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A REVIEW OF OPTICAL FLOW MODELS APPLIED FOR FLUID MOTION ESTIMATION

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ABSTRACT

Fluid flow estimation from image sequences is a challenging due to the large and complex nature of fluid flow. In this work, 36 papers on optical flow models applied for fluid motion estimation from 1980 to 2015 have been reviewed. The advantages and weaknesses of the optical flow models are discussed in detail. To improve the accuracy of the optical flow models, the principle of two multi-resolution schemes is discussed. The employment of higher order models provides more accurate motion vectors than the use of the standard optical flow model; however, it suffers from computational complexity. The use of the multi-resolution framework such as wavelet decomposition will reduce this complexity, as well as improve estimation accuracy.

Keywords: fluid motion, differential method, multi-resolution scheme, optical flow.

INTRODUCTION

Fluid motion estimation from image data has a variety of applications in fluid dynamics, meteorology, river/ocean motion, and underwater oil spill. Due to the complex nature of these kinds of flow, it has been difficult to extract their motion from image sequences. The turbulence behaviour of fluid flow usually causes large spatial and temporal image intensity deformation when captured by optical sensors. Several approaches for fluid motion estimation are available in the literature, but the most commonly used are the cross correlation method and the optical flow method. The cross correlation method relies on measuring the similarity between two pixels or regions in image sequences in order to estimate the image velocity. This approach has been extensively used for fluid dynamic applications and image velocimetry such as particle image velocimetry [1] and optical plume velocimetry [2]. Optical flow estimates motion based on the brightness constancy theory and are commonly applied in the meteorological field [3, 4]. As well as in fluid dynamic field such as, the previous work of the Authors [5] which compared two optical flow algorithms for complex turbulent jet flow estimation. However, the drawback of optical flow is that not appropriate for fluid motion which comprises of complex turbulence behaviour. Another, it is more appropriate for small rigid motion estimation. To overcome this problem, the behaviour of fluid flow has to be considered. The main objective of this paper is to review optical flow models that are used for fluid applications. Comparison between optical flow and cross correlation based methods can be found from the work by Liu [6].

FUNDAMENTALS OF OPTICAL FLOW

Optical flow has been widely used in various fields for estimating the motion of object from image sequences. For instance, they are used for motion estimation and motion tracking [7, 8]. The general form of the optical flow model is shown in Equation. (1). It consists of two terms: the data term (E_{data}) to relate the image information to the physical object motion and the regularization term (E_{Req}) for considering the physical flow properties of the fluid in the estimation. This equation contains the optical flow vectors *(u,v)*; both are unknown and cannot be solved. This is known as the aperture problem. The regularization term helps in finding a solution to aperture problem. This equation has the similar idea of intensity constancy constrains used for optical flow from images, which stated that for any closed system, the mass of the system must remain constant over time [9], which is analogous to with the intensity constancy assumption. Therefore, image motion can be formulated as the following objective function:

$$E(u,v) = \int E_{data}(u,v) + E_{reg} \tag{1}$$

where, E is the energy function to be minimized and *u,v* the velocity vectors.

Figure-1 shows the fundamentals of optical flow between two consecutive images. The basic assumption is that, the brightness of each pixel in the first frame is constant over small time intervals *(t),* and that any change over this time frame is caused by turbulence. By applying Equation. (1) for each pixel between the two frames, the horizontal and vertical motion *(u,v)*, can be obtained.

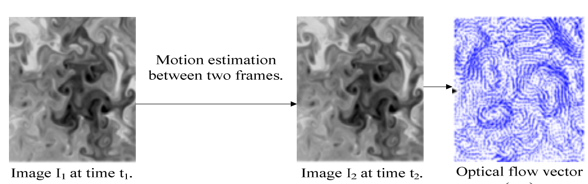

Figure-1. Motion estimation between two frames (with the resultant optical flow).

REVIEW OF OPTICAL FLOW MODELS

The performance of some of the optical flow algorithms is discussed in [9]. The classical optical flow estimation models was first proposed by Horn and Schunck [10] for estimating the motion of rigid bodies.

