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SPIRALING MOTION OF AN UNDERWATER GLIDER: DYNAMIC MODELING

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ABSTRACT

An underwater glider is a class of autonomous underwater vehicles. While these gliders typically move in a saw-tooth pattern, a spiral motion, which may more effective for specific applications, is considered here. The spiral motion of glider may extend its possible applications such as delivery or recovery equipment for subsea installation. In this paper, a spiral glide path for the glider is considered, and the corresponding dynamic model based on Lagrangian principle and analytical expressions determined. The steady-state spiraling equations were derived and solved recursively using the fsolve algorithm. The results compare well with simulation results based on Newton's method. The spiraling motion is highly maneuverable, with less than 1 m turning radius.

Keywords: underwater glider, dynamic model, spiraling motion.

INTRODUCTION

In 1989, Henry Stommel [1] published a revolutionary article about buoyancy driven floats for oceanography. Since then, various underwater gliders such as ALBAC [2], Slocum [3], Spray [4], Seaglider [5]were developed for marine applications at different depths of ocean.

An underwater glider is a special type of autonomous underwater vehicles (AUV) that is primarily used in oceanographic sensing and data collection. In these applications, they are attractive because of their low cost, autonomy and capability for long-range, extended duration deployments. These gliders are buoyancy driven low energy consumption autonomous underwater vehicles with fixed wings and rudder, a cylindrical hull and internal moving masses [6, 7].

The operating principle of an underwater glider is that at deployment it is negative buoyant and therefore tends to dive, during which its wings convert downward motion into the horizontal plane, thus producing a forward force. Once a predetermined depth is reached, the vehicle changes its buoyancy to become neutrally buoyant [8, 9].
Thor I. Fossen [10, 11] developed a nonlinear 6-DOF dynamic model for marine vehicles based on Euler–LaGrange system including control motions of internal components. Leonard and Graver [12, 13] derived a generalize dynamic model of underwater glider based on first principles including nonlinear coupling between internal moving mass and glider. Zhang [14] derived the dynamic model of gliding robotic fish based on Newton principles by simplifying its dynamics motion at sagittal plan. Zhang *et al* [15-17] were proposed the spiral motion of gliding robotic fish by deflecting its tail. Leonard and Bhatta [18] proposed the numerical simulation of spiral motion by changing the position of internal moving masses. It's required more complicated control systems to manage the position of internal moving masses during spiral motion. However, The spiral motion approach that used by Zhang [15, 16] is more appropriate to achieved high maneuverability. This work is an extension of Zhang's, with emphasis on the dynamics of gliders in a spiral glide path.

This paper is organized as follows: Section II outlines the derivation of a complete non-linear dynamic model of an underwater glider based on Euler–LaGrange principle with an internal moving mass in a unidirectional movement to control the pitch angle of glider. In Section III the simplified nonlinear dynamic model for a steady state 6-DOF spiraling motion of glider is derived and solved using the fsolve recursive algorithm. In Section IV, the results are validated with previously published experimental and simulation results by Zhang [15, 16].

DYNAMIC MODELING OF UNDERWATER GLIDER

A simplified point mass dynamic model of a glider with internal moving mass for pitch control is used to describe the gliders 6-DOF motion. In this work, an underwater glider, including all internal moving mass moments and external forces, is considered as a rigid body. The position of internal moving mass will control the glide angle and speed. The mass distribution an underwater glider is shown in Figure-1.

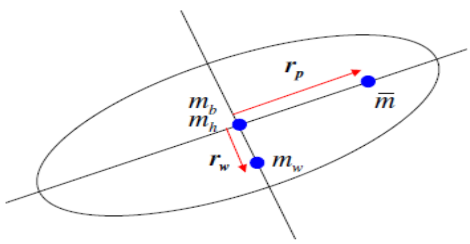

Figure-1. Glider mass distribution [19].

