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A COMPREHENSIVE REVIEW ON WAVELET TRANSFORM AND ITS APPLICATIONS

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ABSTRACT

This paper attempts to discuss the prominence and impact of Wavelet Transform and its core applications based on its striking features and to state properties and other special aspects of it. Various types of wavelets, their variations and applications in signal/image processing are reviewed. Wavelet Transform has erupted as a means of revolutionizing the world of transform domain. In this paper, thorough analysis on various types of wavelet transform for beginners, to get acquainted with and to explore their interest is presented. A survey on privileged areas in radar, Fingerprint image authentication, biomedical image processing is presented. This transform is a promising tool to redefine the probabilistic and statistical analysis of numerical series, image compression musical tones and denoising data.

Keywords: wavelet transform, numerical series, biomedical image

1. INTRODUCTION

In the past few decades Wavelet Transform (WT) has drawn attention by mathematicians and researchers for its multiple outputs at different frequency bands called scales. Wavelet theory involves representation of general functions in terms of simpler, fixed building blocks at different scales and positions and predicted for analysing non-stationary signals. This was realised as a useful approach in several different areas, like subband coding, quadrature mirror filters, pyramid schemes, etc. and then study each component with a resolution matched to its scale.. They have advantages over traditional Fourier methods in analysing physical situations where the signal contains discontinuities and sharp spikes. The purpose of this paper is to focus on two axis 1) Types of Wavelet Transform and their distinct features 2) Wide applications in challenging domains like bio medical fields where signals tend to have extreme variability. The rest of the survey is organized as follows, Section II provides introduction to wavelets and their types Section III presents various categories of wavelets Section IV presents on applications.

2. II WAVELET TRANSFORM

A wavelet is a wave like oscillation with amplitude that starts out at zero, increases, and then decreases back to zero. Wavelets can be combined, using a "reverse, shift, multiply and sum" technique called convolution, to detect discontinuities in a signal. Wavelet Transform domain has gained lots of importance with the subject to understand, in part, the enthusiasm of its proponent's toward its potential application to various numerical problems. Furthermore its properties like good localization in both time and frequency, simplicity, and ease of construction and characterization, invariance under certain elementary operations such as translation, smoothness, continuity and differentiability and good moment properties, zero moments up to some order.A comprehensive review of literature on various issues related to fingerprint image authentication, image steganography, image compression, edge detection techniques and for solving issues like surveillance arepresented. We discuss here various types of wavelets which includes Discrete Wavelet Transform, Continuous Wavelet Transform (CoWT), ℂomplex Wavelet Transform (ℂWT) Fractional Wavelet Transform (FrWT), Fractional Random Wavelet Transform (FrRnWT), Steerable Wavelet Transform (StWT), Quanterion Wavelet Transform and Quincunx Wavelet Transform.

3. PROPERTIES OF WAVELET TRANSFORM

In this section basic properties of the wavelets are discussed. One of them is related to the fact that reconstruction of the signal from its wavelet transform is analysed. This property involves the resolution of identity, the energy conservation in the time-scale space and the wavelet admissible condition.

Any square integrable function which has finite energy and satisfies the wavelets admissible condition can be a wavelet. The second basic property is related to the fact that the wavelet transform should be a local operator in both time and frequency domains. Hence, the regularity (smoothness) condition is related to decay of its Fourier Transform. Aneccasary condition for regularity is for Low pass Filter to have atleat one zero at aliasing frequency$\omega = \pi$. The third basic property is related to the fact that the wavelet transform is a multiresolution signal analysis

4. DISCRETE WAVELET TRANSFORM

DWT is a type of wavelet transform in which the wavelets are discretely sampled. In wavelet analysis the use of a fully scalable modulated window solves the signal-cutting problem. The window is shifted along the signal consequently and for every position, the spectrum is calculated. Then this process is repeated many times with a shorter or longer window for every new cycle. In the end the result will be a collection of time-frequency representations of the signal, all with different resolutions. Because of these collections of representations,

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multiresolution concept emerge s[G.K.Rajini, *et al*, 2012]. The horizontal and vertical filtering along rows and columns yields approximation subimage and detail coefficients like horizontal, vertical and detail subimages.