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The standard regularization term proposed by Horn and Schunck [10] for rigid body motion estimation has been improved to consider the divergence and rotation of the fluid velocity field, and reformulated as first order div-curl regularizer. In this context, Richard *et al* [11], applied the continuity equation as a data term with the first-order regularization to relate image parameters with the real cloud motion. Nevertheless, this approach had some drawbacks: firstly, it was not suitable for large scale of motion which is expected in cloud image data. Therefore, higher error propagation is expected when applying the model for large motion scale in cloud domain. This problem can be solved by over-smoothing the large motion using the appropriate filtering technique, but this process is also a source of error propagation. Again, the differential form of the continuity equation limited its use in large fluid motion. Despite these drawbacks, the application of this model (continuity equation plus first order regularizer) for fluid motion estimation has a degree of advantage over the use of the standard regularizer of Horn and Schunck [10].

The complex nature of fluid flow has made the use of this model difficult for capturing the fluid motion without developing some constraints to consider the physical properties of fluid flow. In the context of fluid motion estimation, Chen [4] and Corpetti [12] have conducted studies to improve this differential algorithm for fluid flow estimation. This paper compiles past and current modifications for improving optical flow model especially for fluid flow estimation. Several data and regularization terms have been applied in the literature to improve the accuracy of optical flow for fluid motion estimation. Table-1 summarizes the optical flow models that are applied for fluid flow estimation.

The early works by Wildes [11] and Corpetti [13] applied the standard data term with first-order regularization for cloud motion estimation. However, this model failed to represent the cloud motion well and inaccurate motion measurement resulted. The model only considered translation of the cloud in one direction without considering rotation and scaling behaviour of cloud motion. On the other hand, the optical flow model in Equation. (1) can be improved by modifying the regularization term to consider the physical properties of fluid motion. Therefore, several regularizations, either in first-order or higher-order forms have been proposed and applied for the purpose of fluid motion estimation.

Corpetti [14] derived the integrated continuity equation in order to overcome this limitation (use for large motion estimation), and applied it to estimate the motion in free turbulent shear layer [12] with second order div-curl smoothing. The higher order regularizations (second order div-curl) are required for capturing the physical behaviour of fluid motion; however, higher order means higher computational time. The second order div-curl regularizer is used for fluid application because of its advantage of maintaining translation, rotation, and scaling properties of the fluid dynamics. It suffers from implementation difficulty because of the higher order differential parts involved. To overcome this problem, Corpetti in [12] proposed to add two auxiliary variables ξ and ω, as approximations of true divergence and curl part to reduce the higher order behaviour of the model and consider the model as linear approximation.

Corpetti [14] estimated cloud motion from recorded image sequences by employing continuity equation as data term and second order div-curl regularization. This method has advantages over the use of standard optical flow constraint which utilizes first-order regularization in well-recovering of the divergent and rotation of cloud flow and for predicting its motion direction.

The div-curl regularization was first proposed by Gupta and Prince [15]. Some flow properties such as spatial distribution of velocity fields along image and vorticity map were well-extracted, which are important for describing the flow behaviour and are difficult to extract using the standard model.

Table-1. Summary of mathematical models of some data and regularization terms.

Mathematical model	**Selected literature**
Data term:	
$\frac{dI}{dt} + I\nabla .u = 0$	Horn, 1981 [9].
$E(d) = \int f(I(x+d(x),t+\Delta t)\exp(\nabla .d(x,t))dx$	Richard *et al.* 2000, [11]
Regularization term:	
$E(u,v) = \int \left\|\nabla u\right\|^2 + \left\|\nabla v\right\|^2 dx$	Dérian 2010 [26], 2012 [28]
$E(u,v) = \int \left\|\nabla^2 u\right\|^2 + \left\|\nabla^2 v\right\|^2 dx$	Ribeiro 2011, [16]

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$E(u) = \int \alpha \left\|\nabla divu\right\|^2 + \beta \left\|\nabla curlu\right\|^2 dx$	Corpetti, 2003, [10].
$E(u,\xi,\omega) = \int \begin{matrix} \lambda\left\|\nabla divu - \xi\right\|^2 + \beta\left\|\nabla curlu - \omega\right\|^2 \\ + \mu(\left\|\nabla\xi\right\|^2 + \left\|\nabla\omega\right\|^2)dx \end{matrix}$	Corpetti, 2003, [10], 2000 [13], 2001 [20], & X. Chen 2015 [4].

