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UNSTEADY MHD FREE CONVECTIVE FLOW PAST A VERTICAL POROUS PLATE WITH VARIABLE SUCTION

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ABSTRACT

An attempt has been made to study the unsteady MHD free convective flow past a vertical plate with variable suction. The Governing equations are transformed into a set of nonlinear coupled partial differential equations and solved using a numerical technique using appropriate boundary conditions for various physical parameters. The velocity temperature and concentration profiles are shown graphically for different physical parameters.

Keywords: MHD flow, porous plate, schmidt number, suction velocity.

INTRODUCTION

In recent years the problem of free convective flows in porous media have gained significant attention because of their possible applications in many branches of science and technology, such as its applications in transportation, cooling of re-entry vehicles and rocket boosters, solid matrix heat exchangers ,thermal insulations, oil extraction and store of nuclear waste materials, underground coal gasification, Ground water hydrology, .In view of these applications many researchers have studied MHD free convective heat and mass transfer flow in a porous medium, some of them are Raptis and Kafoussias [1], Hossain and Begum[2].

Recently the study of free convective mass transfer effects are dominant features in many engineering applications such as rocket nozzles, cooling of nuclear reactors, high sinks in turbine blades high speed aircrafts and their atmospheric re-entry, chemical devices and process equipments. The unsteady free convective flow with radiation effect was investigated by Cogley [3]. Unsteady effect on MHD free convective and mass transfer flow through porous medium with constant suction and constant heat flux in rotating system studied by Sharma [4].But in all these papers thermal diffusion effects have been neglected, where as in a convective fluid when the flow of mass is caused by a temperature difference, thermal diffusion effects cannot be neglected. Gebhart and Mollendorf [5] have shown that when the temperature difference is small or in high prandtl number fluids or when the gravitational field is of high intensity viscous dissipative heat should be taken into account in free convection flow past a semi infinite vertical plate. In recent years progress has been considerably made in the study of heat and mass transfer in magneto hydrodynamic flows due to its application in many devices, like the MHD power generator and Hall accelerator. The influence of magnetic field on the flow of an electrically conducting viscous fluid with mass transfer and radiation absorption is also useful in planetary atmosphere research. Yih [6] numerically analyzed the effect of transportation velocity on the heat and mass transfer characteristics of mixed convection about a permeable vertical plate embedded in a saturated porous medium under the coupled effects of thermal and mass diffusion. The study of free convective flow with Soret effect was studied by G.V. Ramana Reddy [7].

The objective of the present paper is to study the chemical reaction effect on unsteady magneto hydro dynamic flow past a vertical porous plate with variable suction.

Formulation of the problem

An unsteady two-dimensional laminar free convective boundary layer flow of a viscous, incompressible, electrically conducting and the chemical reaction effects on an unsteady magneto hydrodynamic free convection fluid flow past a semi-infinite vertical plate embedded in a porous medium with heat absorption is considered. The x'-axis is taken along the vertical plate and the y' axis normal to the plate. It is assumed that there is no applied voltage, which implies the absence of an electric field. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and Hall Effect are negligible. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with other chemical species which are present, and hence the Soret and Dufour are negligible. Further due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and t' only. Now, under the usual Boussinesq approximation, the governing boundary equations of the problem are:

Continuity equation: $$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty^1) + g\beta^*(C' - C'_\infty) - (\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K'})u' \tag{2}$$

Energy Equation:

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$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

Diffusion equation:

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr^*(C' - C'_\infty) \quad (4)$$

Where C_p -specific heat at constant pressure, g - Gravity due to Acceleration, k-Thermal conductivity, T' - Temperature of the fluid at the plate, T'_∞ -temperature of the fluid at infinity, u', v' are the Velocity components in x', y' direction respectively, ρ - Density of the fluid.

ν -Kinematic viscosity, β and β^* -the thermal and concentration expansion coefficient respectively,

B_0 -magnetic induction, α-the fluid thermal diffusivity, K' -the permeability of the porous medium,

C' -is the dimensional concentration, C'_∞ -is the concentration of free stream, μ -coefficient of viscosity, D-the mass diffusivity.Kr-chemical reaction parameter.

The boundary conditions for the velocity, temperature and concentration fields are:

$$u' = 0,\ T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{\text{in}'t'},\ C' = C'_w \text{ at } y' = 0$$
$$u' \to 0, T' \to T'_\infty, C' \to C'_\infty \text{ as } y' \to \infty \quad (5)$$

Where T'_w and C'_w are the wall dimensional temperature and concentration respectively.

