VOL. 11, NO. 23, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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FORM FACTOR AND EFFICIENCY COEFFICIENT OF THE EXTINCTION FOR A PARALLELEPIPED (OR CUBIC) PARTICLE IN THE WKB APPROXIMATION

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ABSTRACT

In this work, we determine the analytical expression of the form factor for a parallelepiped in the WKB approximation. We will focus, in this paper, to the study for the scattering of an incident ray in the perpendicular plane to the particle. Adapting some variables (size parameter, refractive index, the scattering angle) the other approximations such as the Rayleigh-Gans-Debye (RGD) and anomalous diffraction (AD) are easily deduced from our general formula. Furthermore, the closed form expression of the efficiency coefficient of the extinction is also given. For illustration, some numerical examples are be analyzed.

Keywords: scattering, form factor, parallelepiped , WKB approximation.

1. INTRODUCTION

Studies on the scattering of light by small particles have a long and interesting history in physics. Nevertheless, they still bear new ideas and applications. But there are many potential future applications, particularly, in optical devices and technologies of solar energy [1-2]. Based on electromagnetic theory, Mie theory, published in 1908, gives a rigorous solution to the spread of a monochromatic plane wave by a spherical particle [3], the extension to other forms of particles are less than ideal proved to be a difficult problem. As these systems are of great interest in many scientific disciplines such as astronomy and chemistry. There are two general approaches available:

- numerical techniques : the invariant embedding T-matrix approach, the digitized Green's-function method (DFG) and an improved version of the extended boundary condition method (EBCM). [4-5-6]
- The use of approximate analytical methods: Rayleigh-Gans-Debye (RGD), anomalous diffraction (AD) that are generally valid for a number of situations.

Amongst these analytical methods, the WKB technique, this technique introduced by Rayleigh in 1912 for the solution of wave propagation problems [7], was first applied to quantum mechanics by Jeffreys in 1923 [8], such applications continue up to the present day. Saxon applied the WKB approximation for the scattering of electromagnetic waves by a dielectric sphere [9]. Deirmendjian in 1959 introduced this method for the calculation of the light scattering by spheres and proved its interest [10].

In the WKB approximation, the internal field is equal to the incident field modulated by a phase delay factor which corresponds to an additional phase shift of the wave that propagates inside the particle. Therefore, the WKB approximation is a refinement of the RGD approximation [11].

Because of the complexity of the general scattering theory for nonspherical particles, the possibility of applying the WKB approximation to modeling the scattering of light by parallelepiped objects is worth investigating. This approach is applied on spheres, cylinders and spheroids [12-13-14].

This work is devoted to a theoretical study of scattering of light by parallelepiped, regular for dimensions a×a×l, in the WKB approximation. Within the framework of the scattering theory, we investigate the form factor for this approximation. The content of the article is the following. The second section introduces the principles of the WKB approximation for the calculation of form factor of particles. In the third section, we derive a general analytical expression for the form factor expression for a parallelepiped. In the fourth section, by varying some particle parameters, the RGD and the AD are deduced from our general formula. In the last section we give an analytical expression of the extinction coefficient. By using this formalism, numerical calculations are been performed to illustrate the comportment of the scattered light form factor.

2. FORM FACTOR

In the literature, the expression of the amplitude of light scattering in the WKB approximation, in a scalar form, is [12-13]:

$$|f(\vec{o},\vec{\imath})| = \frac{k^2}{2\pi}\sin\aleph\,|(m-1)F(\theta,\varphi)|, \tag{1}$$

where $\vec{o}$ and $\vec{\imath}$ are the unit vectors along the directions of scattering and propagation of light, respectively. $\aleph$ is the angle between the polarization vector $\vec{e_x}$ and the unit vector $\vec{o}$, θ is the scattering angle between $\vec{o}$ and $\vec{\imath}$, φ is the azimuthally angle, k is the wave vector and m is the relative complex refractive index. The quantity $F(\theta,\varphi)$ is known as the form factor which represents the modification of the scattered irradiance due to the finite size of the particle and to its deviation from sphericity.