The total glider mass or body mass can be expressed as $m_v = m_h + m_w + m_b + \overline{m}$. Where m_h represents a uniform glider hull mass, m_w point mass with displacement r_w to the fixed center of gravity and buoyancy, $\overline{m}$ the movable mass with vector position r_p to control the pitch angle during gliding and m_b the variable ballast mass with respect to geometry center (GC). The mass 'm' is the mass of displaced fluid $m_0 = m_v - m$. The glider is neutrally buoyant if the m_0 is positive (float) and vice versa.

KINEMATICS MODEL

Two frames of reference, one body frame and other internal frame are required to define the motion of rigid body as shown in Figure-2. Let e_1, e_2, e_3 denote body frame related to i, j, k inertial frame of references respectively, as shown in Figure-2. e_1 and e_2 lies in the horizontal plane and is perpendicular to the gravity along the wings of the glider respectively. The k axis is positive downwards and lies in the direction of gravity.

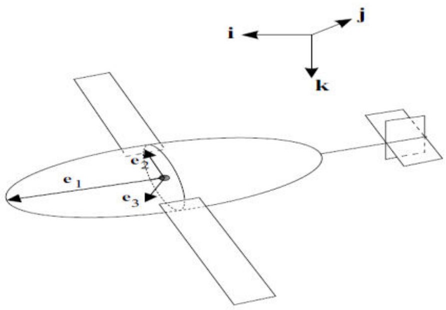

Figure-2. Body frame axis of [12]

In order to model the kinematics of the glider mathematically, the center of gravity (CG), center of buoyancy (CB), rotational matrix R, generalized position and velocity must be identified. The six degrees of freedom kinematic equations for the glider are described in [14] Let the position of the glider from the inertia frame to the origin of body frame be $b = \begin{bmatrix} x & y & z \end{bmatrix}^T$ and orientation $R = \begin{bmatrix} \varphi & \theta & \psi \end{bmatrix}^T$. The gliders longitudinal velocity is $V = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T$ and relative angular velocity $\omega = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$. Thus, the kinematic equations are.

$$\dot{R} = R\omega_b \tag{1}$$

$$\dot{b} = Rv_b \tag{2}$$

$$R = \begin{bmatrix} c\theta * c\varphi & s\varphi * s\theta * c\psi - c\varphi * s\psi & c\varphi * s\theta * c\psi + s\varphi * s\psi \\ c\theta * c\psi & c\varphi * c\psi + s\varphi * s\theta * s\psi & -s\varphi * c\psi + c\varphi * s\theta * s\psi \\ -s\theta & s\varphi * c\theta & c\varphi * c\theta \end{bmatrix}$$

where 's' is sine and 'c' is cosine. $[\theta, \varphi, \psi]$ represents the pitch angle, roll angle and yaw angle respectively.

DYNAMIC MODEL

The dynamic model is derived based on the Lagrangian principle instead of the Newton-Euler formulation adopted by Zhang [14]. The Lagrangian formulation is based on the energy of a dynamic system [20].

The Lagrange's principle in general coordinates is

$$\frac{d}{dt}\left(\frac{dL}{dv_i}\right) - \frac{dL}{dx_i} = Q \tag{3}$$

$$L = K.E - P.E \tag{4}$$

Where, L=Kinetic Energy (K.E) – Potential Energy (P.E) and Q is external forces.

The general kinetic energy expression of dynamic system is

$$K.E = \frac{1}{2} v_i^T M_i v_i + \frac{1}{2} \omega_i^T I_i \omega_i \tag{5}$$

The first term in Equation 5 is the kinetic energy due to translation velocity v_i and the second term is the kinetic energy due to angular velocity ω_i of the dynamic system.