[A. Enis Cetin and Rashid Ansari,1994] presented Signal Recovery from Wavelet Transform Maxima.This paper presents an iterative algorithm for signal recovery from discrete-time wavelet transform maxima. The signal recovery algorithm is developed by using the method of projections onto convex Convergence of the algorithm is assured.

[Ivan W. Selesnick, 1998] described using filter banks and discrete-time properties, this paper examined the differences between traditional wavelet bases and multiwavelet bases with equal approximation order. The properties (preservation and annihilation of discrete-time polynomials) are important for processing discrete-time signals because without them, the transform based on the associated filter bank gives a representation that is not necessarily sparse.

Ivan W. Selesnick, 2006, described a new set of dyadic wavelet frames with two generators. The spectrum of the first wavelet is concentrated halfway between the spectrum of the second wavelet and the spectrum of its dilated version. In addition, the second wavelet is translated by half-integers rather than whole-integers in the frame construction. The wavelet frames presented in this paper are compactly supported and have vanishing moments.

[A.S.Lewis and G.Knowles,1992]explained compression schemes using 2D Wavelet Transform indicated that the HVS compatible filters with finite support and quantizer provides significant improvement in compression in terms of image quality over block based transform methods and currently employed in video CODEC with VLSI technology.

[Stephene Mallat, 1991] discussed completeness, stability application to pattern recognition multiscale representation based on zero crossing. This theme is highly suitable and compact for signals with smooth and sparse singularities such as discontinuities.

Bo Yu Su., *et al,* 2015, proposed Wavelet transform (WT) to detect human falls using a ceiling mounted Doppler range control radar. The radar senses any motions from falls as well as nonfalls due to the Doppler effect. The proposed radar fall detector consists of two stages. The prescreen stage uses the coefficients of wavelet decomposition at a given scale to identify the time locations in whichfall activities may have occurred. The classification stage extractsthe time–frequency content from the wavelet coefficients at manyscales to form a feature vector for fall versus nonfall classification. Experimental results using the data from the laboratory and real in home environments validate the promising and robust performance of the proposed detector.

[Y.Ding and I.W. Selesnick, 2015] proposed an optimized denoising algorithm for signal that combine wavelet-domain sparsity and total variation regularization which are relatively free of artifacts, such as pseudo-Gibbs oscillations, normally introduced by pure wavelet thresholding. This paperformulates wavelet-total variation denoising as a unified problem.To strongly induce wavelet sparsity, the proposed approach usesnon-convex penalty functions.

[H. Xie, *et al*, 2002] developed algorithm for denoising and Markov random field modelingand discussed appearance of speckle noise in synthetic aperture radar (SAR) imagery makes it very difficult to visually and automatically interpret SAR data. Therefore, speckle reduction is a prerequisite for many SAR image processing tasks. In this paper, we develop a speckle reduction algorithm byfusing the wavelet Bayesian denoising technique with Markovrandom- field-based image regularization. The Expectation–Maximization algorithm is used to estimate hyper parameters and specify the mixture model, and theiterated-conditional-modes method is implemented to optimize the state configuration. It also achieves better performance than the refined Lee filter.

[Ali Al-ataby et al., 2010] provided a modified high capacity image steganography technique based on wavelet transform. The proposed method pre-adjusts the original cover image in order to guarantee that the reconstructed pixels from the embedded coefficients would not exceed its maximum value and hence the message will be correctly recovered. Then, it uses Wavelet transform to transform both the cover image and the hidden message. Wavelet transform allows perfect embedding of the hidden message and reconstruction of the original image. It was found that the proposed method allows high payload in the cover image with very little effect on the statistical nature of it. The evaluation of performance parameters like PSNR and MSE were discussed with respect to cover image and extracted image.

[Hemalatha. S *et al.*, 2013] proposed a secure image steganography technique to hide a secret image using a key. The secret image is not hidden, instead a key is generated and the key is hidden in the cover image and using the key the secret image can be extracted where Integer Wavelet Transform (IWT) is used to hide the key. This technique is secure and robust because none can realize the hidden information and it cannot be lost due to noise or any signal processing operations. Experimental results show very good PSNR, which is a measure of performance and security. In this technique the secret information is hidden in the middle bit-planes of the integer wavelet coefficients in high frequency sub-bands.

