ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

EFFECT OF RADIATION ON MHD MIXED CONVECTION FLOW PAST A SEMI INFINITE VERTICAL PLATE

D. R. V. S. R. K. Sastry[1], V. Venkataraman[1], M. Srinivasu[2], S. Balaji[1] and R.Seethalakshmi[1]
[1]Department of Mathematics, SASTRA University, Tirumalaisamudram, Thanjavur, Tamil Nadu, India
[2]Department of Mathematics, Aditya College of Engineering, Surampalem, Andhra Pradesh, India
E-Mail: drvsrksastry@gmail.com

ABSTRACT

In this paper we discuss the effects of radiation, double dispersion and MHD on heat and mass transfer in a mixed convective Newtonian flow over a semi infinite vertical plate embedded in a non-Darcy porous medium. The basic governing equations are made dimensionless by introducing similarity variable and transformations. The resulting equations are then solved numerically by fourth order Runge-Kutta method coupled with shooting technique. Velocity, Temperature and Concentration profiles against the similarity variable are shown graphically. Also the profiles of heat and mass transfer against the mixed convection parameter are shown graphically.

Keywords: mixed convection, double dispersion, heat transfer, mass transfer, radiation.

INTRODUCTION

In the case of a fluid through a porous medium, Thermal and Solutal transport are of a great interest from both theory and application point of view. Heat transfer in the case of homogeneous fluid-saturated porous media has been studied with relation to different applications like dynamics of hot underground springs, terrestrial heat flow through aquifer, hot fluid and ignition front displacements in reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials, and heat exchanges with fluidized beds. Mass transfer in isothermal conditions has been studied with applications to problems of mixing of fresh and salt water in aquifers, miscible displacements in oil reservoirs, spreading of solutes in fluidized beds and crystal washers, salt leaching in soils, etc. An integral approach to the heat and mass transfer by natural convection from vertical plates with variable wall temperature and concentration in porous media saturated with an electrically conducting fluid in the presence of transverse magnetic field has been studied by [1]. The unsteady free convective MHD flow and mass transfer of a viscous, incompressible, electrically conducting fluid past an infinite vertical, non-conducting porous plate with variable temperature was analyzed by [2]. Natural convection from a permeable sphere embedded in a variable porosity porous medium due to thermal dispersion was analyzed by [3]. Dual solutions in mixed convection flow near a stagnation point on a vertical porous plate were investigated by [4]. Combined heat and mass transfer by natural convection under boundary layer approximations has been studied by [5], [6]. Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous medium has been analyzed by [7]. The problem of thermal dispersion and radiation effects on non-Darcy natural convection in a fluid saturated porous medium were studied by [8]. Thermal radiation heat transfer effects on the Rayleigh of gray viscous fluids under the effect of a transverse magnetic field have been investigated by [9]. The steady two-dimensional stagnation-point flow of an incompressible fluid over a stretching sheet by taking into account radiation effects using the Rossell and approximation to model the heat transfer has been investigated by [10]. Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid was studied by [11]. The study of melting effect is considered by many authors. [12] Studied the velocity and temperature fields, the heat transfer rate and the melting layer thickness. The non-linear behavior of non-Newtonian fluids in a porous medium is quite different from that of Newtonian fluids in porous media. The conjugate mixed convection conduction heat transfer of a non-Newtonian power law fluid on a vertical heated plate which is moving in an ambient fluid was noticed by [13]. Analytical method to investigate transient free convection boundary layer flow along a vertical surface embedded in an anisotropic porous medium saturated by a non-Newtonian fluid was presented by [14]. Effect of non-Newtonian natural convection at a melting front in a permeable matrix was discussed by [15]. These results documented the dependence of the local heat transfer rate at the melting front on the type of power law fluid saturating the porous matrix. [16] presented a numerical study for non-Darcy hydro magnetic free convection flow of an electrically conducting and heat generating fluid over a vertical cone and a wedge adjacent to a porous medium. [17] performed an analysis for non-Darcy free convection flow of an electrically conducting fluid over an impermeable vertical plate embedded in a thermally stratified, fluid-saturated porous medium for the case of power law surface temperature. The effect of suction or injection on the free convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium was discussed by [18].

In this paper we studied the effects of radiation, double dispersion and MHD on heat and mass transfer in a mixed convective Newtonian flow over a semi infinite vertical plate embedded in a non-Darcy porous medium.