Moreover, extracting some statistical parameters such as the derivative of the vorticity depth is an indicator of the evolution of turbulent structures and expansion factor. The later form (i.e. continuity equation plus second order div-curl regularization) has been used in Corpetti [3] for estimating the motion of fluid in mixing layer, and the near wake of a circular cylinder. A similar result was found when compared with the classical cross correlation algorithm. This approach extracted accurately the velocity vectors when compared to the one by cross correlation based approach proving that differential method can be used for real fluid flow estimation. However, this approach is computationally expensive and difficult to use for the large size of images. The employment of higher order regularizations leads to more numerical instability [17] of the algorithm. One way to reduce the computational time and the stability of the higher order based model is to solve the model in the multi-scale framework.

Isambert *et al.* [18] formulated a mathematical model for fluid motion estimation based on performing the continuity equation on control points of spline vectors by regularizing the data using second order div-curl. The model was embedded in the multi-scale framework in order to improve the spatial resolution of the captured images. Isambert *et al.* [17] developed an approach called "Partition of Unity and Optical Flow" (PUOF) to estimate the wind motion captured by satellite images. The PUOF approach relies on merging local motion into the whole spatial domain using local spline vectors. The advantages of this approach are the numerical stability it gives because more control points are used to solve the motion model and its capability of handling large images within a small time. When compared with classical differential method, the spline vector method yielded similar results [18].

Generally, the spline-based approach has been a good alternative for fluid motion estimation for several reasons: first of all, it has an efficient implementation to assess fluid turbulent flows for local estimation whereas first-order and second-order without spline framework are global solutions. Again, it easily allows taking conservation law of fluid flow into account, and this leads to an accurate motion field. Furthermore, its implementation with the second order div-curl regularization is more efficient with no complexity for turbulent fluid estimation. It is very advantageous as well in extracting the divergence and curl of the motion field and require no iterative minimization steps which complicate the computational issues [17]. However, the direct implementation of spline-based approaches in fluid motion estimation have some drawbacks: firstly, the estimation from small-scale image results in an over-smooth estimation of motion fields and error propagations are expected. Secondly, the implementation of these methods in case of large images increase the computational cost, besides being sensitive to local minima.

Liu [16], proposed a thin plate spline was proposed for motion estimation, but this approach was bias to lower-order flow fields. Therefore, the multi-scale spline scheme by Isambert [18] is appropriate for flow field of large images. It enables the capture of the fluid rotational motion field than Corpetti's approach [13]. Additionally, a comparison of multi-scale spline method with the thin-plate spline and Horn &Schunck approaches resulted in a minimum angular error. This showed that it was capable of well-presenting the fluid central vortices when tested in meteosat image sequences. Vector splines have been introduced for motion estimation based on the luminance conservation equation [18], and based on mass conservation [19].

Liu and Ribeiro [16] proposed a higher order parametric model to estimate the motion of complicated turbulent flow from image sequences. The method was based on representing flow field as holomorphic functions. Fluid flow properties were clearly extracted, as well as less biased towards a certain fluid flow phenomena compared to classical optical flow approach [9], and first-order and second-order regularization [12]. A comparison between the method by Liu [16] and the work of Isambert [18] and Horn and Schunck [9] proved its robustness.

On the other hand, the decomposition of fluid flow field is important for understanding the behaviour of the flow. A Helmholtz decomposition approach conducted by Kohlberger [20] estimated the motion of non-rigid dense structure in image sequences. This approach decomposes the fluid flow into its two components: divergence free (solenoidal) and curl free (irrotational). Accordingly, from a mathematical point of view, the fluid motion estimation by decomposition of the flow fields have several benefits. Firstly, the irrotational and the solenoidal components of the velocity fields can be provided from its gradients; secondly, basic velocity fields components such as vorticity and divergence can be accessed from its Laplacian. Again, the velocity potential streamlines can be directly extracted. Finally, the several fluid motion components can be extracted such as sources, sinks and vortexes. Therefore, by comparing the work in Kohlberger [20] with the method proposed by Corpetti [21], a similar conclusion was obtained for the fluid motion.

