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APPROXIMATE CIRCUIT MODEL FOR ZERO-SEQUENCE CURRENT ESTIMATION IN ASYMMETRICAL THREE-WIREPOWER NETWORKS

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ABSTRACT

Electrical power systems in steady-state conditions can be effectively analyzed by means of the symmetrical component transformation leading to three uncoupled sequence circuits. The main assumption underlying such an approach consists in the phase symmetry of the power system. Such hypothesis, however, is not always met in modern power systems. In fact, three-phase lines can be asymmetrical because of the asymmetrical geometric arrangement of the three conductors. Short lines cannot be transposed and therefore line asymmetry must be taken into account in the system analysis. When complex power systems are considered, the zero-sequence circuit comes into play even if the lines consist of three wires. This is the case of interconnected three-wire lines. In fact, in case of line asymmetry, a zero-sequence current can circulate within the loop involving such line. In this paper an approximate and effective circuit approach is derived to evaluate the zero-sequence current due to line asymmetry in interconnected three-wire lines. The approximate circuit model derived in the paper is validated by means of numerical simulation of a power system including interconnected lines with asymmetrical behavior.

Keywords: power quality, power system analysis, power system simulation, transmission lines.

INTRODUCTION

It is well known that electrical power systems in steady-state conditions can be effectively analyzed by means of the symmetrical component transformation leading to three uncoupled modal circuits (i.e., the so-called positive, negative, and zero-sequence circuits) [1]-[4]. Indeed, such an approach can be effectively extended to the case of non-sinusoidal steady state which is a very common condition for modern power systems [5]-[6]. The main assumption underlying the above mentioned approach consists in the phase symmetry of the power system, i.e., each of the three phases must be equal to the others and the mutual coupling of each phase pair must be the same. Such hypothesis, however, is not always met in modern power systems for several reasons. First, three-phase lines can be asymmetrical because of the asymmetrical geometric arrangement of the three conductors. This is the case of a line with horizontal arrangement of the three conductors. In fact, in this case mutual coupling between the outer conductors is lower than mutual coupling involving the inner conductor. To face this well-known problem, long lines are segmented in shorter stretches where the three conductors are geometrically transposed. However, this solution is not implemented for short lines, and therefore in this case line asymmetry is an actual problem to be taken into account in the system analysis [7]-[16]. Another reason leading to power system asymmetry is related to the loads. Indeed, modern power systems foresee significant single-phase loads beside conventional three-phase loads. The most important example of large single-phase load is given by high-speed railway lines [17]. In any case, system asymmetry results in coupling of sequence circuits. In case of large system asymmetry the sequence circuit coupling leads to a much more complicated analysis where the advantage of symmetrical component transformation is lost. In case of small asymmetry, however, some approximate methods can be introduced leading to simpler and satisfactory analysis. In this paper, line asymmetry is investigated, and an approximate approach is introduced in order to evaluate the impact of line asymmetry on the zero-sequence current. Indeed, in previous works, coupling of sequence circuits due to line asymmetry was already investigated with reference to positive and negative sequence circuits [7]-[9]. In such works a radial structure of the power system was assumed, and therefore in case of three-wire lines the zero-sequence circuit does not require investigation because it is an open circuit. When more complex power systems are considered, however, the zero-sequence circuit comes into play even if the lines consist of three wires. This is the case of interconnected three-wire lines. In fact, in case of line asymmetry, a zero-sequence current can circulate within the loop involving such line. In this paper an approximate and effective circuit approach is derived to evaluate the zero-sequence current due to line asymmetry in interconnected three-wire lines. The proposed approach is based on a principle commonly used in other sectors of electrical engineering, such as, for example, the approximate analysis of crosstalk in the transmission lines context [18]-[20]. The main idea underlying the approximate approach consists in assuming a unidirectional nature of coupling between the source and the victim circuits. More specifically, assuming weak coupling, the variables of the source circuit are evaluated regardless the mutual coupling. Such variables are then used to evaluate the impact on the victim circuit by assuming no feedback on the source circuit. In this paper it is shown that the positive-sequence circuit acts as the source circuit, whereas the zero-sequence circuit is the

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victim circuit. The approximate circuit model derived in the paper is validated by means of numerical simulation of a power system consisting in two interconnected lines, one symmetrical and one asymmetrical. The zero-sequence current circulating in the loop, due to the line asymmetry, is evaluated through the proposed approximate approach and compared with numerical simulations

BACKGROUND

Symmetrical three-phase power systems can be effectively analyzed through the well-known symmetrical component transformation [1]-[4]. Indeed, even in the case of mutual coupling between the phases, the assumption of symmetrical system results in three uncoupled sequence circuits.

