



INFLUENCE OF TRANSPIRATION AND HALL EFFECTS ON UNSTEADY MHD FREE CONVECTION FLUID FLOW OVER AN INFINITE VERTICAL PLATE

R. Srinivasa Raju¹, N. Venktesh², M. Anil kumar² and G. Jitender³

¹Department of Mathematics, GITAM University, Hyderabad Campus, Rudraram (v), Telangana State, India ²Department of Mathematics, Anurag Group of Institutions, Venkatapur, Ghatkesar, RR District, Telangana State, India ³VNR Vidya Jyothi Institute of Engineering and Technology, Bachupally Hyderabad, India E-Mail: <u>nvenkatesh.nitw@gmail.com</u>

ABSTRACT

In this paper, we studied the influence of Hall current and transpiration on an unsteady MHD free convection electrically conducting incompressible fluid flow along past a infinite vertical plate embedded in a porous medium with presence of viscous dissipation. An efficient Finite Element Method (FEM) is employed to solve non-dimensional PDE's for primary, secondary velocity and temperature of fluid flow with influence of emerging dimensionless parameters, Transpiration cooling parameter, Hall current, Magnetic parameter, Prandtl Number and Eckert number. The velocity, temperature and concentration fields are physically interpreted through graphical forms.

Keywords: MHD Flows, hall current, viscous dissipation and FEM.

1. INTRODUCTION

The Hall Effect is making of the a voltage difference across an electrical conductor. transverse to an electric current in the conductor and an electromagnetic field is perpendicular to the current. It is found by Edwin Hall et al. [1]. The current development of magnetohydrodynamics application is toward a strong magnetic field and toward a low density of the gas. Under this condition, the Hall current becomes important. That importance studied by many researchers Singh et al. [2], Mbeledogu et al. [3], Abuga et al. [4], seth et al. [5]. Recently Srinivasaraju et al. [6] analyzed the influence of hall current, thermal diffusion and diffusion thermo on magnetohydrodynamic fluid flow past an infinite moving vertical plate using. Finite element method.

Reynolds's [7] experimental and Maxwell's [8] independent theoretical studies were the first to describe the phenomenon of thermal transpiration (creep), where the fluid starts creeping in the direction from cold towards hot. Due to consequence of this cold to hot flow, a pressure difference between the hot and cold ends of the capillary can be established and a pressure return flow will occur, partially or completely balancing the thermal creep flow.

Thermal transpiration was originally investigated experimentally by Reynolds [7] in 1880, based on earlier experimental observations from 1831 to 1863 by Graham [9]. In 1910, Knudsen [10] reported the first multistage thermal transpiration pump, which successfully achieved a pressure increase of a factor of ten. Recently Nishanth *et al.* [11] demonstrate an effective mean free path modeling in the Knudsen layer and investigate the fluid flow characteristics associated with thermal transpiration of rarefied gas in the slip and transition flow regimes.

In view of the above investigation, influence of Hall current and Transpiration effect on an unsteady MHD free convection, an electrically conducting incompressible fluid flow along an infinite vertical plate embedded in a porous medium studied with presence of viscous dissipation. An efficient Finite Element Method (FEM) is employed to solve non-dimensional PDE's for primary, secondary velocity and temperature of fluid flow with influence of emerging dimensionless parameters, Transpiration cooling parameter, Hall current, Magnetic parameter, Prandtl Number and Eckert number. The flow field is physically interpreted through graphical forms.