From the continuity equation, it can be seen that v' is either a constant or a function of time,

So assuming suction velocity to be oscillatory about a non-zero constant mean, one can write

$$v' = -v_0(1 + \varepsilon A e^{\text{int}})$$

Where v_0 is the mean suction velocity and ε, A are small such that εA <<1.The negative sign indicates that the suction velocity is directed towards the plate. n is a constant.

In order to write the governing equations and boundary condition in dimension less form the following non dimensional quantities are introduced.

$$u = \frac{u'}{v_o},\ t = \frac{t' v_o^2}{4\nu},\ y = \frac{y' v_0}{\nu},\ T = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$
$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}\ \ Gm = \frac{\nu g \beta^*(C'_w - C'_\infty)}{v_0^3},$$

$$Gr = \frac{\nu g \beta(T'_w - T'_\infty)}{v_0^3},\ K = \frac{v_0^2 K'}{\nu^2},\ \Pr = \frac{\nu \rho c_p}{k},\ M = \frac{\sigma B_0^2 \nu}{\rho v_0^2},\ Sc = \frac{\nu}{D} \quad (6)$$

$$K_r = \frac{K_r^* \nu}{v_0^2},\ n = \frac{4\nu n'}{v_0^2}$$

In view of equation (6), the equations (2), (3), (4) reduced to the following dimensionless form

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{\text{int}})\frac{\partial u}{\partial y} = GrT + GmC + \frac{\partial^2 u}{\partial y^2} - (M + \frac{1}{K})u \quad (7)$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - (1 + \varepsilon A e^{\text{int}})\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \quad (8)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{\text{int}})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KrC \quad (9)$$

And these corresponding boundary conditions are

$$t > 0:\ \ u=0,\ T = 1 + \varepsilon e^{\text{int}},\ C=1 \text{ at } y=0,$$
$$u \to 0, T \to 0, C \to 0 \text{ as } y \to \infty \quad (10)$$

NUMERICAL SOLUTION USING FINITE ELEMENT METHOD

Finite element technique is used to solving the non-dimensional momentum and energy equations (7) (8) and (9) along with the imposed boundary conditions (10).

The Galerkin expression for the differential equation (7) becomes

$$\int_{y_j}^{y_k} \left\{ N^T \left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{1}{4}\frac{\partial u^{(e)}}{\partial t} + B\frac{\partial u^{(e)}}{\partial y} - Ru^{(e)} + P \right] \right\} dy = 0$$

Where $N^T = [N_j \quad N_k]^T = \begin{bmatrix} N_j \\ N_k \end{bmatrix}$, R=M+$\frac{1}{K}$, P= $(G_r)T + (G_m)C$;

Let the linear piecewise approximation solution

$$u^{(e)} = N_j(y)u_j(t) + N_k(y)u_k(t) = N_j u_j + N_k u_k$$

The element equation is given by

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$$\int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N_k' N_k' \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy + \frac{1}{4} \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \right\} dy - B \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j' & N_j N_k' \\ N_j' N_k & N_k' N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy +$$

$$\mathrm{R} \int_{y_j}^{y_k} \left\{ \begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \right\} dy = \mathrm{P} \int_{y_j}^{y_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy$$

Where prime and dot denotes differentiation w.r.to 'y' and 't' respectively
Simplifying we get

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{1}{24} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \frac{B}{2l^{(e)}} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{R}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

where $l^{(e)} = y_k - y_j = h$

In order to get the differential equation at the knot x_i, we write the element equations for the elements $y_{i-1} \leq y \leq y_i$ and $y_i \leq y \leq y_{i+1}$ assemble two element equations, we obtain

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{24} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} - \frac{B}{2l^{(e)}} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{R}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix}$$

$$= \frac{P}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

We put the row equation corresponding to the knot 'i', is

$$\frac{1}{l^{(e)^2}} \left[-u_{i-1} + 2u_i - u_{i+1} \right] + \frac{1}{24} \left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1} \right] - \frac{B}{2l^{(e)}} \left[-u_{i-1} + u_{i+1} \right] + \frac{R}{6} \left[u_{i-1} + 4u_i + u_{i+1} \right] = P$$

Applying Crank-Nicholson method to the above equation then we get

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^n + A_5 u_i^n + A_6 u_{i+1}^n + 24Pk$$