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ARPN Journal of Engineering and Applied Sciences

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$F(\theta,\varphi) = \iiint_{\vartheta} \exp\left(ik\vec{r}(\vec{i}-\vec{o})\right)\exp(ikW)d\vartheta$.
(2)

Wehre , $\vec{r}$ is the position vector of any point within the scattering object,
and

$$W = \int_{Z_e}^{Z}[m(Z')-1]dZ' = (m-1)(Z-Z_e) \quad . \qquad (3)$$

W is the optical path for homogenous particle which is introduced by the scattering object, Z is the Z-coordinate of the scatter element inside the particle and Z_e is the z-coordinate of the initial position of penetration of the object (see Figure-1).

We consider the Cartesian coordinate system orthonormal $R(X',Y',Z')$, the origin coincides with the center of regular parallelepiped which is illuminated with a flat monochromatic wave of wave number k, polarized in the direction $\overrightarrow{e_x}$. We assume that an electromagnetic wave is incident in the plane $Y'OZ'$ at an angle α to the $Z'-$ axis (see Figure-1(a)).

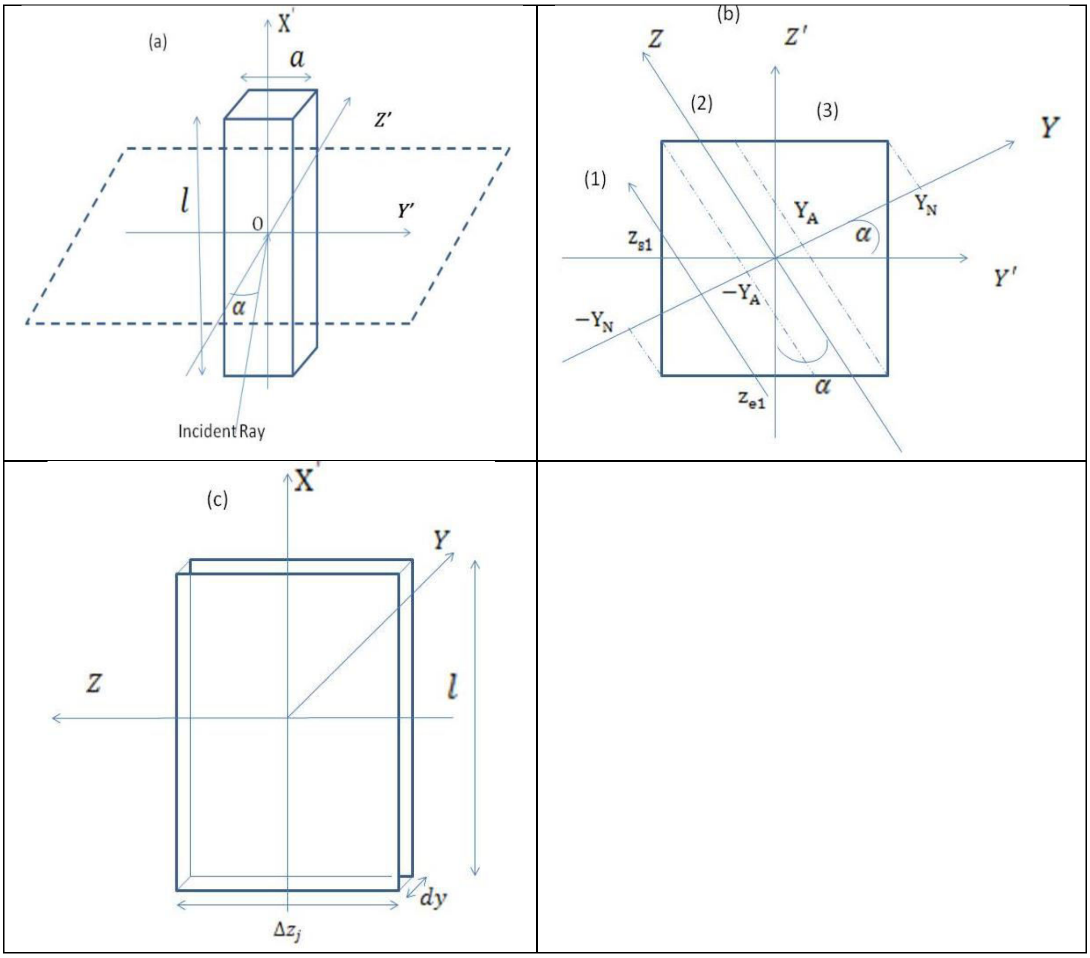

Figure-1. Decomposition of the parallelepiped.

