METHOD OF DETERMINATION OF VIBRATING SCREENS' OSCILLATION'S AMPLITUDE IN A CHARACTERISTIC POINT FOR PLANE MOTION

Michail Dmitrievich Gerasimov¹, Oleg Vitalievich Mkrtchyan², Sergey Nikolaevich Glagolev¹, Dmitriy Michailovitch Gerasimov¹ and Sergey Sergeevich Latyishev¹
¹Belgorod State Technological University named after V.G. Shukhov Russia, Belgorod, Kostyukov, Russia
²Belgorod State Technological University named after V.G. Shukhov Russia, Branch in Novorossiysk City, Russia

ABSTRACT

Traditional designs of vibrating screens have a uniform field of amplitude over the entire surface of the sorting surface generally. Thus, the oscillation amplitude has the same value over the entire sieving surface, or a constant value in each point of the sieving surface. This phenomenon poses a number of problems of sorting of bulk materials, which, ultimately, do not allow to obtain maximum efficiency of the screening process. In recent years developers and manufacturers of industrial screening equipment strive to create the conditions under which it is possible to change the point of application of the driving force, the direction of its operation and value. This sorting process allows you to create amplitude fluctuations of different size on the plane of the sieve of change of state of the sorted material. The grading efficiency of such screens is expected to be much higher both in absolute and specific indicators.

Keywords: vibration, screen, sieving surface, vibration's amplitude, plane-parallel motion, the equation of motion.

INTRODUCTION

In the technical literature [1-6] is widely reflected the issues of study of the kinetics of bulk materials' sorting on traditional screens. Theoretical basis of calculation is the simulation of the sorting process taking into account various factors of that process' efficiency. Recently, there is a tendency for a new direction in the creation of vibrating screens [7-10].

Thus there is regulation of the location of the vibroexciter, the regulation of driving force's direction, receiving driving force with aimed direction and the asymmetry of the working and idle component of the driving force [11], Figure-1.

Main kinematic characteristics of the sieving surface's movement are important to determine the parameters of the sorting process on the vibrating screens.

Knowledge of the vibration amplitude, velocity and acceleration of chosen screen's points allows to simulate the behavior of the material sorting process in the context of a selected model. In this paper we study plane-parallel motion of the sieve.

Let us consider in a general way the movement of the sieve (Figure-2) $A_1A_2$ under the action of the applied forces: the forces of the springs $F_1$ и $F_2$, the force of gravity $mg$, the resistance force $F_r$, the vibroexciter's driving force $F_v$, the resistance torque $M_r$ and the vibroexciter's driving torque $M_v$. 
The solution uses Lagrange's equations of the second kind.

Symbols used in the article (Figure-2):

- \( m \), \( I_c \) – the mass and the moment of inertia of the sieve relative to the center of mass;
- \( C \) – the center of mass of the sieve (at the initial moment the position \( C_0 \) coincides with the beginning of the reference system);
- \( l_1, l_2 \) – shoulders of elastic forces (i.e., distance from the center of mass to the points of application of elastic forces);
- \( A_1, A_2 \) – the points of attachment of the springs to the sieve (the points of application of elastic forces \( F_1, F_2 \));
- \( M_1, M_2 \) – the points of attachment of the springs to the base (lines \( M_1A_1, M_2A_2 \) are the lines of action of elastic forces \( F_1, F_2 \)). It is assumed that initially the lines \( M_1A_{10}, M_2A_{20} \) are vertical;
- \( L_1, L_2 \) – the length of the springs in undeformed state;
- \( c_1, c_2 \) – the stiffness of the springs;
- \( \alpha \) – the angle of the screen to the horizon;
- \( \phi \) – the deflection angle of the sieve from the initial position;
- \( \gamma_1, \gamma_2 \) – the deflection angle of the direction of forces \( F_1, F_2 \) from the vertical;
- \( F_v \) – the vibroexciter's driving force;
- \( \beta \) – the deflection angle of \( F_v \) from the plane of the sieve;
- \( F_r \) – the resistance force caused by different reasons;
- \( \gamma \) – the deflection angle of \( F_r \) from the plane of the sieve;
- \( M_v \) – the vibroexciter's driving torque;
- \( M_r \) – the resistance torque.

