control variables:

$$\begin{aligned} [x] : f_x - \lambda g_x \\ [y] : f_y - \lambda g_y \\ [\lambda] : m - g(x, y) \end{aligned}$$

And to obtain the optimal values of the controls, x^* , y^* , λ^* we have to set these equations equal to zero and solve them simultaneously.

Note that here constraints are treated as explicitly. Thus there is no chain rule effect in the first order conditions. In this process first order conditions is that treat all variables except the control variable at hand as constant. That is the reason why we are using partial derivatives and not total derivatives. The solution satisfies the constraint because in this case constraints are also one of the conditions so that must be explicitly satisfied.

3.2 Lagrangian relaxation (LR)

The LR idea is behind to attach Lagrangian multipliers to some of the constraints, relax these into the objective function, and then solve the original problem. When applying LR technique, the problem can in some cases be divided into smaller isolated subproblems, either naturally, i.e. by relaxing "common constraints" or by splitting variables and relaxing their equality binding, provided the objective and the relaxed constraints are additive in these variables. Different approaches can be used to find the values of the Lagrangian multipliers, for instance a sub-gradient method. Lagrangian Relaxation is a procedure to obtain good upper bounds for a problem (maximization problem), as lower bounds usually can be found using a heuristic. The starting point when using the methods is a modified version of the original problem one is trying to solve, which is known as the Lagrangian Relaxation. The main idea is to eliminate complicating constraints from the original problem, but to punish the objective function for breaking the constraints that have been removed. This is done by moving the relaxed

constraints into the objective function and then multiplying them with a Lagrange multiplier.

$$\begin{aligned} \text{Max } cx & \quad \text{max } cx + \lambda(h - Hx) \\ \text{subject to} & \quad \text{subject to,} \\ Ax = b & \quad Ax = b \\ Hx \leq h & \quad x \geq 0 \\ x \geq 0 & \quad \lambda \geq 0 \end{aligned}$$

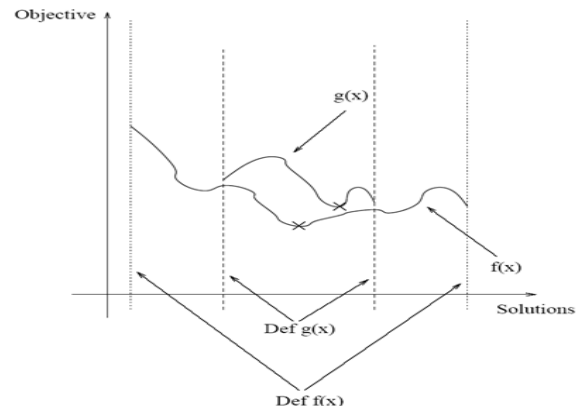


Figure-7. Graphical representation of relaxation.

By adjusting the Lagrange multiplier through a series of iterations, the hope is that a better upper bound can be found by finding the "correct price" of breaking the relaxed constraint(s). The central decisions in an implementation of Lagrange Relaxation are which constraint(s) to relax and how to update the Lagrange multipliers. The general methods to obtain Lagrange multipliers are subgradient optimization and multiplier adjustment. Subgradient optimization method is an iterative procedure that attempts solve the dual to the Lagrangian relaxation. That is, it tries to maximize the lower bound value obtained from the Lagrangian Relaxation. Multiplier adjustment is a heuristic approach that generates a starting set of Lagrange Multipliers, tries to improve them in some systematic way and then repeats the procedure if improvements are made.

4. CONCLUSIONS

This paper reveals an overview of advanced Optimization techniques typically encountered in engineering optimization applications. An analysis is made for various optimization techniques such as Evolutionary algorithms, PSO, GA, ACO, and LR to improve the search process by improving the bounds and the convergence toward the preferred solutions. The main purpose of the hybrid algorithms is to develop the advances of different optimization strategies, avoiding their disadvantages. There are many hybrid techniques that have shown to be successful for different applications. We have provided a review of the hybrid frameworks reported in the literature. The key parameters of heuristic methods are population size, number of generations, crossover rate and mutation rate. PSO provides most of the structure similar with GA but have faster convergence and simpler implementation. However, LR is a tool to find upper bounds on a given



arbitrary maximization problem. Local search and global algorithms are much suitable with smaller number of parameters for solving combinatorial optimization problems. This comparison will be very helpful for industries to determine the optimal parameters and improve the process and quality of products.

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