



VARIOUS MATHEMATICAL MODEL OF TORSIONAL VIBRATION IN OILWELL DRILLSTRING AND THEIR VALIDATION TECHNIQUES

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ABSTRACT

Modern techniques to model torsional drill string vibration have positive impact to the efficiency of oil and gas drilling. The need for torsional model is evident from their detrimental effect to the bottom hole assembly (BHA) fitness. The model can cater planning for more efficient parameter selections for BHA and controlling surface parameter to reduce such vibration. In this paper, the models for torsional vibrations are classified based on their functional concept. The validation techniques are also reported with each classification of the model for torsional vibrations.

Keywords: torsional drill string model, validation techniques, stick-slip model, and mathematical model.

INTRODUCTION

Mathematically, a model maps the future state of a system. It is a Newtonian point of view of how natural system can be calculated and predicted. Analogously, the purpose of modelling vibration is to predict the influence this dynamic phenomena has on any man-made structure. The dynamic load imposed to a drill string while drilling causes various vibrations which can be classified via its modes viz. lateral vibration, axial vibration and torsional vibration. However, amongst the aforementioned modes, torsional vibration has notoriety reputation for causing wear to BHA and increasing non-production time (NPT) [1], [2]. Hence, efforts to model such vibrational behaviour has been extensive with the introduction of various modelling method and with the advancement of remote measurement called measurement while drilling (MWD) tool [3], [4]. The efforts also trailed with validation techniques that reinforce the validity of such model. The validation techniques can be summarized into four, depicted in Table-1.

Experimental techniques	Experimental : large scale test bed	➤ Data collection is online and resembles closely to field data	➤ Lucrative cost for set up
	Experimental : miniature laboratory test bed	➤ Data collection is online and resembles closely to field data with the expense of accuracy	➤ Accuracy of the data depends on experimental model ➤ Requires transformation techniques to replicate experimental data to field data
	Field data comparison	➤ Accurately depict real world scenario	➤ Not readily available

Each of these validation techniques is preceded by a modelling approach to torsional vibrations. The models can be classified into: (a) physical and mathematical modelling techniques and (b) data driven model.

Table-1. Four methods to validate mathematical model of drill string dynamics.

	Method	Advantages	Disadvantages
Simulation techniques	Numerical method via finite element analysis	➤ Readily available techniques and templates ➤ Running process requires minimal cost	➤ Limited observation of result ➤ The validation is purely on simulation and dependent on the degree of which the model reflects real world scenario ➤ Computationally and timely expensive
	Analytical method via simulation	➤ Readily available software ➤ Running process requires minimal cost	➤ Limited observation of result ➤ Parametric dependent ➤ May cause computation and time expense

PHYSICAL AND MATHEMATICAL MODELLING TECHNIQUE

Physical modelling techniques have a reputation to be used in many literatures due to its simplicity. The method depicts a simplified view of a drill string with mechanical element such as spring, damper, and massive body. This technique is often followed by mathematical derivation of the mechanical component of the model. However, the model cannot represent torsional vibration per se, which requires excitation function to mimic the torsional behaviour of a drill string. To fill this gap, analytical model of either bit-rock interaction or self-excited phenomena are introduced to the system which calls for various mathematical theory and techniques.

In physical models, torsional pendulum representation is common. The pendulum can be modelled mathematically to represent one degree of freedom motion [5], [6], or two degree of freedom motion. Patil & Teodoriu (2013) uses multi-segment two degree of freedom (DOF) torsional pendulum system coupled with



damper to represent mud damping effect towards the drill string [7]. Patil & Teodoriu (2013) uses the premise that torsional vibration is caused by bit-formation interaction on the drill [7]. With this the mathematical derivation of their system depends on the nonlinear bit-rock interaction effect. Their model is validated by numeric-analytical through simulation under MatlabTM/Simulink. Their model and simulation techniques produce results that correlate surface parameter which is weight on bit (WOB) and rotational speed of the turn table bottom hole parameters i.e. rate of penetration (ROP) and bit rotational speed. Spanos *et al.* (2002) also uses physical modelling technique to represent the nonlinear stochastic nature of drill string vibration [8]. Their model is based on lumped mass/spring model with 2-DOF: for rotational motion along the axial line and another for the lateral motion of the drill string. Their stiffness model, however, follows mathematical modelling techniques where the derivation of the BHA stiffness is based on Euler-Bernoulli beam theory. Spanos *et al.* (2002) validated their model using Monte Carlo Simulation using Auto Regression Moving Average filtering [8]. The configuration of lumped mass/spring configuration often seen in literatures can be modified to incorporate friction between bit and rock formation. The use of conveyor belt, as described by Abdo & Abouelsoud (2011), shows a torsional drill string model by using a linear spring [9]. Figure-1 explains the configuration of the system.