Underwater gliders work under the influence of gravity force and buoyancy. The gravity force acts in a downward direction along the positive the z axis of the glider. The kinetic energy of underwater glider is

$$K.E = \frac{1}{2} v_i^T \left[M_{rb} + M_f\right] v_i + \frac{1}{2} \omega_i^T \left[I_{rb} + I_f\right] m_i \omega_i \tag{6}$$

Where, $M_{rb} + M_f = M$ is rigid body mass and added mass respectively, $I_{rb} + I_f = I$ rigid body inertia mass and added inertia respectively. The potential energy of dynamic system due to gravity force can be expressed as

$$P.E = -m_i g Z_i \tag{7}$$

where the g is the gravity force and Z_i is the position of center of mass of dynamic system in inertial frame of reference. From equation 7 the potential energy of system in body fixed coordinates is

$$P.E_b = -m_b g\left(R^T Z_i\right) \tag{8}$$

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where ${}_R{}^{T}Z_i$ is represented as [20]. Using the values of K.E and P.E in Equation 4

$$L = \frac{1}{2} v_i^T M_i v_i + \frac{1}{2} \omega_i^T I_i \omega_i - m_b g \left(R^T Z_i\right) \tag{9}$$

Differentiating Equation. (9) with respect to translational velocity and angular velocity and then differentiating with respect to time.

$$\frac{d}{dt}\left(\frac{dL}{dv}\right) = \frac{d}{dt}\left(\frac{d}{dv}\left(\frac{1}{2} v_i^T M_i v_i + \frac{1}{2} \omega_i^T I_i \omega_i - m_b g \left(R^T Z_i\right)\right)\right) \tag{10}$$

After simplification, the translation velocity of the glider is

$$M\dot{v}_b + \omega_b \times M v_b + m_b g R^T k = F \tag{11}$$

Here, F is total external force acting on the dynamic system.

$$F = m_v g R^T k - \overline{m} g R^T k + F_{ext} - u \tag{12}$$

Here, k is the unit vector along the gravity in Z-axis direction, F_{ext} is the external hydrodynamic forces acting on the glider body, expressed in the body-fixed frame, and 'u' is the total force exerted on the movable mass $\overline{m}$ by the glider structure, expressed in the body fixed frame. m_v is glider mass acting along the gravity of the glider.

The control input force 'u' for moveable mass is

$$u = -\overline{m} g R^T k + \overline{m} \dot{v}_i - u_0 \tag{13}$$

$$\dot{v}_b = \frac{1}{M + \overline{m}I}\left(\left(M + \overline{m}I\right) v_b \times \omega_b + m_0 g R^T k + F_{ext} - u_0\right) \tag{14}$$

Here $M + \overline{m}I = M_{rb} + M_f + \overline{m}I$

Similarly, the equation of angular moments is

$$J\dot{\omega}_i = T$$

(15)

$$J\dot{\omega}_b + \omega_b \times J\omega_b = T$$

The external moment can be described including movable mass and external force as

$$T = Mv \times v + T_{ext} + m_w g r_w \times (R^T k) - r_p \times u \tag{16}$$

The buoyancy mass and internal moving mass control force u is

$$u = -\overline{m} g \times R^T k + u_0$$

$$\dot{\omega} = \frac{1}{J}(J\omega \times \omega + Mv \times v + T_{ext} + m_w g r_w \times (R^T k) + \overline{m} g r_p \times (R^T k) - r_p \times u \tag{17}$$

$u_0 = \dot{m}_0$ is buoyancy control. u_0 will be simplified as the voltage applied to the pump.

HYDRODYNAMIC MODEL

In order to study the hydrodynamic behaviors, all the velocity fixed coordinates are first transferred to the body frame of reference. For this purpose, the rotational matrix R_{bv} is used.

$$R_{bv} = \begin{bmatrix} C\alpha S\beta & -C\alpha S\beta & -S\alpha \\ S\beta & C\beta & 0 \\ S\alpha C\beta & -S\alpha S\beta & C\alpha \end{bmatrix} \tag{18}$$

where 's' is sine and 'c' is cosine.