Where

$A_1 = 1 + 6Brh + 2Rk - 12r$; $A_2 = 8Rk + 24r + 4$; $A_3 = 1 + 2Rk - 12r - 6Bhr$;
$A_4 = 1 - 2Rk - 6Brh + 12r$; $A_5 = 4 - 8Rk - 24r$; $A_6 = 1 - 2Nk + 6Brh + 12r$; (11)

$$P = (G_r) T_i^j + (G_m) C_i^j ;$$

Applying similar procedure to equation (8), we get

$$B_1 T_{i-1}^{n+1} + B_2 T_i^{n+1} + B_3 T_{i+1}^{n+1} = B_4 T_{i-1}^n + B_5 T_i^n + B_6 T_{i+1}^n \quad (12)$$

Where

$B_1 = -12r+6B\,(Pr)rh+(Pr)$; $B_2 = 24r+4Pr$; $B_3 = -12r-6B(Pr)rh+Pr$;
$B_4 = 12r-6B(Pr)rh+(Pr)$; $B_5 = -24r+4Pr$; $B_6 = 12r+6B(Pr)rh+(Pr)$

Applying similar procedure to equation (9), we get

$$J_1 C_{i-1}^{n+1} + J_2 C_i^{n+1} + J_3 C_{i+1}^{n+1} = J_4 C_{i-1}^n + J_5 C_i^n + J_6 C_{i+1}^n \quad (13)$$

Where

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J_1 = -12r+6B (Sc)rh+(Sc)+(2KrSck);

J_2 = 24r+4Sc+(8KrSck);

J_3 = -12r-6B(Sc)rh+(Sc)+(2KrSck);

J_4 = 12r-6B(Sc)rh+(Sc)-(2KrSck);

J_5 = -24r+4Sc-(8KrSck);

J_6 = 12r+6B(Sc)rh+(Sc)-(2KrSck);

Here $r = \frac{k}{h^2}$ and k, h are mesh sizes along y- direction and time-direction respectively index 'i' refers to space and 'j' refers to time .The mesh system consists of h=0.1 and k=0.001.

In the equations (11), (12), and (13), taking i = 1(1)n and using boundary conditions (10), then we get the following system of equations are obtained:

$A_i X_i = B_i$ for i=1(1)n

Where A_i's are matrices of order n and X_i, B_i's are column matrices having n-components. The solutions of above system of equations are obtained by using Thomas Algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C++ program. In order to prove the convergence and stability of Galerkin finite element method, the same C++ program was run with smaller values of h and k, no significant change was observed in the values of u, T and C. Hence the Galerkin finite element method is stable and convergent.

Skin- friction

Skin-friction coefficient (τ) at the plate is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Nusselt number (N_u) at the plate is

$$N_u = \left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Sherwood number (S_h) at the plate is

$$S_h = \left(\frac{\partial C}{\partial y}\right)_{y=0}$$

RESULTS AND DISCUSSIONS

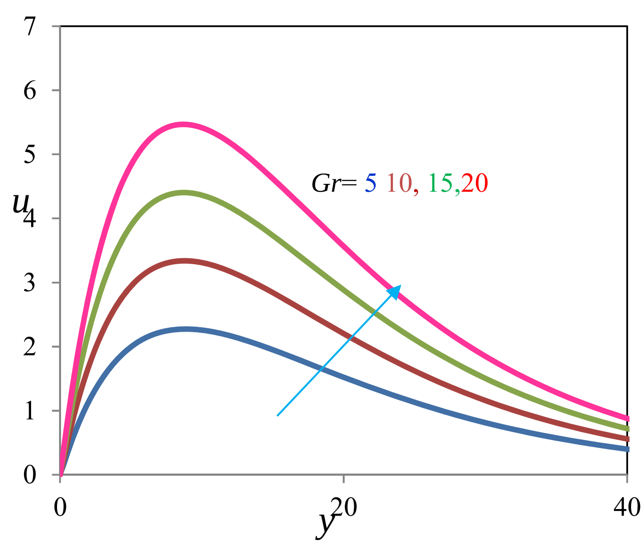

Figure-1. Velocity profile for different values of Gr when Gm =5.0, M=1.0, K=5.0, Pr=0.71, Sc=0.6, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

The effect of Grashof number on the velocity profiles are shown in Figure-1. Increase in Grashof number (Gr) contributes to the increase in velocity, while all other parameters are kept at some fixed values. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly approaches the free stream value.