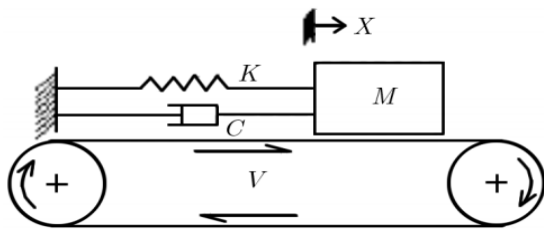


Figure-1. The configuration of lumped mass/spring model with contact between mass and conveyor belt surface replicating friction between bit and formation (Abdo & Abouelsoud, 2011)

The system introduced by Abdo & Abouelsoud, (2011) is analytically derived based on Liapunov second method to derive the oscillatory nature of the system [9]. Their simulation is based on analytically solved second order equation of motion with reports on their models limit cycles based on multiple conditions. Lumped parameter mass/spring model are often used when new parameters are being studied in torsional drill string model. Tang *et al.* (2015) studied the effect of surface rotary table speed on the torsional vibration of a drilling system [10]. Their research uses lumped parameter mass/spring constant to represent their torsional drill string model with bit-rock interaction is modelled based on Coulomb. The derivation of the stick-slip motion of the drill string is governed by Euler-Lagrange equations of motion. The solution to their Euler-Lagrange equations of motion are based on the

boundary condition of a stick slip where at stick phase, the motion of the bit is regarded as zero and at slip phase, the bit relative angular response of the bit is considered constant. Their simulation is based on the numerical method where Tang *et al.*, (2015) managed to produce results that shows an attracting cycle limit of the drill string torsional behaviour for certain range of the rotary table speed. The same approach was adopted in papers done by Zhu & Tang (2015) where they look into the formation of limit cycle for friction induced vibration [11].

Multiple degree of freedom torsional drill string modelling has also been reported based on lumped mass/spring system. Liu *et al.* (2014) has proposed 32 drill string segments based on the lumped mass/spring system [12]. The system takes regard of the dry friction into the model system and shows that the axial-torsional vibration induced by the system is caused by state-dependent delay which is observed on polycrystalline diamond compact (PDC) bit. Their finite element validation method uses 128 states induced by the 32 segments in the drill string model. It is thus far that the mathematical model of tool-bit interaction has been deterministic in nature. However, tool-bit interaction observed in the field is stochastic in nature. To address this problem, Ritto, Soize & Sampaio (2009) simulated uncertain model on a drill string system that realistically represent the torsional drill string model of a drilling system [13]. Ritto *et al.* (2009) uses nonlinear Timoshenko beam theory. Often enough, with highly nonlinear system, the method of validation is often in the form of finite element method as is the case for Ritto *et al.* (2009). Highly nonlinear system also calls for numerical method for solution of drill string vibration under different parameter consideration. In the report presented by Yigit & Christoforou (1998), a beam model of drill string was used to prove the self-excitation of drill string vibration [2]. The paper explained from their validation method of the model that the geneses of other vibrations mode are highly likely caused by the interaction of the bit-rock formation which in turn excites the drill string to produce varying torque on the bit (torsional vibration) [2]. Their experimental set up validated further the finding of the numerical simulation that was done based on the model.

DATA DRIVEN MODEL

Data driven model are based on streams of offline or online data that is used to generate either parameters of a system or the output for an agitator. This type of method of modelling technique is often used for highly dynamic system with online data to optimize control parameters of the system.

Stream of data that can produce parametric value of a system are useful in determining torsional characteristic of a drill string. The method are known as system identification and has been utilised in a full scale rig experimental setup [6]. The parameters of a drilling process are obtained based on the input and the output of a drilling process through system identification method and produce a model of drill string torsional model. The model is used to find the optimal surface parameter through



stability analysis. Bayliss, Pachal & Whidborne (2012) has used parametric identification to model the torsional vibration of a directional drilling exercise [14]. The parameters produced in their recursive least square identification method are then used to model the torsional vibration. Their system is simulated using symbolic-numerical method via Matlab™.

CONCLUSION

The model of drill string vibration is a two tier process. First aspect of the model deals with the model of the mechanical nature of the drill string which is a commonplace for researcher today to use lumped parameter mass/spring model. The motion of this drill string is govern by the mathematics that we give to the motion, either it moves freely in one, two, to six DOF. The more freedom the model of the drill string is based on, the more complex and nonlinear the equation of motion is. Thus the coupling of the two or more modes of vibration in drill string model is only practically validated through numerical method or finite element method with underlying linearization technique. The second aspect of the model delves into the interaction of the bit and the rock formation. The model can be based on various friction theories or any stochastic method. The coupling of the friction to the drill string motion model gives a complete representation of the drill string dynamics. In this paper various model techniques and their validation method has been elucidated; one being the mathematical and mechanical modelling technique, and another on data driven modelling technique. Both produce results that are commonly utilised in different ways. Mathematical and modelling techniques are more often used in offline or non-real-time purposes while data driven modelling technique are used in real-time system for optimising system control.

ACKNOWLEDGEMENTS

This research is funded under ERGS Grant Scheme (O153AB-I16) by Ministry of Higher Education, Malaysia

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