2. MATHEMATICAL FORMULATION

An unsteady free convection flow of an electrically conducting viscous incompressible fluid along a porous flat plate has been considered.

a) In Cartesian coordinate system, let x'-axis is taken to be along the plate and the y'-axis normal to the plate. Since the plate is considered infinite in x'-direction, hence all physical quantities will be independent of x'-direction.

b) Let the components of velocity along x' and y' axes be u' and v' which are chosen in the upward direction along the plate and normal to the plate respectively.

c) A uniform magnetic field of magnitude B_o is applied normal to the plate. The transverse applied magnetic field and magnetic Reynold's number are assumed to be very small, so that the induced magnetic field is negligible.

d) The polarization effects are assumed to be negligible and hence the electric field is also negligible.

e) Initially, for time $t' \leq 0$, the plate and the fluid are maintained at the same constant temperature (T'_{∞}) in a stationary condition.

f) When t' > 0, the wall is maintained at constant temperature (T'_w) higher than the ambient temperature (T'_w) respectively.

g) Using the relation $\nabla .\overline{H} = 0$ for the magnetic field $\overline{H} = (H_x, H_y, H_z)$, we obtain $H_y = \text{constant} =$ H_o (say) where H_o is the externally applied transverse magnetic field so that $\overline{H} = (0, H_o, 0)$. The equation of conservation of electric charge $\nabla .\overline{J} = 0$ gives $j_y =$ constant, where $\overline{J} = (j_{x_x}, j_{y_y}, j_z)$. We further assume that the plate is non – conducting. This implies $j_y = 0$ at the plate and hence zero everywhere.

h) When the strength of magnetic field is very large, the generalized Ohm's law in the absence of electric field takes the following form:

$$\overline{J} + \frac{\omega_e \tau_e}{B_o} \overline{J} \times \overline{H} = \sigma \left(\mu_e \overline{V} \times \overline{H} + \frac{1}{e n_e} \nabla P_e \right)$$
(1)

i) Under the assumption that the electron pressure (for weakly ionized gas), the thermo - electric pressure and ion - slip conditions are negligible, equation (1) becomes:

$$j_x = \frac{\sigma \mu_e H_o}{1 + m^2} (mu' - w') \text{ and } j_z = \frac{\sigma \mu_e H_o}{1 + m^2} (mw' + u')$$
 (2)

Where u' is the x'-component of \overline{V} , w' is the z'component of \overline{V} and $m(=\omega_e \tau_e)$ is the hall parameter.



Figure-1. Geometry of the problem

Within the above framework, the equations which govern the flow under the usual Boussinesq approximation are as follows:

$$\frac{\partial v'}{\partial y'} = 0 \tag{3}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \mu^2 {}_e H_o^2}{\rho (1+m^2)} (u'+mw') + g\beta (T'-T_{\infty}') (4)$$

$$\frac{\partial w'}{\partial t'} + v' \frac{\partial w'}{\partial y'} = v \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma \mu^2 H_o^2}{\rho (1 + m^2)} (w' - mu')$$
(5)

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial {y'}^2} + \frac{v}{C_p} \left(\frac{\partial u'}{\partial y'}\right)^2 \tag{6}$$

The corresponding initial and boundary conditions are:

$$\begin{aligned} t' &\leq 0: \ u' = 0, \quad w' = 0, \ T' = T'_{\infty} \ for \ all \quad y' \\ t' &> 0: \begin{cases} u' = 0, \quad w' = 0, \ T' = T'_{w} \ at \quad y' = 0 \\ u' = 0, \ w' = 0, \ T' = T'_{\infty} \ as \quad y' \to \infty \end{cases}$$
 (7)

The non - dimensional quantities introduced in the equations (3) - (6) are:

$$t = \frac{t'U_{o}^{2}}{\nu}, \quad y = \frac{y'U_{o}^{2}}{\nu}, (u, v, w) = \frac{(u', v', w')}{U_{o}}, \\ \theta = \frac{(T' - T'_{w})}{(T'_{w} - T'_{w})}, \quad M = \frac{\sigma\mu_{e}^{2}H_{o}^{2}U}{\rho U_{o}^{2}}$$

$$Ec = \frac{U_{o}^{2}}{C_{p}(T'_{w} - T'_{w})}, \quad Gr = \frac{\nu g\beta(T'_{w} - T'_{w})}{U_{o}^{3}}, \\ Pr = \frac{\mu C_{p}}{k}$$

$$(8)$$

Where U_o is the reference velocity. The governing equations can be obtained in the dimensionless form as:

$$\frac{\partial v}{\partial y} = 0 \tag{9}$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{M}{(1+m^2)} (u+mw) + (Gr)\theta \quad (10)$$

$$\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{\left(1 + m^2\right)} \left(w - mu\right) \tag{11}$$

$$\frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + (Ec) \left(\frac{\partial u}{\partial y}\right)^2$$
(12)