Subsequently, Cuzol [22, 23] used the concept of Helmholtz decomposition to estimate a lower dimensional fluid motion. It was applied to analyse the motion components (vortices, divergence) in real image sequences

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of meteosat meteorological and experimental fluid mechanics. Again, the continuity equation and second order div regularizer was utilized in data-term and smoothness-term respectively. By seeding a less number of particles, a better accuracy and faster computation resulted. However, by increasing the number of particles with low-dimensional representation of velocity vectors will lead to higher computational cost.

MULTI-RESOLUTION SCHEMES

The differential based methods have not been able to capture large motion fluid flow. The higher order models are one way to improve its accuracy. However, the higher order schemes are also limited for higher motion estimation and are also associated with increasing computational cost. So to overcome this problem, multi-scale frameworks have been used to increase the accuracy of capturing large motion estimation of turbulence and reduce the computational complexity as well. Various of these frameworks were applied such as the classical coarse-to-fine [24, 25]. The main idea behind this framework is to find the motion vectors by resizing the image sequences in a shape of pyramid as shown in Figure-2. The motion estimation usually starts from the coarse (i.e. large image size) to a fine level (i.e. small image size) individually and thus the name coarse to fine. The main advantage of this approach is to solve the non-linearity of optical flow algorithms caused by large displacement between the consecutive images [26] which may not fall under the linearity of brightness constancy assumption.

Despite the advantage of embedding any algorithm in a coarse-to-fine framework, inaccurate motion vectors are expected in higher levels, as well as inaccurate results, as observed, in tracking the motion at finer resolution in Dérian *et al.* [27] work. Moreover, the coarse-to-fine framework approximates the motion in each level as linear relationship, which again limits its use for only small motion estimation. Zille *et al.* [28], employed Lucas-Kanade algorithm in a coarse-to-fine scheme for estimating the optical flow, and this outperformed the classical optical flow algorithm with first order regularizer.

In the context of using a multi-scale framework for fluid motion estimation, wavelet transform has been used in several works to improve the accuracy of the estimation and reduce the computational time of motion estimation algorithm [29]. The wavelet transform has several applications in fluid domain, such as turbulent coherent structure analysis [30, 31], fluid motion simulation [32], and motion estimation [33]. More information about the relation between fluid and wavelet is available from [31, 34].

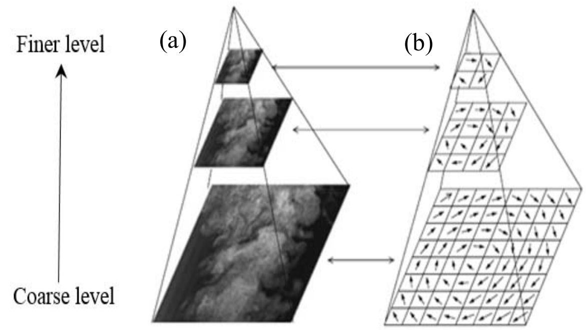

Figure-2. Coarse-to-fine concept: (a) the input image resized down to present coarse to finer level; (b) the motion vectors estimated at each level [20].

The basic concept behind the wavelet is to decompose the original image or signal into approximation part and detailed parts using a high pass filter and low pass filter respectively. Each level of wavelet decomposition will result in four sub-images: one approximation image with three detailed images named as horizontal, vertical, and orthogonal. Figure-3(a) shows the three level wavelet decompositions, and Figure-3(b) illustrates how to generate and decompose the sub-images from original images. First, the original image is decomposed into horizontal approximation and horizontal detail using a lower pass filter and a high pass filter. Then the horizontal approximate is decomposed into approximate image and a vertical by applying a low pass and high pass filter to the horizontal approximate generated. Again, by filtering the detail approximation using low and high pass filters, the horizontal and orthogonal details are generated.