The transformation matrix, in its rational form, is given by

$$\mathbf{S} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

where

$$\alpha = e^{j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{2}$$

and $\alpha^2 = \alpha^*$. The transformation matrix is a Hermitian matrix, i.e., $\mathbf{S}^{-1} = \mathbf{S}^{T*}$.

The symmetrical components transformation when applied to phasor voltages provides

$$\begin{bmatrix} V_+ \\ V_- \\ V_\mathbf{0} \end{bmatrix} = \mathbf{S} \cdot \begin{bmatrix} V_\mathrm{a} \\ V_\mathrm{b} \\ V_\mathrm{c} \end{bmatrix} \tag{3}$$

where V_+, V_-, and V_0 are the positive, negative, and zero sequence voltages. Of course, the same transformation applies to phasor currents.

Symmetrical three-phase passive components (i.e., lines and loads) can be described in terms of an impedance matrix with the following structure

$$\mathbf{Z} = \begin{bmatrix} Z & Z_m & Z_m \\ Z_m & Z & Z_m \\ Z_m & Z_m & Z \end{bmatrix} \tag{4}$$

By defining the column vectors

$$\mathbf{V}_s = \begin{bmatrix} V_+ \\ V_- \\ V_0 \end{bmatrix}, \; \mathbf{I}_s = \begin{bmatrix} I_+ \\ I_- \\ I_0 \end{bmatrix}, \; \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}, \; \mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{5}$$

the transformed current/voltage relationship for a symmetrical passive component can be written

$$\mathbf{V}_s = \mathbf{S}\mathbf{Z}\mathbf{S}^{-1}\mathbf{I}_s = \mathbf{Z}_s\mathbf{I}_s \tag{6}$$

where

$$\mathbf{Z}_s = \begin{bmatrix} Z_+ & 0 & 0 \\ 0 & Z_- & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \tag{7}$$

and

$$\begin{aligned} Z_+ = Z_- &= Z - Z_m \\ Z_0 &= Z + 2Z_m \end{aligned} \tag{8}$$

The diagonal form of the sequence impedance matrix (7) leads to the above-mentioned uncoupled sequence circuits when the transformation is applied to the whole three-phase system.

Power system lines are usually transposed along their length such that each of the three conductors can be considered geometrically equivalent to the others on average basis. In some cases, however (e.g., when the length of the line does not allow transposition), the mutual inductance between each conductor pair cannot be considered the same for each conductor pair [7]-[16]. A typical example is given by a line with a horizontal arrangement of the three conductors. In this case, the mutual inductance between the outer conductors is lower than the two pairs including the inner conductor. This arrangement results in an asymmetric impedance matrix (4) in the out-of-diagonal coefficients 1-3. The impedance matrix in this case can be written

$$\tilde{\boldsymbol{Z}} = \boldsymbol{Z} + j\delta X_m \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{9}$$

where Z is the symmetric part of the impedance matrix, and δX_m is the deviation of the mutual reactance 1-3 due to geometric asymmetry of conductor arrangement.

CIRCUIT MODEL FOR ZERO-SEQUENCE CURRENT DUE TO LINE ASYMMETRY

By applying the symmetrical component transformation to (9), the symmetric part Z of the impedance matrix results in the diagonal matrix (7), whereas the asymmetrical part proportional to δX_m results in a full matrix. This means that line asymmetry results in coupling of sequence voltages/currents in vectors V_S and I_S. In particular, from a rigorous viewpoint, evaluation of zero-sequence current I_0 would require the solution of three mutually coupled circuits. In this paper an approximate approach is introduced in order to avoid the solution of such a demanding coupled problem.