The initial and boundary conditions (7) in the non - dimensional form are:

$$t \le 0 : u = 0, \ w = 0, \ \theta = 0 \quad for \ all \quad y \\ t > 0 : \begin{cases} u = 0, \ w = 0, \ \theta = 1 & at \ y = 0 \\ u = 0, \ w = 0, \ \theta = 0 & as \ y \to \infty \end{cases}$$
(13)



From equation (9), we see that ν is either constant or a function of time *t*. Similarly solutions of equations (10)-(12) with the boundary conditions (13) exists only if we take

$$\nu = \lambda t^{\frac{-1}{2}} \tag{14}$$

Where λ is a non - dimensional transpiration parameter? For suction $\lambda > 0$ and for blowing $\lambda < 0$. From (14), it can be observed that the assumption is valid only for small values of time variable.

3. METHOD OF SOLUTION

3.1. Finite element method: The finite element method (FEM) is a numerical and computer based technique for solving a variety of practical engineering problems A number of physical problems are solved by transforming into a matrix equation. The prime feature of FEM is its ability to advise the geometry or the media of the problem to be analyze with great flexibility. This is because the discretization of the domain of the problem which is performed using highly flexible uniform or non uniform or elements that can be easily to illustrate the complex shapes. The method primarily consists in assuming the piecewise continuous function for the solution and obtain the parameters of the functions in a way that reduces the error in the solution. The steps are involved in the FEM as follows.

Step 1: Discretization of the domain: The basic concept of the FEM is to divide the domain or the region of the problem divided into small connected patches is called finite elements. The collection or group of elements is called the finite element mesh. These finite elements are connected in a non overlapping manner, such that they completely cover the entire space of the problem.

Step 2: Generation of the element equations: A

typical element is isolated from the mesh and the variational formulation of the given problem is constructed over the typical element.

- a) Over an element, an approximate solution of the variational problem is supposed, and by substituting this in the system, the element equations are generated.
- b) The element matrix, which is also called as stiffness matrix, is constructed by using the element interpolation functions.

Step 3: Assembly of the element equations: The system of algebraic equations so obtained are assembled by imposing the inter element continuity conditions. This yields a number of algebraic equations called as the global finite element model, which governs the entire domain.

Step 4: Imposition of the boundary conditions: On the assembled algebraic equations, the Dirichlet and Neumann boundary conditions (13) are imposed.

Step 5: Solution of assembled equations: The assembled equations are obtained and this can be solved by any of the numerical techniques, namely, Gauss elimination method, LU decomposition method, and the final matrix equation can be solved by a direct or indirect (iterative) method. For computational purposes, the coordinate y is varied from 0 to $y_{max} = 1$, where y_{max} represents infinity *i.e.*, external to the momentum, energy boundary layers.



Figure-2. Effect of magnetic parameter M on primary velocity and secondary velocity profiles.

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Figure-3. Effect of Hall parameter m on primary velocity and secondary velocity profiles.

In one-dimensional space, linear, quadratic, or element of higher order can be taken. The complete flow domain is divided into 10000 quadratic elements of equivalent size. Each element is three-noded, and therefore the entire domain contains 20001 nodes. Four functions are to be calculated. After assembly of the element equations, we obtain a system of 80004 equations which are nonlinear. After applying the boundary conditions, a system of linear equations are obtained which are solved by using the Gauss elimination method, maintaining an accuracy of 0.00001. A convergence principle based on the comparative difference between the current and previous iterations are employed. When these differences are satisfy the desired accuracy, the solutions are assumed to have been converged and the iterative process is terminated. The Gaussian quadrature is used for solving the integrations. The desired code of the algorithm has been executed in MATLAB. Excellent convergence is achieved for all the results.