The subscript zero indicates the corresponding value at the initial position.
Generalized coordinates of the considered motion is the coordinates of the center of mass of the sieve \( x_c \) and \( y_c \) and the angle of the sieve to the horizon \( \alpha \). This produces the following system of motion equations:

\[
\begin{align*}
\dot{X}_0 &= X_1 + X_2 + X_3, \\
\dot{Y}_0 &= Y_1 + Y_2 + Y_3, \\
\dot{\theta}_0 &= A_1 + A_2 + A_3,
\end{align*}
\]

where

\[
\begin{align*}
X_1 &= \frac{c_1 \omega}{\sqrt{(l_1 \cos \alpha + x_0 - y_0 + \theta_0)^2 + (l_1 \sin \alpha + y_0 - \theta_0)^2}} \left( (l_1 \cos \alpha + x_0 - x_{M1}) \times \right. \\
&\quad \left. (l_1 \sin \alpha + y_0 - y_{M1}) \right) \\
&\quad + c_1 \left[ 2(l_1 \cos \alpha + x_0) - x_{M1} \right], \\
X_2 &= F_x \cos(\beta - \alpha) - F_x \sin \alpha, \\
Y_1 &= \frac{c_1 \omega}{\sqrt{(l_1 \cos \alpha + x_0 - y_0 + \theta_0)^2 + (l_1 \sin \alpha + y_0 - \theta_0)^2}} \left( (l_1 \sin \alpha + y_0 - y_{M1}) \times \right. \\
&\quad \left. (l_1 \cos \alpha + x_0 - x_{M1}) \right) \\
&\quad - c_1 \left[ 2(l_1 \sin \alpha + y_0) - y_{M1} \right], \\
Y_2 &= F_y \sin(\beta - \alpha) - F_x \cos \alpha - mg, \\
A_1 &= \frac{c_2 \omega^2}{\sqrt{(l_1 \cos \alpha + x_0 - y_0 + \theta_0)^2 + (l_1 \sin \alpha + y_0 - \theta_0)^2}} \left[ -\sin \alpha (l_1 \cos \alpha + x_0 - x_{M1}) + \right. \\
&\quad + \cos \alpha (l_1 \sin \alpha + y_0 - y_{M1}) \right] \times \\
&\quad \left\{ (-l_1 \cos \alpha + x_0 - y_0 + \theta_0)^2 + \right. \\
&\quad \left. (l_1 \sin \alpha + y_0 - \theta_0)^2 \right\} \\
&\quad - 2c_1 \omega (l_1 \cos \alpha + x_0) + x_{M1} \sin \alpha + 2 \cos \alpha (-l_1 \sin \alpha + y_0) + y_{M1} \cos \alpha + \\
&\quad + 2c_1 \omega \sin \alpha (l_1 \cos \alpha + x_0) - 2c_1 \omega \cos \alpha (-l_1 \sin \alpha + y_0) - \\
&\quad - c_1 l_2 \omega \sin \alpha - c_1 l_2 \omega \cos \alpha.
\end{align*}
\]
In the derivation of (1) the angle \( \gamma \) (the deflection angle of \( F_r \) from the plane of the sieve) is assumed to be equal 90°. The system (1) has no analytical solution and can be solved only by numerical methods.

Simplify the form of equations. Consider the vector model of the mechanism and use several different notation (Figure-2). To derive the Lagrange's equations introduce other generalized coordinates — the coordinates of the center of mass of the sieve \( x_c \) and \( y_c \) and the deflection angle of the sieves from the initial position \( \phi \). These generalized coordinates are more convenient because they can be considered as small quantities \((x_c, y_c, \phi << 1)\) and the expansion of functions in power series in these coordinates make possibility to consider into account only the members containing first degree of these variables. Taking into account that \( \phi = \alpha - \alpha_0 \), we obtain from vector model of the mechanism (Figure-3):

\[
A_2 = \frac{c_2 l_2 \omega}{\sqrt{(-l_3 \cos \alpha + x_c)^2 + (l_3 \sin \alpha + y_c)^2}} \left(\sin \alpha (-l_2 \cos \alpha + x_c - x_{N2}) + \cos \alpha \left(l_2 \sin \alpha + y_c - y_{N2}\right)\right) \times
\]
\[
\times \left\{ \frac{-l_2 \cos \alpha + x_c}{(-l_3 \cos \alpha + x_c - x_{N2})^2 + (l_3 \sin \alpha + y_c - y_{N2})^2} \left[l_2 \cos \alpha + c_2 l_2 \omega \sin \alpha (-l_2 \cos \alpha + x_c) + 2c_2 l_2 \omega \cos \alpha (l_2 \sin \alpha + y_c) + c_2 l_2 \omega y_{N2} \cos \alpha \right] + \right.
\]
\[
+ 2 \sin \alpha (-l_2 \cos \alpha + x_c - x_{N2} \sin \alpha + 2 \cos \alpha (l_2 \sin \alpha + y_c - y_{N2} \cos \alpha) - 2c_2 l_2 \omega \sin \alpha (-l_2 \cos \alpha + x_c) - 2c_2 l_2 \omega \cos \alpha (l_2 \sin \alpha + y_c) + c_2 l_2 \omega y_{N2} \cos \alpha \right)
\]

\[
A_2 = F_\gamma \omega \sin (\beta - \alpha) - F_{\gamma} \omega \cos \alpha - F_\gamma \omega \cos (\beta - \alpha) + F_\gamma \omega \sin \alpha - (M_1 + M_2) \omega.
\]

\[
\omega = \frac{dx}{dt}.
\]