Where α is the angle of attack $\alpha = \tan^{-1}(v_z/v_x)$ and β is the sideslip angle $\beta = \sin^{-1}(v_y/v)$. The hydrodynamic forces (drag, lift and side force) and moment (roll moment M_x, pitch moment M_y and yaw moment M_z) are transferred from velocity frame to body frame of reference. Hence,

$$F_{ext} = R_{bv}\left[-D \quad SF \quad -L\right]^T \tag{19}$$

$$T_{ext} = R_{bv}\left[M_x \quad M_y \quad M_z\right]^T \tag{20}$$

The hydrodynamic forces and moments are generally dependent on the angle of attack, sideslip angle and velocity.

$$D = \left(K_{D0} + K_D^{\alpha}\alpha^2 + K_D^{\delta}\delta^2\right)V^2 \tag{21}$$

$$SF = \left(K_{sf}^{\beta}\beta + K_{sf}^{\delta}\delta\right)V^2 \tag{22}$$

$$D = \left(K_{L0} + K_L^{\alpha}\alpha\right)V^2 \tag{23}$$

$$M_x = \left(K_{mR}^{\beta}\beta + K_{q1}\omega_x\right)V^2 \tag{24}$$

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$$M_y = \left(K_{M0} + K_M \alpha + K_{q2}\omega_y\right)V^2 \tag{25}$$

$$M_z = \left(K^{\beta}_{my}\beta + K_{q3}\omega_z + K^{\delta}_{my}\delta\right)V^2 \tag{26}$$

Here δ is the rudder deflection, K_{q1}, K_{q2}, K_{q3} are the rotational damping coefficients. The 'K' coefficients are based on the CFD simulation [6, 17, 21].

STEADY-STATE SPIRALING MOTION

In this study, the steady spiraling motion of underwater glider is investigated numerically through an iterative method. A numerical algorithm based on the fsolve recursive algorithm was used to solve the 6-DOF dynamic equations of glider. The spiraling motion of glider will be manipulated by three control inputs ($\delta, \bar{m}, m_0$) deflection of rudder, position of internal mass to control the pitch angle and net buoyancy rate respectively. The hydrodynamic angles (α, β) are considerably effected by the hydrodynamic forces and moments during the motion of glider under the influenced of fixed control inputs. The angular velocity along z-axis in body fixed coordinates is

$$\omega_b R^T K = \omega_z R^T K = \begin{bmatrix} -\omega_z \sin(\theta) \\ \omega_z \sin(\varphi)\cos(\theta) \\ \omega_z \cos(\varphi)\cos(\theta) \end{bmatrix} \tag{27}$$

The translational velocity in body fixed coordinates is

$$v_b = R_{bv}\begin{bmatrix} V & 0 & 0 \end{bmatrix}^T \tag{28}$$

$$v_b = \begin{bmatrix} V\cos(\alpha)\cos(\beta) \\ V\sin(\beta) \\ V\sin(\alpha)\cos(\beta) \end{bmatrix} \tag{29}$$

The steady state spiraling motion dynamic equations were obtained by setting the derivatives and all control forces in Equation. 14 and Equation. 17 to zero.

$$0 = \left(Mv_b \times \omega_b + m_0 g R^T k + F_{ext}\right) \tag{30}$$

$$0 = \begin{pmatrix} J\omega_b \times \omega_b + Mv_b \times v_b + T_{ext} + m_w g r_w \times R^T k \\ + \bar{m} g r_p \times R^T k \end{pmatrix}$$

The 6-DOF dynamic equations at steady state conditions are obtained by substituting all the values in the dynamic model.