In rectangular coordinates the form factor can be written as

$$F(\theta,\varphi) = \iiint_{\vartheta} \exp[ik(-x\sin\theta\cos\varphi - y\sin\theta\sin\varphi)]\exp[ik(m-\cos\theta)z]\exp[-ik(m-1)z_e(y)]\,d\vartheta \ . \qquad (4)$$

After some algebraic manipulations, the form factor can be expressed in a simple form,

$$F(\theta,\varphi) = A(\theta,\varphi)\int_{-Y_N}^{Y_N} \mathrm{ampl}\left(z_e(y),z_s(y)\right)\exp[-iky\sin\theta\sin\varphi]dy, \qquad (5a)$$

where,

$$A(\theta,\varphi) = \frac{l}{ik(m-\cos\theta)}\frac{\sin(d)}{d}, \qquad (5b)$$

$$d = \frac{kl}{2}\sin\theta\cos\varphi \ , \qquad (5c)$$

and

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$$\mathrm{ampl}(z_e(y), z_s(y)) = \exp[-ik(2m-1-\cos\theta)z_e(y)] - \exp[ik(1-\cos\theta)z_s(y)], \quad (5d)$$

with $z_e(y)$, $z_s(y)$ are the z-coordinates of the intersection of the incident ray and the body surfaces, and

$$Y_N = \frac{a}{2}\cos(\alpha) + \frac{a}{2}\sin(\alpha) \quad (6)$$

From the symmetry of the particle, we can decompose the parallelepiped into three regions. Each region in turn is divided into longitudinal slices with thickness dy and width Δz_j (see Figure-1). Therefore, the form factor can be written as:

$$F(\theta,\varphi) = F_1(\theta,\varphi) + F_2(\theta,\varphi) + F_3(\theta,\varphi), \quad (7)$$

where,

$$F_1(\theta,\varphi) = A(\theta,\varphi)\int_{-Y_N}^{-Y_A} \mathrm{ampl}\left(z_{e1}(y), z_{s1}(y)\right)\exp[-iky\sin\theta\sin\varphi]dy, \quad (8)$$

$$F_2(\theta,\varphi) = A(\theta,\varphi)\int_{-Y_A}^{Y_A} \mathrm{ampl}\left(z_{e2}(y), z_{s2}(y)\right)\exp[-iky\sin\theta\sin\varphi]dy, \quad (9)$$

$$F_3(\theta,\varphi) = A(\theta,\varphi)\int_{Y_A}^{Y_N} \mathrm{ampl}\left(z_{e3}(y), z_{s3}(y)\right)\exp[-iky\sin\theta\sin\varphi]dy, \quad (10)$$

with

$$Y_A = \frac{a}{2}\cos(\alpha) - \frac{a}{2}\sin(\alpha), \quad (11)$$

and $z_{e\,i}(y)$, $z_{s\,i}(y)$ (with $i = 1, 2, 3$) are the z-coordinates of the intersection of the incident ray and the body surfaces for each region.
Finally, one obtains

$$F_1(\theta,\varphi) = -i\frac{a^2 l\tan\alpha}{2u}\frac{\sin d}{d}\exp\left[-i(2q\sin\alpha - 2t)\frac{Y_A}{a}\right][I_1 - J_1], \quad (12)$$

$$F_2(\theta,\varphi) = 2\frac{a\,lY_A}{\cos\alpha}\frac{\sin d}{d}\frac{\sin u}{u}\frac{\sin\left((2q\sin\alpha - 2t)\frac{Y_A}{a}\right)}{(2q\sin\alpha - 2t)\frac{Y_A}{a}}e^{i\frac{\rho}{2}}, \quad (13)$$

and

$$F_3(\theta,\varphi) = i\frac{a^2 l\tan\alpha}{2u}\frac{\sin d}{d}\exp\left[i(2q\sin\alpha - 2t)\frac{Y_A}{a}\right][I_3 - J_3], \quad (14)$$

where I_1, J_1, I_3 and J_3 are given by

$$I_1 = e^{ig}\left[\frac{\exp[-2i(q\sin^2\alpha + u - t\sin\alpha)] - 1}{-2i(q\sin^2\alpha + u - t\sin\alpha)}\right], \quad (15)$$

$$J_1 = e^{iq}\left[\frac{\exp[-2i(q\sin^2\alpha - t\sin\alpha)] - 1}{-2i(q\sin^2\alpha - t\sin\alpha)}\right], \quad (16)$$

$$I_3 = e^{ig}\left[\frac{\exp[-2i(q\cos^2\alpha + u + t\sin\alpha)] - 1}{-2i(q\cos^2\alpha + u + t\sin\alpha)}\right], \quad (17)$$

$$J_3 = e^{iq}\left[\frac{\exp[-2i(q\cos^2\alpha + t\sin\alpha)] - 1}{-2i(q\cos^2\alpha + t\sin\alpha)}\right]. \quad (18)$$