[Shiva Kumar K.B. *et al.*, 2011] presented a Hybrid Domain in LSB steganography in which communication using public channel steganographic method is adopted for maintaining privacy and secrecy of data. The proposed algorithm is integrated to have both spatial and frequency domain features. The cover image and payload are divided into two cells each and RGB components of cover image cell I are transformed using DWT and Discrete Cosine Transform(DCT) by retaining the components of cell II in spatial domain, This algorithm has better PSNR and less MSE when compared to other algorithms with improved security.

[Rajini.G. K and Ramachandra Reddy. G., 2012] presented an block based approach of DWT algorithm for fingerprint authentication. To test the efficiency of the proposed algorithm to recognize the rotated images, it is compared with algorithm based on ℂomplex wavelet processing. The algorithm proves to be better as its energy in both original and block based rotated images were equal. The minimal relative error between the energy of an original image and the block based rotated image shows the higher performance of the proposed algorithm based on image retrieval. The Euclidean distance is computed for complex wavelets coefficients as it is preferred for its computational simplicity. The block based approach using real wavelets consumes less time, since it computes the mean values of the blocks of the image. The reliability and simplicity involved in our algorithm has high potentiality in face recognition and motion estimation. The computational advantages of the proposed real wavelets over complex wavelets were particularly significant and produce good results.

5. CONTINUOUS WAVELET TRANSFORM (CoWT)

5.1. Poisson wavelet

[Karlene. A, *et al,* 1996, 1997] formulated functional analysis, several different wavelets are known by the name Poisson. In one context, the term "Poisson wavelet" is used to denote a family of wavelets labeled by the set ofpositive, the members of which are associated with the Poisson Probability function.

5.2 Morlet wavelet

[Turhan,1983]developed a wavelet named the Morlet wavelet or Gabor wavelet is a wavelet composed of a complex exponential(carrier) multiplied by a Gaussian envelope). This wavelet is closely related to human perception, both hearing and vision.

5.3 Mexihan hat

The Ricker wavelet is frequently employed to model seismic data, and as a broad spectrum source term in computational electrodynamics. It is usually only referred to as the Mexican hat wavelet obtained through negative normalization of second derivative of a Gaussian function, i.e., up to scale and normalization, the second Hermite function.

5.4 Shannon wavelet transform

Another exiting function called Shannon scaling function or the Sinc function is the starting point for the definition of the Shannon wavelet family. It can be shown that the Shannon wavelets coincide withthe real part of the harmonic wavelets which are the band-limited complex function.

[Carlo Cattani, 2008] described theconcepts of Shannon wavelets and its functions which are sharply localized in frequency. The derivativeof the Shannon wavelets has been computed by a finite formulaboth for the scaling and forthe waveletfor any order derivative. Computation of coefficients is obtained for the basis derivatives as a finite series for any order derivatives.

5.5 Meyer wavelet

It is a modified version of Shannon wavelet The scaling function is more smoothed, and it lacks the sharp continuities seen in the Shannon Wavelet.

6. COMPLEX WAVELET TRANSFORM

Complex Wavelet Transform has been crafted to eliminate the demerits of DWT like Lack of shift invariance and poor directionality. This ℂWT has an ability to generate real and complex coefficients which are nearly shift invariance and good directionality as complex filters separates frequency space but suffers from redundancy 2:1 for signals and 4:1 for images as real and complex coefficients are produced. This transform that contain carefully designed filters of different delays that minimize the aliasing effects due to decimation.

The Dual-Tree ℂomplex Wavelet Transform (DT-ℂWT) is comparatively enhanced version of the DWT, with important additional properties: It is nearly shift invariant and directionally selective in two and higher dimensions. It achieves this with a redundancy factor of only $2^d$for d-dimensional signals, which is substantially lower than the undecimated DWT. The multidimensional dual-tree ℂWT is no separable but is based on a computationally efficient, separable filter bankℂomplex Wavelets Transforms (ℂWT) use complex-valued filtering (analytic filter) that decomposes the real/complex signals into real and imaginary parts in transform domain. The real and imaginary coefficients are used to compute amplitude and phase information required to describe the energy localization of oscillating functions (wavelet basis).It possesses extraordinary features like shift invariance, rotation invariance, good directional selectivity and has considerably low redundancy but at the expense of extra computational cost and system overhead, yet it allows perfect reconstruction of the signal. The six sub bands of 2D dual tree gives information strongly oriented at (30º, 0º, -30º, 60º, 90º, and 120º.