$$0 = m_y v\sin(\beta)\cos(\varphi)\cos(\theta)\omega_z - m_z v\sin(\alpha)\cos(\beta)\sin(\varphi)\cos(\theta)\omega_z - m_0 g\sin(\theta) - D\cos(\alpha)\cos(\beta) - SF\cos(\alpha)\sin(\beta) + L\sin(\alpha) \quad (31)$$

$$0 = -m_z v\sin(\alpha)\cos(\beta)\sin(\theta)\omega_z - m_x v\cos(\alpha)\cos(\beta)\cos(\varphi)\cos(\theta)\omega_z - m_0 g\sin(\varphi)\cos(\theta) - D\sin(\beta) - SF\cos(\beta) \quad (32)$$

$$0 = m_x v\cos(\alpha)\cos(\beta)\sin(\varphi)\cos(\theta)\omega_z - m_y v\sin(\beta)\sin(\theta)\omega_z + m_0 g\cos(\theta)\cos(\varphi) - D\sin(\alpha)\cos(\beta) - SF\sin(\alpha)\sin(\beta) - L\cos(\alpha) \quad (33)$$

$$0 = (J_2 - J_3)\sin(\varphi)\cos(\theta)^2\cos(\varphi)\omega_z^2 + (m_2 - m_3)\sin(\beta)\sin(\alpha)\cos(\beta)V^2 - m_w g r_w \sin(\varphi)\cos(\theta) + M_x\cos(\alpha)\cos(\beta) - M_y\cos(\alpha)\sin(\beta) - M_z\sin(\alpha) \quad (34)$$

$$0 = (J_x - J_z)\cos(\varphi)\sin(\theta)\cos(\theta)\omega_z^2 + (m_z - m_x)\cos(\alpha)\sin(\alpha)\cos(\beta)^2 V^2 - m_w g r_w \sin(\theta) - m_0 g r_p \cos(\varphi)\cos(\theta) + M_x\sin(\beta) + M_y\cos(\beta) \quad (35)$$

$$0 = (J_y - J_x)\sin(\varphi)\sin(\theta)\cos(\theta)\omega_z^2 + (m_x - m_y)\cos(\beta)\cos(\alpha)\sin(\beta)V^2 + m_0 g r_p \sin(\varphi)\cos(\theta) + M_x\sin(\alpha)\cos(\beta) - M_y\sin(\alpha)\sin(\beta) + M_z\cos(\alpha) \quad (36)$$

The six states: pitch angle, heading angle, angular velocity along z-direction, glider velocity, angle of attack and side angle $(\theta, \varphi, \omega_z, V, \alpha, \beta)$ was solved numerically to define the spiraling motion of underwater glider for three inputs forces (internal moving mass position, net buoyancy and rudder deflection). The dynamic of spiraling motions is highly nonlinear due to trigonometry functions involved in these equations. We write these equations in steady state function to obtain the solution numerically. In this study, the spiraling motions of a prototype gliding robotic fish-like underwater glider [17] with parameters in Table 1 are considered.

The spiraling motion of glider involves two parameters. The rotational motion around the z-direction with radius R and glide speed in vertical plane $V_{vertical}$

$$V_{vertical} = V\sin(\theta - \alpha) \tag{37}$$

$$R = V\cos(\theta - \alpha)/\omega_z \tag{38}$$

RESULTS AND DISCUSSIONS

Initial values $\begin{pmatrix} \theta = -10, \varphi = -10, \omega_z = 0.1, \\ V = 3, \alpha = 0, \beta = 0 \end{pmatrix}$ were chosen for the fsolve iteration algorithms. The numerical

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solution of the given states and initial conditions are shown in Table-1.

The glide velocity increases with increasing net buoyancy while at fixed buoyancy and position of moveable mass with change of ruder deflection is decreased the turning radius of spiral motions. The numerical simulation results are consistent with the Newton iteration method and experimental results by Feitian [15-17] as shown in Figure-3 and Figure-4.

The Newton method requires the Jacobian/derivatives for the evaluation of the nonlinear equations, which is time consuming and difficult for 6-DOF nonlinear equations [22], although a Jacobian may be desirable for a poorly scaled problem. However, Newton's method cannot be used if the Jacobian is a singular matrix. On the other hand, the Matlab's fsolve function using Levenberg-Marquardt Algorithm is computationally efficient and does not require a Jacobian to solve nonlinear equations.