Figure-3. The vector model of the mechanism.
In this case Lagrange's equations will have the form:

\[
\begin{align*}
\sum_{i} m_i \ddot{y}_i &= k_1 \varphi + k_2 \varphi, \\
\sum_{i} m_i \ddot{y}_i &= k_3 \varphi + k_4 \varphi + k_3 \varphi,
\end{align*}
\]

Where

\[
\begin{align*}
k_1 &= F_y \sin (\beta - \alpha_0) - F_z \sin (\gamma - \alpha_0), \\
k_2 &= F_y \cos (\beta - \alpha_0) - F_z \cos (\gamma - \alpha_2), \\
k_3 &= -c_1 - c_2, \\
k_4 &= (c_1 + c_2) \cos \alpha_0 - F_y \sin (\beta - \alpha_0) + F_z \sin (\gamma - \alpha_2), \\
k_5 &= (c_1 + c_2) \cos \alpha_0 + F_y \sin (\beta - \alpha_0) + F_z \sin (\gamma - \alpha_2) - mg, \\
k_6 &= -c_1 l_1 + c_2 l_2 \cos \alpha_0, \\
k_7 &= -c_1 l_1 + c_2 l_2 \cos \alpha_0, \\
k_8 &= c_1 l_1 \cos \alpha_2 + c_2 l_2 \cos \alpha_2 - c_1 l_1 \cos \alpha_2 + c_2 l_2 \cos \alpha_2.
\end{align*}
\]

If we divide the first and second equation of the system (2) by weight, and the third at the moment of inertia of the sieve, you get a system of non-homogeneous differential equations. In matrix form the system looks like this:

\[
\dot{\mathbf{x}} = A \mathbf{x} + \mathbf{b},
\]

where \( \mathbf{x} = [x_1, y_1, \varphi]^T \), \( A \) – the coefficient matrix of the system:

\[
A = \begin{pmatrix}
0 & 0 & k_1 & \cdots \\
0 & 0 & k_2 & \cdots \\
0 & 0 & k_3 & \cdots \\
0 & 0 & k_4 & \cdots \\
0 & 0 & k_5 & \cdots \\
0 & 0 & k_6 & \cdots \\
0 & 0 & k_7 & \cdots \\
0 & 0 & k_8 & \cdots \\
\end{pmatrix}
\]

\( \mathbf{b} \) – column matrix of free terms:

\[
\mathbf{b} = \begin{pmatrix}
k_1 & k_2 & k_3 & \cdots \\
k_4 & k_5 & k_6 & \cdots \\
k_7 & k_8 & \cdots & \cdots \\
\end{pmatrix}
\]

Lower the order of the resulting system of ODE of second order for the numerical solution. Let \( x_1 = x_1, x_2 = y_1, x_3 = \varphi, x_4 = \dot{x}_1, x_5 = \dot{y}_1, x_6 = \dot{\varphi} \). Now the system (3) can be written in the form:

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{pmatrix} =
\begin{pmatrix}
k_1 & k_2 & k_3 & k_4 & k_5 & k_6 \\
k_4 & k_5 & k_6 & k_7 & k_8 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{pmatrix} +
\begin{pmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4 \\
k_5 \\
k_6
\end{pmatrix}
\]

From the equations the balance of sieve in initial moment follows static condition of communicative between initial deformations:

The system (5) is convenient for numerical solution.