With

$$\rho = \frac{ka}{\cos\alpha}(m-1), \quad (19)$$

$$u = \frac{ka}{2\cos\alpha}(m - \cos\theta), \quad (20)$$

$$g = \frac{\rho}{2} + u, \quad (21)$$

$$q = \frac{\rho}{2} - u, \quad (22)$$

and

$$t = \frac{ka}{2}\sin\theta\sin\varphi. \quad (23)$$

Therefore, $F(\theta,\varphi) = F_1(\theta,\varphi) + F_2(\theta,\varphi) + F_3(\theta,\varphi)$ is a closed-form of the form factor for a parallelepiped (or cubic). This expression is valid for any value of the phase delay of the wave penetrating through the center of the homogenous absorbing parallelepiped. For illustration, we show in Figures 2-3 the behavior of the form factor as a function of the scattering angle θ, for two special cases ($\frac{l}{a} = 0.2$; parallelepiped plat and $\frac{l}{a} = 2$; parallelepiped column). These Figures show that the form factor exhibits some lobes with intensity decreasing with scattering angle.

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ARPN Journal of Engineering and Applied Sciences

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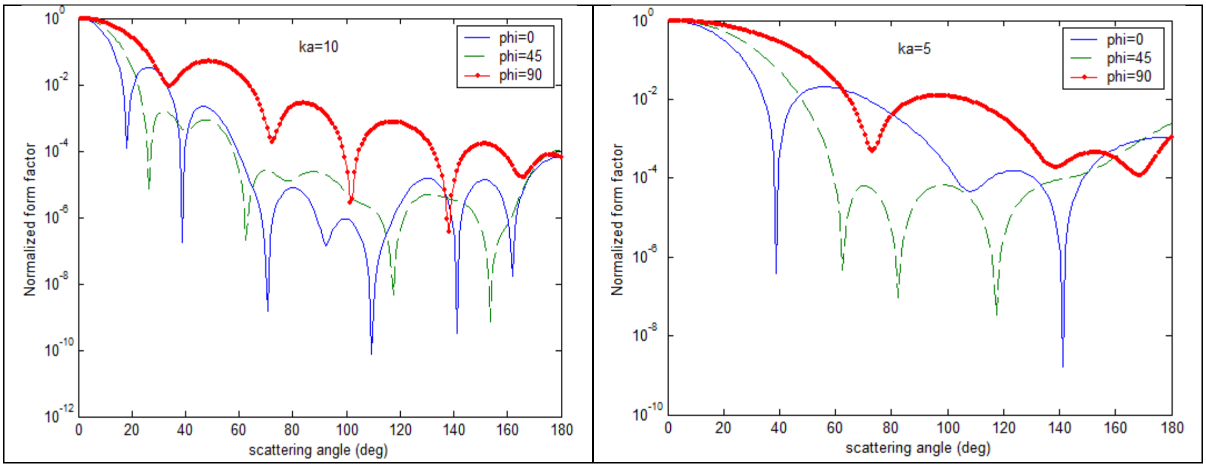

Figure-2. Normalized form factor as a function of scattering angle θ for absorbing parallelepiped, for two values of the parameter ka. with $\frac{l}{a} = 2$, $m = 1.15 + 0.01i$, $\alpha = 30°$.

