



OPTIMALLY TUNED ACTIVE DAMPED DYNMIC VIBRATION ABSOBER

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ABSTRACT

Vibrations can be initiated in a number of ways, and if resonances are involved, can result in very significant effects. Vibration can cause reliability problems on equipment and fatigue failures. A dynamic vibration absorber, or vibration neutralizer, is a tuned spring-mass system which reduces or eliminates the vibration of a harmonically excited system. When dynamic vibration absorber is tuned to the frequency of forced vibration of a structure, the device can completely eliminate the vibration by creating an anti-resonance at the frequency of vibration. However, any change in the frequency of vibration from the tuned frequency renders the device largely ineffective. The major drawback of the passive dynamic vibration absorber is that it is suitable only for a narrow band width of operation and therefore, it is useful in eliminating single frequency resonant vibrations. An active vibration absorber comprising a spring-mass system attached to a rigid base and an actuator utilizing the feedback taken from the absorber mass is proposed. The actuator is controlled by the feedback taken from the absorber mass itself. Optimization design of an active vibration absorber for the minimization of the resonant vibration amplitude of a single degree-of-freedom vibrating structure is derived. The effects of optimum tuning parameters on the vibration reduction of the primary structure are revealed based on the analytical model. Design parameters an active vibration absorber are optimized for the minimization of the resonant vibration amplitude of the vibrating structure.

Keywords: dynamic vibration absorber, active control and vibrations.

INTRODUCTION

Machines experience excessive vibration under the action of force whose excitation frequency nearly coincides with a natural frequency. Under such conditions, a dynamic vibration absorber can be used to reduce the vibrations of the machines. The dynamic vibration absorber is designed such that the natural frequencies of the machines with absorbers are away from the excitation frequency. Most vibration absorbers commonly used in machinery that operate at constant speed are undamped [1]. The amplitude of vibration of main mass is nonzero in a damped dynamic vibration absorber. Damping is added when the frequency band is narrow for the absorber to be effective for operation. Liu and Liu [2] presented the analysis of optimum damped dynamic vibration absorber. Wong and Cheung [3] derived optimum tuning condition of dynamic absorber based on the fixed-points theory. Cheung *et al.* [4] presented optimization design of a hybrid vibration absorber for the minimization of the resonant vibration amplitudes. Chatterjee [5-6] proposed an active, standalone vibration absorber utilizing the PD control without requiring any feedback from the main structure. As the absorber runs on the PD control of the absorber mass, this standalone feature makes it user-friendly. The standalone active absorber with the optimal parameter PD control has shown to have performed better than a passive absorber [5].

This study investigates the proportional control for undamped vibration absorber and derivative control for damped vibration absorber. The influence of active control on the resonant frequencies in the case of undamped vibration absorber and on the response of main mass in the case of damped vibration absorber are presented.

ANALYSIS

Active Undamped Dynamic Vibration Absorber

The schematic of two-degree-of freedom active undamped vibration absorber is shown in Figure-1, wherein an auxiliary mass m_2 is attached to a machine mass m_1 through a spring of stiffness k_2 .

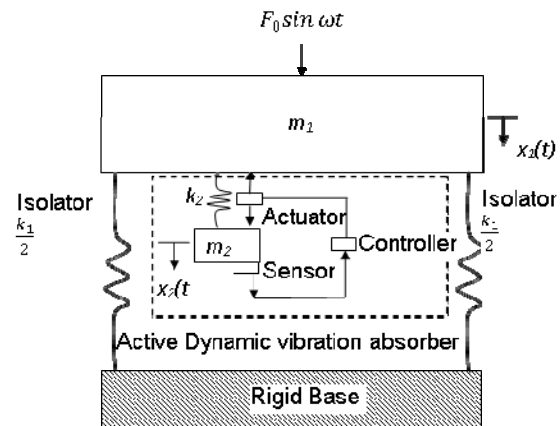


Figure-1. Active undamped vibration absorber.

Using an active control system with proportional gain to control the vibration of mass m_2 , the equations of motion are expressed as

$$m_1 \ddot{x}_1 + k_1 x_1 - k_2 (x_2 - x_1) = F_0 \sin \omega t - F_c \quad (1)$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) = F_c \quad (2)$$



Where the actuator will be designed to exert a control force F_c such that $F_c = -k_p x_2$.