Felix.c, *et al*., 2003., introduced two-stage mapping-based complex wavelet transforms that consist of a mapping onto a complex function space followed by a DWT of the complex mapping using ℂomplex Double Density DWT (ℂDDWT).This transform is simultaneously directional and non-redundant and flexibility to use *any* DWT in the transform implementation.

[Kingsbury.N.G, 1999] proposed shift invariant properties of DT-ℂWT which employs dual filters to obtain real and imaginary parts of ℂomplex wavelet coefficients while preserving the usual properties of perfect reconstruction and computational efficiency with well-balanced frequency responses and describes how to estimate the accuracy of this approximation and design of suitable filters to achieve the shift invariant property.

[Kingsbury.N.G, 2001] presented “ℂomplex Wavelets for shift invariant analysis and filtering of signals” and discussed two different variants of this

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complex transform, based on odd/even and quarter-sample shift (Q-shift) filters and later described briefly how the dual tree may be extended for images and other multi-dimensional signals, and later summarized a range of applications of the transform that took advantage of its unique properties like motion estimation, compensation, denoising and deconvolution, texture analysis and synthesis, image segmentation, classification and watermarking.

[Ivan W. Selesnick, 2001; 2002] demonstrated the design of approximate Hilbert transform pairs of wavelet bases and significant improvements for wavelet-based signal processing by utilizing a pair of wavelet transforms where the wavelets form a Hilbert transform pair. This paper describes the design procedures based on spectral factorization, for the design of pairs of dyadic wavelet bases where the two wavelets form an approximate Hilbert transform pair.

[Mallat.S, 1987] suggested that wavelet representation can be efficiently implemented with a pyramid architecture using quadrature mirror filters and the original signal can also be reconstructed with a similar architecture. The numerical stability was well illustrated by the quality of the reconstruction and the orientation selectivity of this representation which was useful for many applications. These applications of the wavelet representation include signal matching, data compression, edge detection, texture discrimination and fractal analysis. Computer vision applications have been emphasized and its representation can also be used for pattern recognition.
[Mallat. S, *et al.*, 1992] proposed singularity detection and processing of wavelets with fast oscillations. The local frequency of such oscillations is measured from the Wavelet Transform Modulus Maxima (WTMM). It has been shown numerically that one dimensional (1D) and two dimensional (2D) signals can be reconstructed with a good approximation; from the local maxima of their wavelet transform modulus. As an application, an algorithm is developed that removes white noises from signals by analyzing the evolution of the wavelet transform maxima across scales. In 2D, the wavelet transform maxima indicate the location of edges in images and the denoising algorithm is extended for image enhancement.

7. FRACTIONAL WAVELET TRANSFORM

I.W.Selesnick *et al* formulated the DT-ℂWT which possesses different sets of PR filter banks at each stage with half sample delay condition. To avoid different sets of PR filter banks at each stage Thierry Blu and Micheal Unser [9] proposed Fractional Wavelet. Transform which have identical filter banks at all stages which ease the design aspect. Fractional Wavelet transform introduced by Thierry Blu and Micheal Unser [1, 3] is a new family of scaling functions, the size and location of which are governed by the two parameters namely α (order) and τ (shift).It also involves binomial distribution, approximation theory and numerical analysis.
[Unser. M. and Blu.T., 2003] revisited wavelet theory starting from the representation of a scaling function as the convolution of a B-spline (the regular part of it) and a distribution (irregular or residual part).Later proved that the B-spline component is entirely responsible for five key wavelet properties: order of approximation, reproduction of polynomials, vanishing moments, multiscale differentiation property, and smoothness (regularity) of the basis functions.

[Ivan W. Selesnick *et al.*, 2005] enhanced the real Wavelet Transform to Dual-Tree ℂomplex Wavelet Transform with additional properties like nearly shift invariant, nearly rotation invariance and directionally selective in two or more higher dimensions. The magnitude and phase of the DT-ℂWT coefficients are exploited for developing effective wavelet based algorithms. The applications of these sophisticated algorithms are promising in the field of image classification, segmentation, denoising, deconvolution, image sharpening, motion estimation, texture synthesis and analysis.