The first assumption is that the voltage source of the power system under analysis is characterized by a

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dominant positive-sequence component. This is a reasonable assumption in most of actual systems. In the ideal case where the negative-sequence component of the source is equal to zero, the whole negative-sequence circuit is not fed. This leads to the approximate assumption of neglecting the impact of the interaction of the negative-sequence circuit with the positive and the zero-sequence circuits. As far as only positive and zero-sequence circuits are considered, a second approximation is introduced. Following a principle used in other sectors of electrical engineering (e.g., in some electromagnetic compatibility issues such as crosstalk in transmission lines [18]-[20]), the interaction of a source of interference with a victim is described by assuming no feedback from victim to source. More specifically, an approximate analysis is performed by solving the circuit of the source of interference by neglecting the presence of the victim. Then, the calculated interfering variable is used to evaluate the impact on the victim circuit. By using this principle for the problem investigated in this paper, the source of interference is the positive-sequence circuit, whereas the victim is represented by the zero-sequence circuit. From the above mentioned assumptions, by applying the symmetrical component transformation to (9) we can readily derive the following approximate results:

$$\begin{aligned} V_+ &\cong Z_+ I_+ \\ V_0 &\cong Z_0 I_0 - j\alpha^* \frac{\delta X_m}{3} I_+ \end{aligned} \tag{10}$$

The circuit representation of (10) is shown in Figure-1 where the influence of the positive-sequence circuit on the zero-sequence circuit is taken into account by a controlled voltage source driven by the positive-sequence current evaluated as if the system was symmetrical. From a practical viewpoint, application of the approximate model (10) requires two steps. First, solution of the positive-sequence circuit by assuming symmetrical power system. Second, solution of the zero-sequence circuit by placing a controlled source in each place where an asymmetrical line is located. Clearly, the proposed model can be used also in the case of more than one asymmetrical line belonging to the same power network. Future work could be devoted to treat such multiple line-asymmetries as noise sources in the zero-sequence circuit [21].

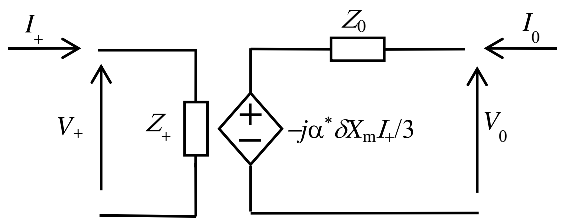

Figure-1. Approximate circuit model for the interaction between positive and zero-sequence circuits due to line asymmetry.

NUMERICAL VALIDATION

Predictions through the approximate model (10) were validated by means of a Simulink® implementation of a three-wire power network including one stretch of an asymmetrical (untransposed) line (see Figure-2). The power system is based on the network described in [9], [11], [14]. In particular, the source was a 12.47 kV balanced three-phase source, whereas the stretch of asymmetrical line was a three-phase line with length 3.2187 km and horizontal arrangement of conductors such that $\delta X_m = -0.1683\ \Omega$. The per-unit-length impedance matrix of the asymmetrical line is reported here for reference:

$$\tilde{\mathbf{Z}} = \begin{bmatrix} 0.2494 + j0.8748 & 0.0592 + j0.4985 & 0.0592 + j0.4462 \\ 0.0592 + j0.4985 & 0.2494 + j0.8748 & 0.0592 + j0.4985 \\ 0.0592 + j0.4462 & 0.0592 + j0.4985 & 0.2494 + j0.8748 \end{bmatrix} \left[\frac{\Omega}{km}\right] \tag{11}$$

The parallel symmetrical line included in the network shown in Figure-2 was the same in length and such that $\delta X_m = 0$. The load was a floating star-connected load characterized by lagging power factor 0.9, and apparent power ranging from 10 MVA to 100 MVA. Figure-3 shows the positive and the zero-sequence circuits, including the controlled voltage source in the zero-sequence equivalent of the asymmetrical line according to (10). Analytical estimation of the zero-sequence current circulating in the zero-sequence impedance loop in Figure-3 is straightforward. In fact, by taking into account that for the specific example under analysis $Z_{10} = Z_{20} = Z_0$ and $Z_{1+} = Z_{2+} = Z_+$ we obtain:

$$|I_0| = \left| \frac{-j\alpha^* \frac{\delta X_m}{3} I_{2+}}{2Z_0} \right| = \left| \frac{\delta X_m E_+}{6Z_0(Z_+ + 2Z_L)} \right| \tag{12}$$

The approximate analytical results provided by (12) (solid line) are compared in Figure-4 with numerical results (markers) obtained through the Simulink implementation shown in Figure-2. Agreement between analytical and numerical results is very good, even though the accuracy is worsening as the load power increases.