Using harmonic solution $x_j = X_j \sin \omega t$, $j=1,2$, the steady-state amplitude of masses m_1 and m_2 are obtained as

$$X_1 = \frac{(k_2 + k_p - m_2 \omega^2) F_o}{(k_1 + k_2 - m_1 \omega^2)(k_2 + k_p - m_2 \omega^2) - k_2(k_2 + k_p)} \quad (3)$$

$$X_2 = \frac{k_2 F_o}{(k_1 + k_2 - m_1 \omega^2)(k_2 + k_p - m_2 \omega^2) - k_2(k_2 + k_p)} \quad (4)$$

When the amplitude of machine mass (m_1) is zero, Equation (3) results in

$$\omega^2 = \frac{(k_2 + k_p)}{m_2} \quad (5)$$

Using the undamped dynamic absorber, the amplitude of machine operating at its original resonant frequency will be zero, the natural frequencies of main mass and absorber mass coincides with operating frequency as

$$\omega^2 = \omega_1^2 = \frac{k_1}{m_1} = \omega_2^2 = \frac{(k_2 + k_p)}{m_2} \quad (6)$$

The dynamic vibration absorber introduces two resonant frequencies Ω_1, Ω_2 , while eliminating the vibration at operating frequency ω , and the operating frequency ω , must be kept away from the two resonant frequencies Ω_1, Ω_2 . The values of two resonant frequencies Ω_1, Ω_2 can be found by setting the denominator of Equation (4) to zero. The roots obtained by setting the denominator of Equation (4) to zero, expressed in square of nondimensional resonant frequencies of machine with absorber mass are given as

$$r_1^2, r_2^2 = \left(1 + \frac{\mu}{2(1+p)}\right) \mp \sqrt{\left(1 + \frac{\mu}{2(1+p)}\right)^2 - 1} \quad (7)$$

Optimally tuned active damped dynamic vibration absorber

The active undamped dynamic vibration absorber described in the previous section eliminates vibration at operating frequency ω , of machine but introduces two two resonant frequencies Ω_1, Ω_2 . The amplitude of the machine can be reduced by active damped dynamic vibration absorber as shown in Figure-2.

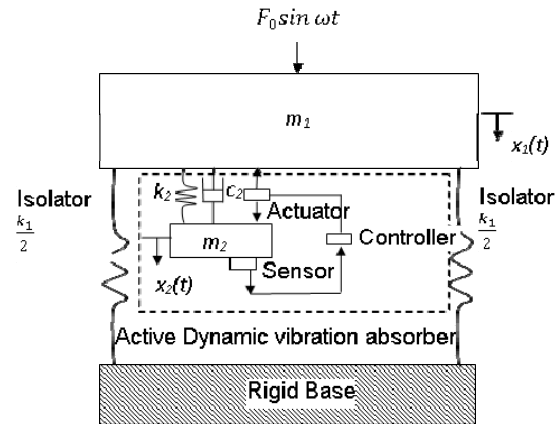


Figure-2. Active damped vibration absorber.

The equations of motion are given as

$$m_1 \ddot{x}_1 + k_1 x_1 - k_2 (x_2 - x_1) - c_2 (\dot{x}_2 - \dot{x}_1) = F - F_c \quad (8)$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) + c_2 (\dot{x}_2 - \dot{x}_1) = F_c \quad (9)$$

where $F_c = -k_d \dot{x}_2$

The general transfer function for the two degree of freedom system can be expressed as

$$\frac{X_1}{\delta_{st}} = \frac{G_{22}}{\Delta} \quad (10)$$

where

$$G_{22} = \hat{s}^2 + 2\zeta \hat{s} + f^2 \quad (11)$$

$$\Delta = \hat{s}^4 + \hat{s}^3 2\zeta \left(1 + \frac{\mu}{1+q}\right) + \hat{s}^2 (f^2 + 1 + \mu f^2) + \hat{s} 2\zeta + f^2 \quad (12)$$

RESULTS AND DISCUSSION

Active undamped dynamic vibration absorber

The parameters considered in the analysis of active undamped dynamic vibration absorber are: ratio of proportional gain to stiffness of absorber mass ($p=0, 1, 2$); mass ratio ($\mu=0-1$). Results of variations in resonant frequencies of two degree of freedom main and absorber mass are compared for different parameters of proportional gain ratio. Figure-3 shows the variation of nondimensional resonant frequencies of machine with absorber mass (r_1 and r_2) as a function of mass ratio (μ). It can be seen that the difference between r_1 and r_2 increases with increase in mass ratio μ . It can also be seen that r_1 is less than and r_2 is greater than the operating speed of the machine.