[Blu. T and Unser. M., 2000] presented the fractional spline wavelet transform its definition implementation and defined a new wavelet transform that is based on a recently defined family of scaling functions: the fractional B-splines. The interest of this family is that they interpolate between the integer degrees of polynomial B-splines and they allow a fractional order of approximation. He also proposed that this transform can be especially useful for the synthesis of fBm (fractional Brownian motion) noises.

[Shi Jun *et al.*, 2011] showed the limitations of Fractional Fourier Transform (FrFT) and Wavelet Transform (WT) and the signal analysis of the former is limited to time-frequency plane whereas the latter fails to obtain the local structures of the signal. The proposed algorithm of FrWT not only inherits the advantages of multiresolution analysis of WT but also the capability of representation of signal in fractional domain. The Novel FrWT (NFrWT) its basic properties, inversion formula and admissibility conditions were also presented.

[Unser. M, *et al.,* 1993] described a family of polynomial spline wavelet transforms of compact support which are natural counterpart of the standard B-splines. The excellent feature of this wavelet is that their time-frequency localization, a property relevant for non-stationary signals. He also extended the Mallat's fast decomposition algorithm which operates by iterated filtering and decimation for evaluation of the class of polynomial spline wavelet transforms. The corresponding analysis and synthesis are not necessarily identical but can take binomial form.

[Strang. G and Nguyen. T, 1996] in their book explained the concepts of wavelets and perfect reconstruction filter bank, orthogonal filter banks, M-channel filter banks, design of two channel filter banks and its applications towards image and video compression, speech, audio, Electrocardiograph(ECG) compression and possible application of wavelets to differential equations. Another most important communication application of wavelets is transmultiplexer and its applications towards adaptive systems were also discussed.

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[Unser. M, 1999] demonstrated Splines as a perfect fit for signal and image processing and explained polynomial splines as a linear combination of B-splines basis functions. Splines are smooth and well-behaved functions and have multiresolution features. The conventional sampling theorem can be easily modified to obtain a spline representation of an analog signal. He demonstrated B-spline application towards contour detection, zooming and visualization, geometric transformations, image compression, multi scale processing and image registration.

[Unser.M, and Blu. T, 1999] extended the B-splines to fractional order and its remarkable feature is that these new functions inherit all the excellent properties of the polynomial B-splines with two exceptions: positivity and compact support. This feature enabled to construct fractional wavelet bases of L_2. The enlarged families of orthogonal and semi-orthogonal spline wavelets with a continuous order indexing rather than a discrete one are possible. These new fractional spline wavelets have explicit formulas in both the time and frequency domain. Their most notable feature is the order of approximation α +1 which is no longer an integer and in addition to this, they behave like fractional derivative operators. The only demerit of these functions is that they are not compactly supported.

[Zhou Wang, *et al*., 2004] proposed objective methods for assessing perceptual image quality traditionally to quantify the visibility of errors (differences) between a distorted image and a reference image using a variety of known properties of the human visual system. Further it was investigated that human visual perception is highly adapted for extracting structural information from a scene, and the authors introduced an alternative complementary framework for quality assessment based on the degradation of structural information.

[G.K.Rajini, *et al* 2015] developed an algorithm for fingerprint authenticationusing Fractional Wavelet Transform (FrWT). It is a new family of wavelet transform that is based on well defined scaling functions, the Fractional B-splines. Fractional wavelet transforms perform better than other wavelets due to their superiority of more energy compactation in approximation coefficients. This paper also presents simulation results of filter banks of FrWT and reveals the suitability and superiority of this transform for fingerprint authentication.

8. FRACTIONAL RANDOM WAVELET TRANSFORM

In this section we describe a new transform called Fractional Random Wavelet Transform (FrRnWT), afamily of wavelet transform that inherits excellent mathematical properties of Wavelet Transform (WT), Fractional Random Transform (FrRnT).It describes the information in spatial and frequency domain with randomness, due to rotation of time frequency plane. The features of FrRnWT are exploited for hiding technique in Image Steganography. The modern secure image steganography presents a challenging task of transferring the embedded image to the destination without being detected in an insecure channel. Animations were used as cover object to transfer information and its robustness justifiedwhen applied to digital image steganography [15]. [Bhatnagar *et al*., 2012] emphasized the wavelet transform, which is an important tool in signal and image processing which has been generalized by coalescing wavelet transform and fractional random transform. The FrRnWT inherits the excellent mathematical properties of wavelet transform and fractional random transform. Possible applications of the proposed transform are in biometrics, image compression, image transmission, transient signal processing, etc. As a primary application, biometric security is explored by proposing an efficient and robust fingerprint encryption technique. To achieve desired goal, the well-known chaotic maps, i.e., logistic and Arnold cat maps are used. Logistic map is used to generate the fractional orders and number of cat map iterations. The experimental results have been carried out with detailed key space, key sensitivity, statistical and numerical analysis which demonstrate the efficiency and robustness of this proposed fingerprint encryption technique.