4. RESULTS AND DISCUSSIONS

The effect of Hall current and transpiration on an unsteady magnetohydrodynamic flow and mass transfer of an electrically conducting incompressible non - Newtonian viscous dissipative fluid along an infinite vertical porous plate has been studied and solved by using finite element method. The effects of material parameters such as Hall parameter (m), Hartmann number (M), Transpiration cooling parameter (λ) , Prandtl number (\Pr) and Eckert number (Ec) separately in order to clearly observe their respective effects on the primary velocity, secondary velocity and temperature profiles of the flow.



Figure-4. Effect of Eckert number Ec on primary velocity, secondary velocity and Temperature profiles.

Figure-2 shows the influence of magnetic field on primary and secondary velocity. Primary velocity decreases and secondary velocity increases as increasing of magnetic parameter. Figure-3 shows the influence of hall current on primary and secondary velocity. Primary and secondary velocity increases in the entire the boundary region as increasing of hall current. Figure-4 shows the influence of Eckert number on primary, secondary velocity and temperature of the fluid. As increasing of Eckert number the heat will be generate. Therefore the Primary velocity, secondary velocity and temperature increases in the entire region as increasing of Eckert number, it is evident in the Figure-4. Figure-5 shows the influence of Prandtl number on primary, secondary velocity and temperature of the fluid. The primary velocity, secondary velocity and temperature of the fluid decreases in the entire boundary region as increasing of Prandtl number. Figure-6 shows the influence of transpiration parameter on primary velocity, secondary velocity and temperature of the fluid. The primary velocity, secondary velocity and temperature of the fluid. The primary velocity, secondary velocity and temperature of the fluid decreases as increasing of transpiration parameter.



Figure-5. Effect of Prandtl number Pr on primary velocity, secondary velocity and temperature profiles.



Figure-6. Effect of transpiration parameter on primary velocity, secondary velocity and temperature profiles.

5. CONCLUSIONS

Influence of Hall current and transpiration on an unsteady MHD free convection, an electrically conducting incompressible fluid flow along an infinite vertical plate embedded in a porous medium with presence of viscous dissipation studied through an efficient computational finite element technique.

The following conclusions are drawn from the above study:

- Primary velocity of the fluid increases as increasing of Hall current Eckert number while decreases as increasing of Magnetic parameter, Prandtl number and Transpiration parameter.
- Secondary velocity of the fluid increases as increasing of Hall current, Eckert number and Magnetic parameter while decreases as increasing of Prandtl number and Transpiration parameter.
- The fluid temperature increases as increasing of Eckert number while decreases as increasing of Prandtl number and Transpiration parameter.

Nomenclature

- \mathcal{E} Porosity of the porous medium
- θ Dimensionless Temperature away from the plate
- *T* Temperature of the fluid
- T_w Temperature of the plate
- T_{∞} Temperature of the fluid far away from
- u' Velocity component in x' the plate direction
- w' Velocity component in z' direction
- x' Spatial co ordinate along the
- ν Kinematics viscosity

- y' Spatial co ordinate normal to the plate
- α Thermal Diffusivity
- k_e Mean absorption coefficient
- \overline{V} Velocity vector
- *k* Thermal conductivity
- *m* Hall parameter
- σ Electrical conductivity,
- μ Viscosity
- μ_e Magnetic permeability
- ρ Density
- *p* Pressure
- ω_{e} Electron frequency
- τ_e Electron collision time
- U_a Reference velocity
- *e* Electron charge
- *M* Hartmann number
- n_e Number density of the electron
- Pr Prandtl number
- P_e Electron Pressure
- *Gr* Grashof Number
- g Acceleration due to Gravity, 9.81 m/s²
- *Ec* Eckert number
- β Volumetric co efficient of thermal
- λ Non dimensional transpiration parameter

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