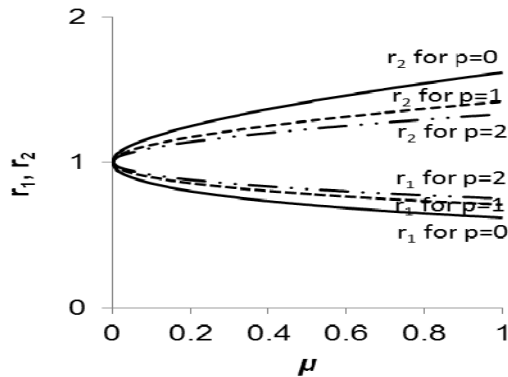


Figure-3. Variation of r_1 and r_2 .

Optimally tuned active damped dynamic vibration absorber

The parameters considered in the analysis of active damped dynamic vibration absorber are: ratio of derivative gain to damping constant of absorber mass ($q=0, -3.2, -3.35$); mass ratio ($\mu=1/5, 1/20$); damping ratio ($\zeta=0.1$); ratio of natural frequencies ($f=1$). Results of nondimensional steady state response of main mass (X_1/δ_{st}) are compared for different parameters of derivative gain ratio and mass ratio. Figure-4 shows the variation of nondimensional steady state response of main mass (X_1/δ_{st}) as a function of forced frequency ratio (g). It has been observed that the most efficient vibration absorber is one for which the ordinates of points of two peaks are equal.

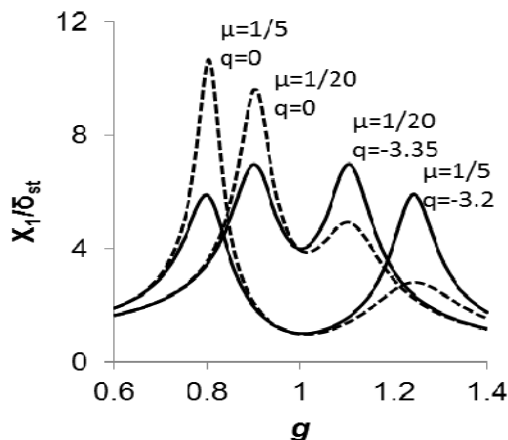


Figure-4. Response of machine under the effect of active damped vibration absorber.

An absorber satisfying this condition is optimally tuned active damped dynamic vibration absorber. The optimal values are found by making the nondimensional response as flat as possible at the two peaks.

CONCLUSIONS

The present study examines the active undamped dynamic vibration absorber under proportional gain control and optimally tuned active damped dynamic vibration absorber under derivative gain control. The range of nondimensional resonant frequencies of machine with absorber mass (r_1 and r_2) as a function of mass ratio (μ) are reduced with proportional gain control for undamped absorber. The optimal values of nondimensional steady state response of main mass (X_1/δ_{st}) as a function of forced frequency ratio (g) are obtained using derivative gain control for damped absorber.

ACKNOWLEDGEMENTS

The authors would like to convey their appreciation for the support of Universiti Teknologi Petronas.

NOMENCLATURE

c_2	Damping constant of absorber mass, Ns/m
c_c	Critical damping constant, Ns/m; $c_c = 2m\omega_n$
f	Ratio of natural frequencies; $f = \omega_a/\omega_n$
F_o	Steady state force, N
k_p	Proportional gain, N/m
k_d	Derivative gain, Ns/m
k_1	Stiffness of main mass, N/m
k_2	Stiffness of absorber mass, N/m
m_1	Main mass, kg
m_2	Absorber mass, kg
r_1, r_2	Nondimensional resonant frequencies of machine with absorber mass; $r_1 = \Omega_1/\omega_2$, $r_2 = \Omega_2/\omega_2$
p	Ratio of proportional gain to stiffness of absorber mass; $p = k_p/k_2$
q	Ratio of derivative gain to damping constant of absorber mass; $q = k_d/c_2$
g	Forced frequency ratio; $g = \omega/\omega_n$
X_1, X_2	Steady state response of main and absorber mass, m
δ_{st}	Static deflection, m; $\delta_{st} = F_o/k_1$
μ	Mass ratio; $\mu = m_2/m_1$
ζ	Damping ratio; $\zeta = (c_2 + k_d)/c_c$
ω_a, ω_2	Natural frequency of absorber, rad/s;
	$\omega_a = \omega_2 = \sqrt{(k_2 + k_p)/m_2}$
ω_n, ω_1	Natural frequency of main mass, rad/s;
	$\omega_n = \omega_1 = \sqrt{k_1/m_1}$

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