The wavelet framework has several advantages for motion estimation that includes: its ability to avoid the linearity problem of classical coarse-to-fine technique, being easy to implement for high-order regularizations in order to improve the accuracy and performance of motion estimation algorithm, and giving accurate motion vectors because the wavelet decomposition process will filter-out the higher frequencies (i.e. noise) from the image sequences. Moreover, the wavelet framework can solve the numerical instabilities of traditional regularizations (i.e. first order, second order, and second order div curl) which is a result of differential form of these regularizations [35].

Dérian *et al*. [27], proposed a higher order regularization for fluid motion estimation based on discrete wavelet decomposition and this resulted in accurate motion estimation with lower complexity. This proposed algorithm performed better than other approaches such as the work of first-order and second div-curl in terms of accuracy and computational time.

Dérian *et al*. [29], applied wavelet strategy for estimating complex, large turbulence motion from image sequences using higher order regularization in order to capture the real physical properties of turbulence. This method captured well the motion estimation in both the fine and coarsest scales with easy computational steps.

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Kadri-Harouna [35], presented a high order wavelet-based regularization for fluid type motion estimation. He presented discrete and continuous form regularizations which produced efficient results with low complexity of computations.

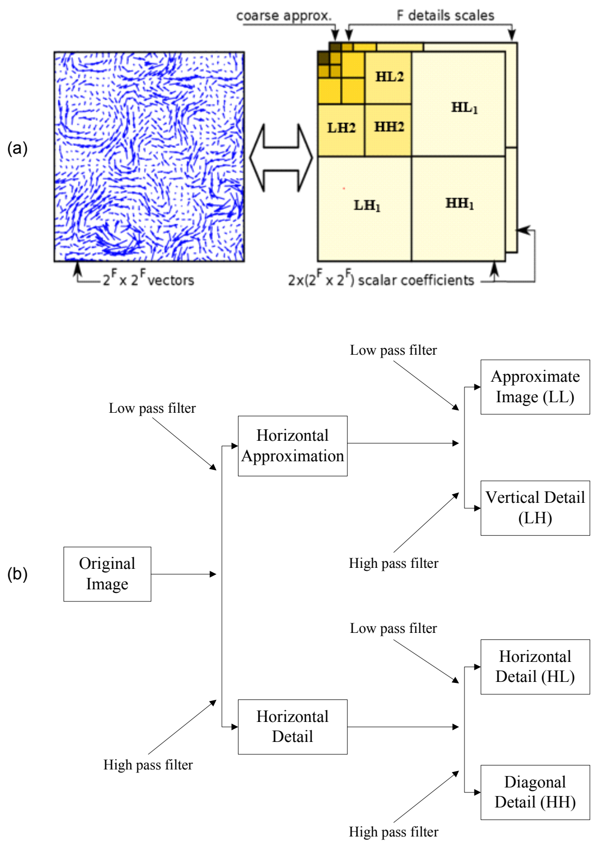

Figure-3. Principle of discrete wavelet decomposition: (a) Sample showing the three level wavelet decompositions with the motion vectors; (b) Basic decomposition process [36].

CONCLUDING REMARKS

In this current paper, differential models for fluid motion applications have been reviewed. The first-order regularization is often used for smoothing the flow field such as rigid body motion and is limited to only fluid flow. The use of higher order regularizations such as second order div-curl for fluid flow estimation has the advantage of being robust against image noise caused by chaotic motion of turbulence. It also considers the physical properties of the flow field. However, higher order means higher computational time and will be difficult to use for real time application such as cloud motion tracking. The physics-based strategies for optical flow computation which is based on using continuity and second order div-curl regularity are more robust for fluid flow estimation. However, these methods are still suffering from the complexity and instability in numerical solution and not appropriate for large scale image such as satellite image processing. Therefore, the accuracy and performance of these higher order regularizations can be improved using multi-scale schemes such as coarse-to-fine and wavelet based approaches.

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