[Abbas Cheddad *et al*., 2009] described the differences between steganography and watermarking schemes. Generally steganography methods struggle in achieving a high embedding rate and the authors suggested video files having many excellent features for information hiding such as large capacity and good imperceptibility. Various challenges to embed group of images and comparison of steganography, watermarking and encryption were discussed with respect to some criteria's like authentication, secret data, key, types of attacks and flexibility. Comparison of common Steganalysis software's like Hide and seek, S-Tools with respect to parameters like PSNR, visual inspection and security related issues were also discussed.

[Rajini. G.K. and Ramachandra Reddy, 2015] presents a novel technique for image steganography based on Fractional Random Wavelet Transform. This transform has all the features of wavelet transform with randomness and fractional order built into it. The randomness and fractional order in the algorithm brings in robustness and additional layers of security to steganography. The stegano image generated by this algorithm contains both cover image and hidden image and image degradation is not observed in it. The steganography strives for security and pay load capacity. The performance measures like PeakSignal to Noise Ratio (PSNR), Mean Square Error (MSE), Structural Similarity Index Measure (SSIM) and Universal Image Quality Index (UIQI) are computed. In this proposed algorithm, imperceptibility and robustness are verified and it can sustain geometric transformations like rotation, scaling and translation and is compared with some of the existing algorithms.

9. STEERABLE WAVELET TRANSFORM

It is often required to use same set of filters for rotation at different angles and different orientation under adaptive control. Freeman (1991) developed the need of designing a filter, with filter response as a function of orientation and named it steerable. The term 'steerable' (rotated) is used to describe a separate category of filters in which a filter of arbitrary orientation is synthesized as linear combination of a set of basis kernels. These filters provide components at each scale and orientation separately and its non- aliased subbands and over complete are good for texture and feature analysis.

Steerable filters are designed for analysis of biomedical imagery which enables the physician to investigate the vascular disease which is of paramount importance in carotid artery stenting procedure. It also enables algorithms in machine vision, pattern recognition and scene analysis.

[Jun Li, 2003] proposed a wavelet approach to edge detection and explained the working mechanism of edge detectors from the point of view of wavelet transforms and developed a new wavelet-based multi-level edge detection method. The proposed wavelet based edge detection algorithm combines the coefficients of wavelet transforms on a series of scales and significantly improves the results. The wavelet transform characterizes the local regularity of signals by decomposing signals into elementary building blocks that are well localized both in space and frequency. This work explains the underlying mechanism of classical edge detectors, and also indicates a way of constructing optimal edge detectors under specific working conditions.

[Simina Emerich, 2008] described that wavelets provide a unified framework for a number of techniques, which were developed independently for various signal and image processing applications. The purpose was to improve some of these techniques by using the B-spline functions and show that the corresponding fractional and generalized B-spline wavelets behave like multiscale differentiation operators of fractional order. This is in contrast with the classical wavelets whose differentiation order is constrained to be an integer and this property is well suited for image edge detection. Singularities detection can be carried out by finding the local maxima of the fractional wavelet spline transform.

[Unser. M *et al*., 2000] presented a brief review of fractional derivatives, splines and tomography and developed a spline calculus for dealing with fractional derivatives. After a brief description of fractional splines they presented main formulas for computing the fractional derivatives of the underlying basis functions.

[Douglas Shy *et al*., 1994] elaborated the X-Y separable pyramid steerable scalable kernels for edge detection and generalized the Pseudo Singular Value Decomposition (PSVD) which reduces computation time and storage requirements. The two major improvements that have been made on existing schemes for generating steerable decompositions such as compact term by term x-y seperability and Laplacian pyramidal implementations.