From the sieve's equilibrium equations in the initial moment of time follows the static link condition between the springs' initial deformations:

\[
\begin{pmatrix}
\lambda_{10} \\
\lambda_{20} \\
\lambda_{10} \\
\lambda_{20} \\
\end{pmatrix} =
\begin{pmatrix}
l_1 \\
l_2 \\
l_1 \\
l_2
\end{pmatrix}
\]

For numerical solution we consider a screen size like GIL-52:

\[
\begin{align*}
m &= 2800 \text{ kg} - \text{mass of screen}, \\
I_{cz} &= \frac{m}{3} (a^2 + b^2) \approx 4000 \text{ kg} \cdot \text{m}^2 \quad \text{(the moment of inertia is approximated provided that} \ a = 2 \text{ and} \ b = 0.1 \text{ m, where} \ a \text{ and} \ b \text{ – the geometrical dimensions of the screen in plane Oxy, modulated as rectangle);} \\
l_1 &= l_2 = 2 \text{ m}; \quad L_1 = L_2 = 16 \text{ cm}; \quad c_1 = c_2 = 156 \text{ kN} \cdot \text{m}^{-1}; \quad \lambda_{10} = \lambda_{20} = 0.3 \text{ cm}; \quad \lambda_{10} = \lambda_{20} = 1.23 \text{ cm} \quad \text{- the extreme value of the elastic deformation of the springs;} \\
\alpha_0 &= -10\ldots-20^\circ; \quad F_y = \Sigma m_o \dot{R} \cos (\alpha_0 + \omega t); \quad \beta = -5\ldots-175^\circ; \quad F_z = m_{\text{load}} e^{-t} = 14 \text{ kN}; \quad \gamma = -70\ldots-110^\circ; \quad M_v = F_v \cdot d_1; \quad M_r = F_r \cdot d_2; \\
\text{The acceleration of each vibroexiter is calculated as the sum of the following summands:} \\
\alpha_i &= \left(\frac{2\pi t}{30}\right)^2 R_i \cos \left(\varphi_0 - \frac{2\pi t}{30} \varphi_0\right)
\end{align*}
\]
Accordingly, the resulting force of vibroexciters will be equal to

\[ F_y = \sum m_i a_i \]

In this case, the rotation frequency associated with the angular velocity

\[ \omega_i = \frac{n_i \pi}{30} \]

Choose \( n_1 = 500 \text{ rpm} \) и \( n_2 = 1000 \text{ rpm} \), \( R_1 = 0.06 \text{ m} \), \( R_2 = 0.06 \text{ m} \), \( m_1 = 20 \text{ kg} \), \( m_2 = 2 \text{ kg} \), where \( m_1 \), \( m_2 \) – the weight of the first and second unbalances, \( n_1 \), \( n_2 \) – the rotation frequency of the first and second unbalance, respectively.

Initial conditions – the position and velocity of the center of mass of the sieve at the initial moment of time we will take the zero, \( IC = [0 0 0 0 0] \).

For given numerical values of the parameters the solution must be a solution of the oscillating system under the action of the driving force. For the period of time 0.15 sec the numerical solution of system (5) with the use of specialized software and Runge-Kutta methods gave the result shown in Figure-4. It is seen that the solution hasn't the expected oscillatory nature, due to the simplification of the system. The oscillatory nature of the solution gets on a different time range 0...200 sec (Figure-5). Note that in this case the oscillatory nature has only a vertical component of the velocity of the center of mass screening.

\[ \text{Figure-4. The solution of the system for time range } t \approx 0...0.15 \text{ sec}. \]

To come to a physically meaningful result, we can assume that the graphs in Figure-5 are only valid for the first half of the time range. And in the second half of the period are changing the direction of its movement so as to come to the end of the period to values of variables at the beginning of this period. Thus one can estimate in first approximation the amplitude of the movements of the center of mass of the sieve \( x_{c,\max} \approx 18 \text{ mm}, y_{c,\max} \approx 10 \text{ mm}, \varphi_{\max} \approx 0.009 \text{ rad} \) in its plane-parallel motion.

Для определения кинематики движения грохотимого материала важно также оценить амплитуды движений крайних точек сита: To determine the kinematics of sieving material it is also important to evaluate the amplitude of movements of the endpoints of the sieve:

\[ x_{\text{extr, max}} = x_{c,\max} + l_1 \varphi_{\max} = 18 + 2000 \cdot 0.009 \approx 36 \text{ mm}, \ y_{\text{extr, max}} = 10 + 2000 \cdot 0.009 \approx 28 \text{ mm}. \]

In this case, both end points have the same amplitude.

\[ \text{Figure-5. The solution of the system for time range } t \approx 0...200 \text{ sec}. \]

\[ \text{CONCLUSIONS} \]

Modern vibrating screens must have the tendency of getting in one design several distinctive features:

- to have a exciter with aimed direction of the force;
- to have the ability to change the driving force's direction in the range 0...360°;
- to be able to move the exciter along the sieve;
- to be able to change the oscillation's amplitude along the sieve;
- to obtain asymmetric driving force with the coefficient of asymmetry within the prescribed limits.

The paper presents the basis of determining the vibration amplitude of the vibrating screen in plane-parallel motion of sieve and subject to the above requirements.
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