Table-1. Glider parameters

Parameters	values	Parameters	Values
m_x	3.88kg	m_y	9.9kg
m_z	5.32kg	$\overline{m}$	0.8kg
K_{D0}	0.45	K_D^{α}	17.59 rad^{-1}
K_{SF}^{β}	-2 rad^{-1}	K_{SF}^{δ}	1.5 rad^{-1}
K_{L0}	0.075	K_L^{α}	19.58 rad^{-1}
I_1	0.8 $kg.m^{-2}$	I_2	0.05 $kg.m^{-2}$
I_3	0.8 $kg.m^{-2}$	K_{M0}	0.0076m
M_{Mr}^{β}	-0.3 m/rad	M_{MP}^{α}	0.57 m/rad
M_{My}^{β}	5 m/rad	M_{My}^{δ}	-0.2 m/rad
K_{q1}	-0.1 m-s/rad	K_{q2}	-0.5 m-s/rad
K_{q3}	-0.1 m-s/rad	S	0.012 m^2

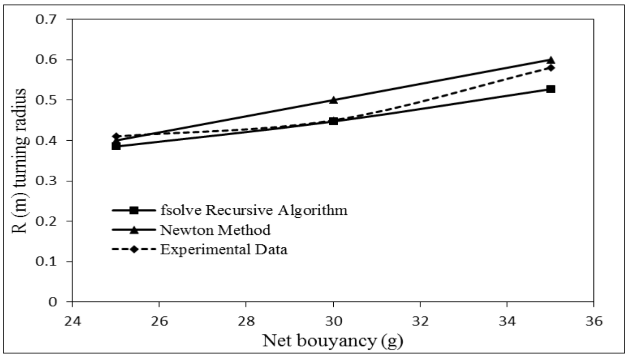

Figure-3. Turning radius of spiraling motion verse net buoyancy at fixed position of internal pitch control mass.

The relationship between the spiral turning radius 'R' and the velocity on vertical plan '$V_{vertical}$' is inversely proportional, as the turning radius and velocity decreases with an increase of tail deflection angle. When the turning radius is increased, the side slips angle 'β' also increases, which influence the velocity of the glider. The sideslip angle, β is an angle between the velocity vector and longitudinal axis of the glider. On the other hand, the steady spiraling motion of glider is also function of net buoyancy. As the buoyancy mass increases, the vertical velocity increases, but the radius of the spiral motion decreases. The position of internal pitch control mass increases or decreases the sideslip angle which influences the glide speed and turning radius.

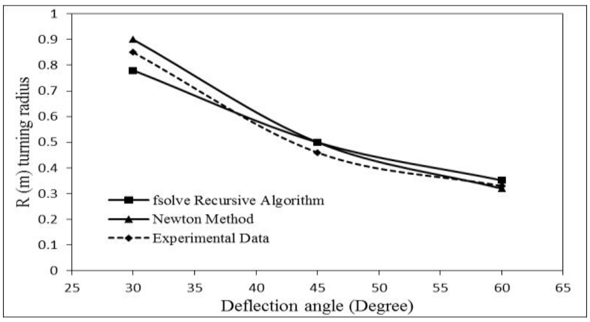

Figure-4. Turning radius of a spiraling motion verse tail deflection at fixed position of internal pitch control mass (5cm) and a net buoyancy of 30 gram.

The experimental results and simulation results differ 0-8%. The experimental results may be subjected to measurements error. These errors may also be due to the glider not fully achieving steady state conditions [17, 23] e.g. in a transient phase. Experimental data acquired through the camera is susceptible to errors.

CONCLUSIONS

In this study, a mathematical model of a glider based on Euler-Lagrangian method is derived. This model is a simplification of Graver and Leonard [12, 13] nonlinear dynamic model and considers all external forces. The steady state spiraling motion equations were derived and numerically solved based on the fsolve recursive algorithm. The results are in close agreement with those based on Newton's method as well as experimental results. The derived model is an alternative to that based on Newton's method, as it can be solved without computing the Jacobian.

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