[William T. Freeman and Edward H. Adelson, 1991] illustrated the design of steerable filters and their use in several tasks like analysis of orientation and phase, angularly adaptive filtering, edge detection and shape from shading. Volumetric spatial data and temporal image requires three dimensional (3D) video processing and these steerable filters are extended to motion analysis.

[Rajini. G.K. and Ramachandra Reddy, 2014] illustrated comparison techniques of spatial and transform domain techniques for edge detection. Edges represent structural information of objects present in an image. In this paper analysis of biomedical imagery which enables physician to investigate the vascular disease which is of paramount importance in carotid artery stenting procedure is discussed. It also enables algorithms in machine vision, pattern recognition and scene analysis. Edge detection through gradient operators like Sobel, Prewitt, Canny and Roberts are compared with transform domain techniques like DWT, FrWT and SWT.

10. QUANTERION WAVELET TRANSFORM

Quaternion Wavelet Transform is established based on the quaternion algebra, quaternion Fourier transform, and Hilbert transform; using four real DWT, the first real discrete wavelet corresponding to quaternion wavelet real part, the other real discrete wavelet is obtained by the first real discrete wavelet transform's Hilbert transform, corresponding to quaternion wavelet three imaginary part, respectively, the four real wavelet composed of quaternion analytic signal. This transform has approximate shift invariance, abundant phase information, and limited redundancy and retaining the traditional wavelet time-frequency localization ability.

[Ming Yin *et al,* 2012] stated the concepts and properties of quaternion wavelet transform, gives quaternion wavelet scale and wavelet functions, and applies the quaternion wavelet in image denoising, puts forward Bayesian denoising method based on quaternion wavelet transform, considering wavelet coefficient's correlation, and generalized Gaussian distribution is used to model the probability distribution function of wavelet coefficients' magnitude and the best range of the Bayesian thresholding parameter is found out. The experimental results shows that this method provides better PSNRthan some denoising methods.

[Sanoj Kumar *et al*., 2012] discussed a novel phase based approach using Fractional Quaternion Wavelet Transform (FrQWT) for computing disparity as optical flow from a given sequence of images. In this paper phases are estimated from fractional quaternion analytic signal using the concept of quaternion algebra. Disparity is estimated as the optical flow using the phase difference method for realistic image sequences.

[Eduardo Bayro-Corrochanothe, 2006] explained the theory and practicalities of the QWT, such that the reader can apply it to current problems with quaternionic phase concept. This transform offers three phases at each level of the pyramid, which can be used for powerful top-down parameter estimation. The experimental part demonstrated QWT for optical flow estimation.

10.1 Quincunx wavelet transform

Quincunx wavelets are more isotropic than seperable wavelets. It is non-separable transforms that allow avoiding using vertical/horizontal wavelets. The scaling grows like $2^{j/2}$ with the scale j instead of 2^j, which can be advantageous. Biorthogonal or orthogonal quincunx wavelets are constructed with perfect reconstruction or conjugate mirror filters defined with quincunx subsampling [Dong Wei, *et al*, 1997]. These Wavelets are implemented with seperable filter banks that increases the scale by 2, by dividing the image grid in a coarse grid that keeps out of four,plus three detail grids of the same size and that correspond to three different wavelets which employ a dilation operation induced by the Quincunx matrix [Bin Han, 2000]. [Manuela Feilne, 2001] formulated 2D orthogonal wavelets refinement filters have a simple analytical expression in the Fourier domain as a function of the order *a,* which may be non-integer and have good isotropy properties.

11. DISCUSSIONS WITH COMPARISON

A comprehensive study of various wavelet transforms for signal and image processing is presented over here. The main purpose of this paper is to analyse how efficiently wavelet transform projects time frequency representations. Since Fourier, STFT lacks in representation of frequency domain information and wavelet transform stands distinguished in terms of simultaneous time/frequency representations are discussed in this paper. The performance of various wavelet transforms based on its properties with various applications is presented.

12. CONCLUSIONS

Wavelet Transforms evolved when time/frequency representation was a challenging phenomenon. Reliable reconstruction of signal with minimum loss with respect to PSNR and MSE are appreciable factors. The use of wavelets in science engineering and medicine has increased because of its interpretation of data and multiresolution feature representation. This paper deals with the survey of various types of wavelets their properties and performances of these wavelets of each were discussed. To summarize, every wavelet transform was deployed to satisfy certain strategies and requirements of signal and image processing with less computation cost.

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