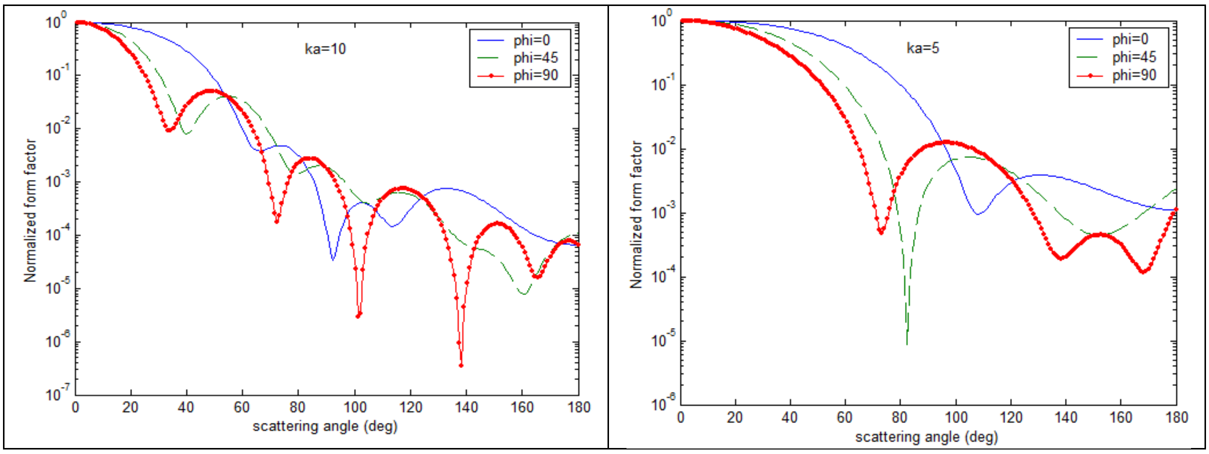

Figure-3. Normalized form factor as a function of scattering angle θ for absorbing parallelepiped, for two values of the parameter ka. with $\frac{l}{a} = 0.2$, $m = 1.15 + 0.01i$, $\alpha = 30°$.

3. SPECIAL CASES

3.1 RGD approximation

In the limit of a small refractive index, the WKB approximation reduces to RGD approximation for scattering from parallelepiped. For this approximation, we assume that $\rho \ll 1$. Which implies that the imaginary part of the form factor is vanishes. In this case the expression of the form factor reduced to.

$$F(\theta,\varphi) = a^2 l \frac{\sin d}{d}\left[\frac{\frac{2Y_A}{a}}{\cos\alpha}\,\frac{\sin u}{u}\,\frac{\sin\left((2u\sin\alpha+2t)\frac{Y_A}{a}\right)}{(2u\sin\alpha+2t)\frac{Y_A}{a}}\,\frac{\tan\alpha}{u}\left(\begin{array}{l}\sin(u+(u\sin\alpha+t)\cos\alpha)\frac{\sin(u\sin^2\alpha+t\sin\alpha)}{u\sin^2\alpha+t\sin\alpha}\\ +\sin((u\sin\alpha+t)\cos\alpha)\frac{\sin(u\cos^2\alpha-t\sin\alpha)}{u\cos^2\alpha-t\sin\alpha}\end{array}\right)\right]. \quad (24)$$

To illustrate our analytical result, we represent in Figure-4 the behavior of the normalized form factor as a function of the scattering angle θ in the case of a non absorbing parallelepiped. The parameters used in the calculation are: $\frac{l}{a} = 2$, m=1.01 and angle $\alpha = 30^{\circ}$ for 3 values of φ.

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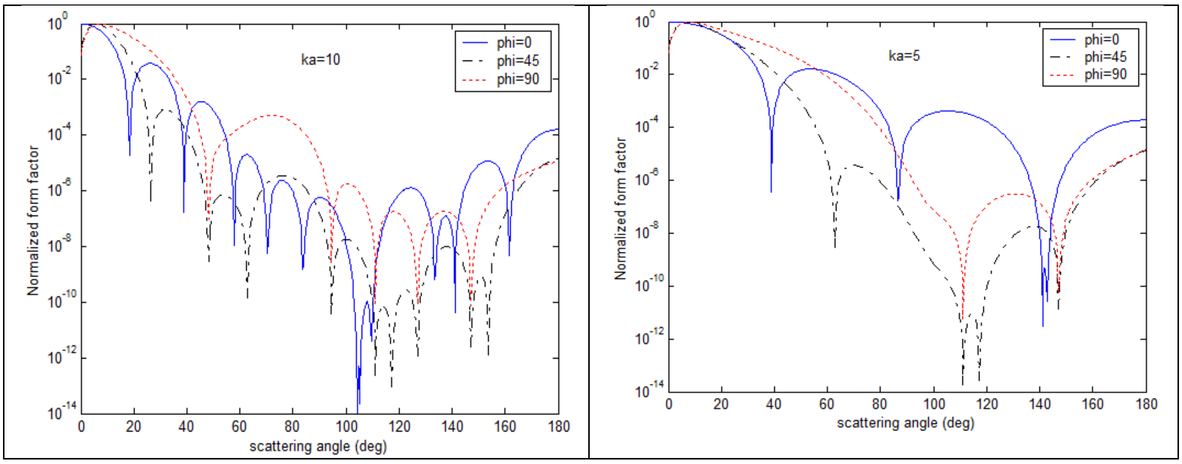

Figure-4. Normalized form factor as a function of scattering angle θ for nonabsorbing parallelepiped, for two values of the parameter ka.