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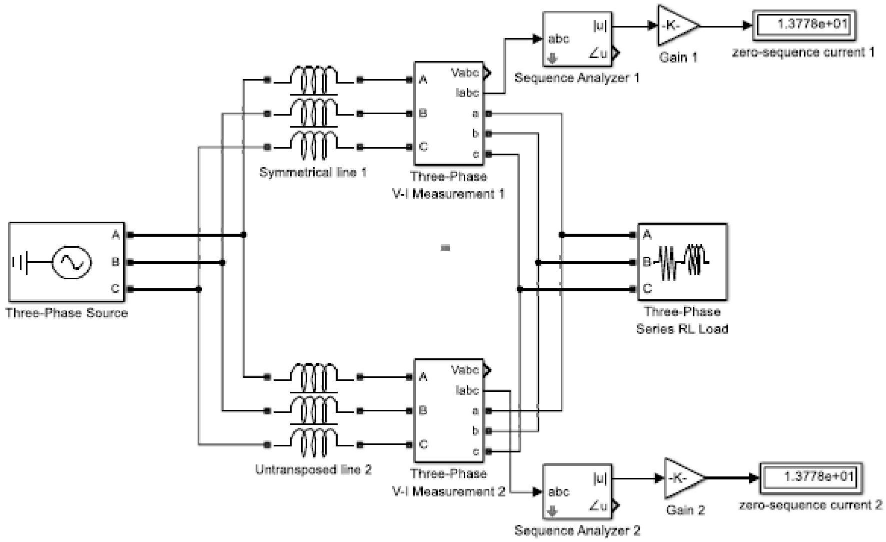

Figure-2. Simulink® implementation of a power network including a stretch of asymmetrical (untransposed) line.

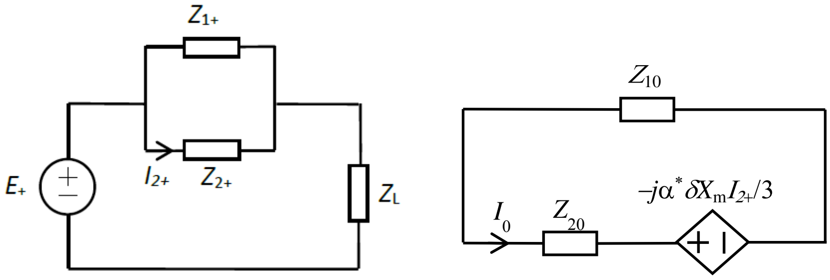

Figure-3. Positive (left) and zero (right) sequence circuits related to the power network represented in Figure-2.The controlled voltage source takes into account line asymmetry according to (10).

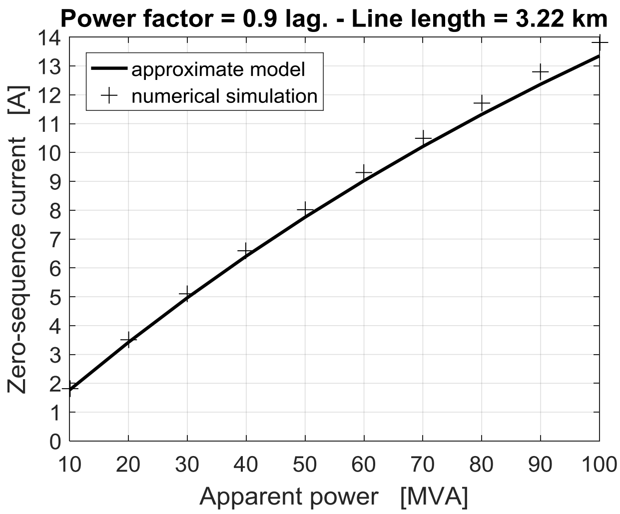

Figure-4. Comparison between analytical (i.e., the circuit model in Figure-3) and numerical (i.e., Simulink implementation in Figure-2) results for the zero-sequence current circulating in the line loop, originating from asymmetry of one of the two lines, as a function of the load apparent power.

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