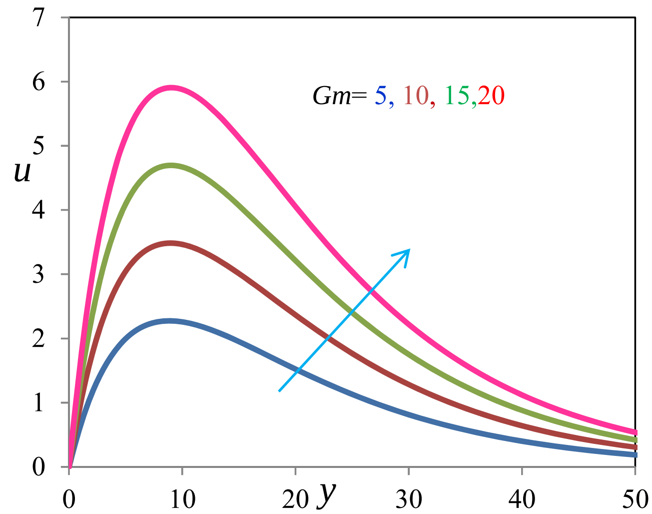

Figure-2. Velocity profile for different values of *Gm* when *Gr* =5.0, M=1.0, K=5.0, Pr=0.71, Sc=0.6, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

The effect of modified *Gr* ashof number *Gm* on the velocity profiles is observed in the above figure. Increase in *Gm* is found to influence the velocity to increase. Also as *Gm* increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

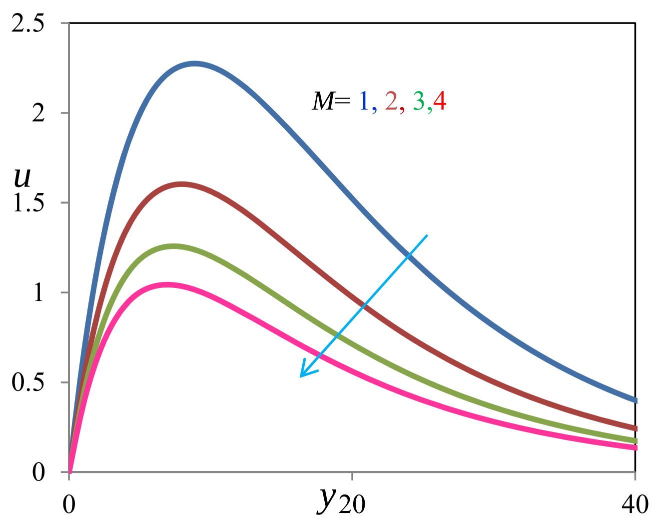

Figure-3. Velocity profile for different values of M when Gr=5.0, *Gm* =5.0, K=5.0, Pr=0.71, Sc=0.6, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

For various values of the magnetic parameter M, the velocity profiles are plotted in Figure-3. It can be seen that as M increases the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the flow.

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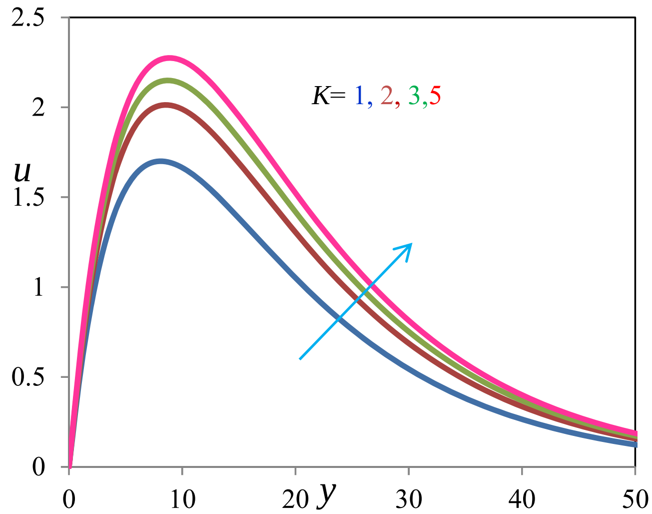

Figure-4. Velocity profile for different values of K when Gr =5.0, Gm = 5.0, Pr=0.71, M=1.0, Sc=0.6, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

The effect of the permeability parameter K on the velocity field has been studied in Figure-4. Increase in the resistance of the porous medium which will tend to increase the velocity.