3.2 Anomalous diffraction

This approximation, called anomalous diffraction, is characterized by low refractive index, in this case, the light passing through the particle (transmitted without reflection) interferes with diffracted light, producing a so-called anomalous diffraction.

Such an approximation is valid if $X = ka \gg 1$, X size parameter, and $|m-1| \ll 1$. This implies that the rays are not deviated when they cross the interface particle-medium and that the reflection at this interface is negligible. We consider in this case intermediary values of ρ and an angle $\theta \ll 1$, so

$$d = \frac{kl}{2}\theta\cos\varphi, \tag{25}$$

$$t = \frac{ka}{2}\theta\sin\varphi, \tag{26}$$

$$u + \frac{\rho}{2} \approx \rho\ , \tag{27}$$

and

$$u - \frac{\rho}{2} \approx 0\,. \tag{28}$$

The expression of the form factor is reduced to

$$F(\theta,\varphi) = -ia^2 l \frac{\sin d}{d}\left[\begin{matrix} \tan\alpha\left(\dfrac{\sin\left(\rho/2 + t\sin\alpha\right)}{\rho/2 + t\sin\alpha}e^{-it\cos\alpha} + \dfrac{\sin\left(\rho/2 - t\sin\alpha\right)}{\rho/2 - t\sin\alpha}e^{it\cos\alpha}\right)\dfrac{e^{i\rho/2}}{\rho} \\ -2\tan\alpha\dfrac{\sin(t\sin\alpha)}{t\sin\alpha}\dfrac{\cos(t\cos\alpha)}{\rho} + i\dfrac{\frac{2Y_A}{a}}{\cos\alpha}\dfrac{\sin(\rho/2)}{\rho/2}\dfrac{\sin(\frac{2Y_A}{a}t)}{\frac{2Y_A}{a}t}e^{i\rho/2} \end{matrix}\right]. \tag{29}$$

We present in Figure-5, the normalized form factor as a function of the scattering angle θ for three values of φ. We respect the conditions imposed on the scattering angle, size parameter and refractive index of the scattering object with $m = 1.1 + 0.01i$, $\frac{l}{a} = 2$, and angle $\alpha = 30^{\circ}$ for two values of the parameter ka.

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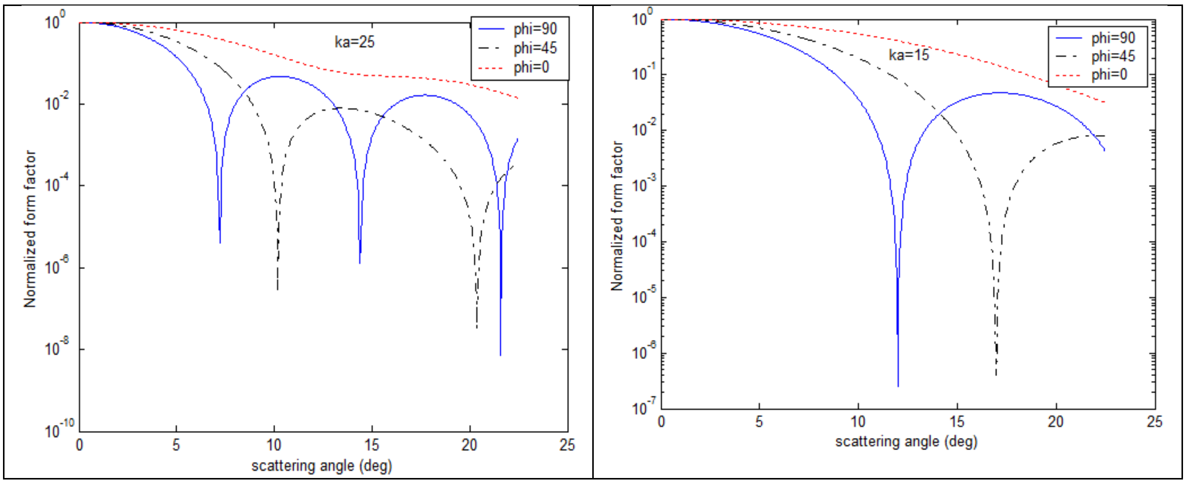

Figure-5. Normalized form factor as a function of scattering angle θ for parallelepiped, for 3 values of φ.