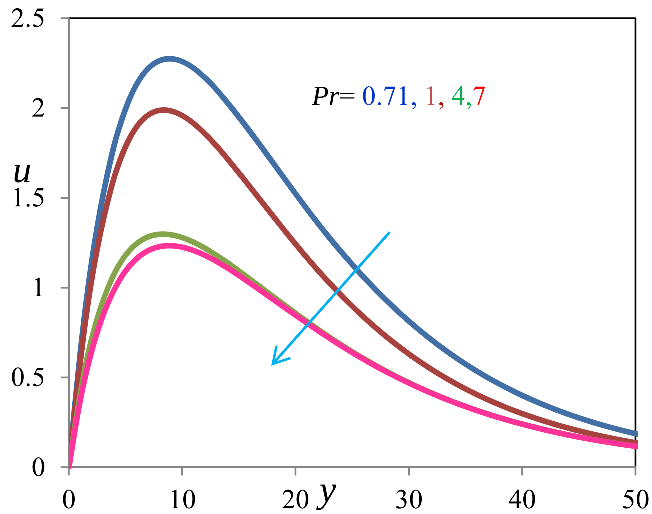

Figure-5(a). Velocity profile for different values of Pr when Gr=5.0, Gm=5.0, M=1.0, K=5.0, Sc=0.6, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

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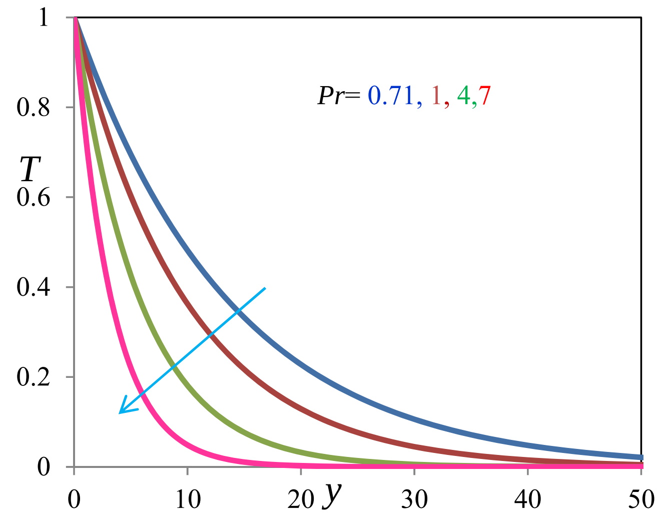

Figure-5(b). Temperature profile for different values of Pr when *Gr* =5.0, *Gm* = 5.0, M=1, K=5.0, Sc=0.6, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

Figure 5(a) and 5(b) illustrate the velocity and a temperature profile for different values of the prandtl number Pr. It is observed that an increase in the prandtl number decreases the velocity and temperature. Also as we move away from the boundary, the prandtl number has not much of significant influence on the temperature.

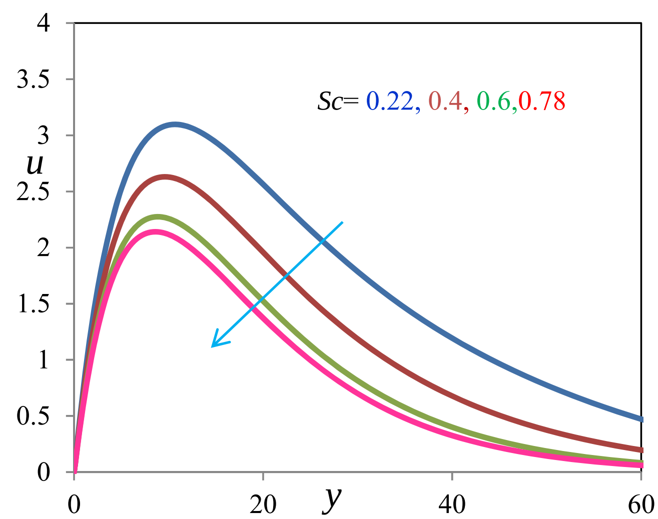

Figure-6(a). Velocity profile for different values of Sc when Gr=5.0, *Gm* =5.0, M=1, K=5.0, Pr=0.71, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

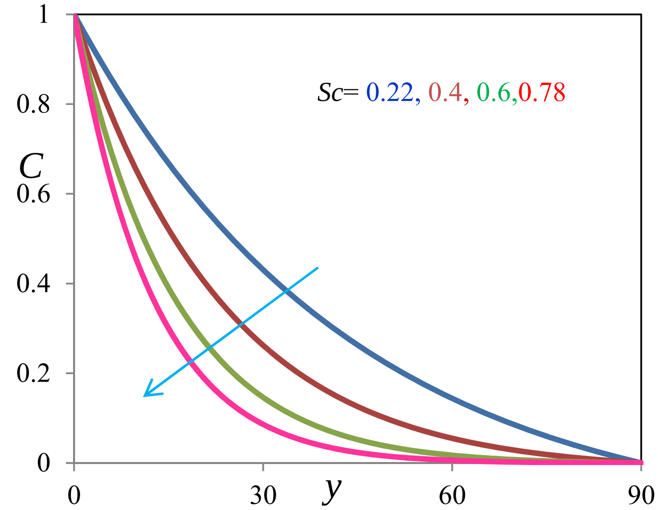

Figure-6(b). Concentration profile for different values of Sc. when *Gr* =5.0, *Gm* =5.0, M=1, K=5.0, Pr=0.71, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Kr=1.0.

The influence of the Schmidt number Sc on the velocity and concentration profiles are plotted in Figure-6(a) and Figure-6(b) respectively. When Schmidt number increases the velocity and concentration decreases. Further it is observed that Sc does not contribute much to the concentration field as we move far away from the boundary surface.

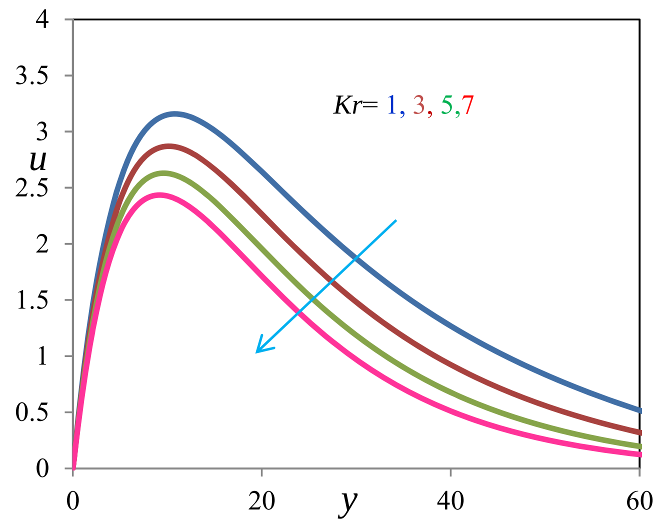

Figure-7(a). Velocity profile for different values of Kr when *Gr* =5.0, *Gm* =5.0, M=1, K=5.0, Pr=0.71, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Sc=0.6.

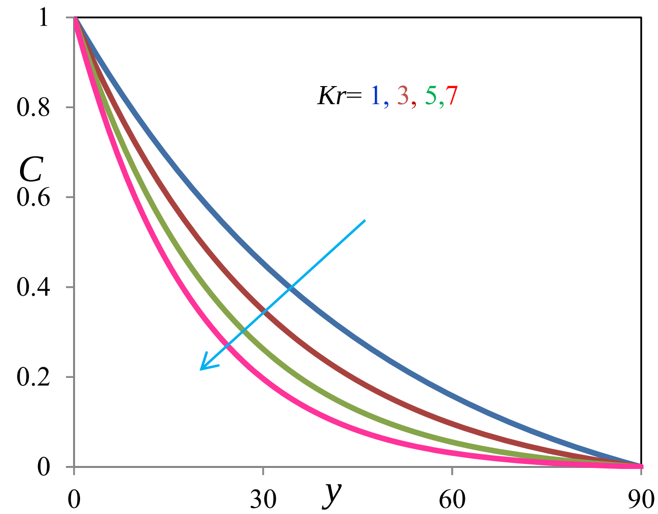

Figure-7(b). Concentration profile for different values of Kr. When *Gr* =5.0, *Gm* =5.0, M=1, K=5.0, Pr=0.71, A=0.01, $\varepsilon = 0.002$, n=1.0,t=1.0,Sc=0.6.

It is clearly observed that the velocity profile and concentration profile decreases by increasing the chemical reaction parameter, shown in Figures 7(a) and 7(b).

CONCLUSIONS

In this paper we have studied the chemical reaction effects on MHD flow past a vertical plate with variable suction. From present numerical study the following conclusions can be drawn:

a) The velocity increases with the increase of *Gr* and *Gm*.
b) The velocity decreases with an increase in the magnetic parameter.
c) The velocity increases with an increase in the permeability of the porous medium parameter.
d) Increasing the prandtl number substantially decreases the velocity and the temperature profiles.
e) The velocity as well as concentration decreases with an increase in the Schmidt number.
f) The velocity as well as concentration decreases with an increase in the chemical reaction parameter.

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