4. EXTINCTION COEFFICIENT

The efficiency factor of extinction is connected to the cross section of extinction by [3]

$$Q_{ext} = \frac{\sigma_{ext}}{P}, \tag{30}$$

where P is the projected air given by the parallelepiped , and σ_{ext} is the cross section of extinction [15]

$$\sigma_{ext} = \frac{4\pi}{K} \mathrm{Im}\{|f(\vec{\imath},\vec{\imath})|. \tag{31}$$

where $|f(\vec{\imath},\vec{\imath})|$ given by Eq.(1) and Im is the imaginary part. Finally, the efficiency coefficient is expressed as:

$$Q_{ext} = \frac{2k}{P}\ \mathrm{Im}(|(m-1)F(0,0)|). \tag{32}$$

The projected air is

$$P = al\ (\sin\alpha + \cos\alpha). \tag{33}$$

The expression of the efficiency coefficient of extinction is reduced to

$$Q_{ext} = \mathrm{Re}\left(\frac{-2}{\tan\alpha+1}\left((e^{i\rho}-1)\left(1-\tan\alpha+\frac{2\tan\alpha}{i\rho}\right)-2\tan\alpha\right)\right). \tag{34}$$

The efficiency coefficient of extinction of parallelepiped, under incidence perpendicular to the particle, for a given angle α, is independent of the height l.

In the following, we give as example, the analytical expression of the efficiency coefficient for real refractive indices in two specials cases of the incidence.

- $\alpha = 0$, we find $Q_{ext} = 2 - 2\cos\rho$. (35)
- $\alpha = \frac{\pi}{4}$, we find $Q_{ext} = 2 - 2\frac{\sin\rho}{\rho}$. (36)

These two last expressions can be verified by using this method for normally incidence.

In Figure-6, we represent the behavior of the efficiency factor of the extinction Q_{ext} versus $X = ka$, for 3 values of angle α. Note that, after some oscillations, this coefficient tends to the value 2 for large quantities of ka.

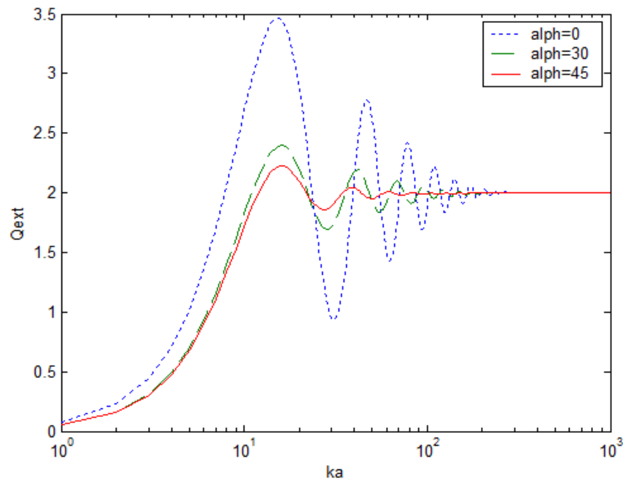

Figure-6. The extinction efficiency factor Q_{ext} as a function of size parameter $X = ka$, for $m = 1.2 + 0.02i$.

5. CONCLUSIONS

In this paper, we have shown the possibility of applying the WKB approximation for nonspherical particle for horizontally incidence. For this, we have determined the analytical expression of the forme factor for homogenous parallelepiped. We have applied this formula without restricting conditions. The other approximations covered by the WKB approach, such as the Rayleigh–Gans–Debye approximation (RGD) and, the anomalous diffraction (AD) are deduced easily from our analytic formula. Furthermore an analytical expression of the extinction efficiency factor is also given. Our goals are to shown that the WKB approximation can be extended to a large number of different shapes as needed is specific problems and to give an analytical formulation of this method for arbitrarily oriented particles. To illustrate the comportment of the scattered light form factor, some practical illustrations are analyzed.

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