VOL. 11, NO. 23, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

MATHEMATICAL FORMULATION

Consider a two dimensional MHD mixed convective heat and mass transfer from a semi infinite vertical plate embedded in a non-Darcy porous medium saturated with a Newtonian fluid. The x -coordinate is measured along the plate from its leading edge and the y -coordinate normal to it. A magnetic field is applied in the y - direction, the wall is maintained at constant temperature T_w and constant concentration C_w. The temperature and mass concentration of the ambient medium are assumed to be T_∞ and C_∞ respectively, under these assumptions and using the Boussinesq approximation, the boundary layer equations can be written as

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$\left[1+\left(\frac{K\sigma\mu_e^2H_0^2}{\mu}\right)\right]\frac{\partial u}{\partial y}+\frac{c\sqrt{K}}{\upsilon}\frac{\partial}{\partial y}\left(u^2\right)=\pm\frac{gK}{\upsilon}\left(\beta_T\frac{\partial T}{\partial y}+\beta_C\frac{\partial C}{\partial y}\right) \tag{2}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right)+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Concentration equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_e\frac{\partial C}{\partial y}\right) \tag{4}$$

Boussinesq approximation:

$$\rho=\rho_\infty\left\{1-\beta_T\left(T-T_\infty\right)-\beta_C\left(C-C_\infty\right)\right\} \tag{5}$$

together with the boundary conditions:

$$\left.\begin{array}{l} y=0\ :\ v=0\ ,\ T=T_w\ ,\ C=C_w \\ y\to\infty : u\to u_\infty\ ,\ T=T_\infty\ ,\ C=C_\infty \end{array}\right\} \tag{6}$$

where u and v are velocity components in x and y directions, T is the temperature, K is the permeability constant, C is an empirical constant, υ is the kinematic viscosity, g is the acceleration due to gravity, β_T and β_C are the coefficients of thermal and solute expansions. Ec is the Eckert number, ρ is the density, C_p is the specific heat at constant pressure, M is the magnetic number. The '+' and '-'signs in equation (2) correspond to the flow with aiding and opposing buoyancies respectively. One can define the velocity, temperature and concentration in terms of similarity space variable, $\eta = (y/x)\sqrt{Pe_x}$, as follows:

$$f\left(\eta\right)=\frac{\psi}{\alpha\sqrt{Pe_x}},\ \theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi\left(\eta\right)=\frac{C-C_\infty}{C_w-C_\infty} \tag{7}$$

where ψ is the stream function. With the help of similarity transformations given by (7), one can transform the governing equations as follows:

$$\left(1+M+f'\right)f''=\pm\frac{Ra_x}{Pe_x}\left(\theta'+N\phi'\right) \tag{8}$$

$$\left(1+\frac{4}{3R}\right)\theta''+\frac{1}{2}f\theta'+\Pr E_C\left(f'\right)^2+D\left(f''\theta'+f'\theta''\right)=0 \tag{9}$$

$$\phi''+\frac{1}{2}Lef\phi'+LeB\left(f'\phi''+f''\phi'\right)=0 \tag{10}$$

where

Local Rayleigh number, $Ra_x=\dfrac{Kg\beta_T\left(T_w-T_\infty\right)x}{\upsilon\ \alpha}$,

Magnetic parameter, $M=\dfrac{K\sigma\mu_e^2H_0^2}{\mu}$

Lewis number, $Le=\dfrac{\alpha}{b}$, Peclet number, $Pe_x=\dfrac{u_\infty x}{\alpha}$,

Prandtl number, $\Pr=\dfrac{\upsilon}{\alpha}$

Schmidt number, $Sc=\dfrac{\upsilon}{b}$, Buoyancy ratio, $N=\dfrac{\beta_C\left(C_w-C_\infty\right)}{\beta_T\left(T_w-T_\infty\right)}$, Eckert number, $E_c=\dfrac{u_\infty^2}{c_p\left(T_w-T_\infty\right)}$

Inertia parameter, $F=\dfrac{c\sqrt{K}\,u_\infty}{\upsilon}$

The corresponding boundary conditions are transformed to

$$f\left(0\right)=0,\theta\left(0\right)=\phi\left(0\right)=1,f'\left(\infty\right)=\theta\left(\infty\right)=\phi\left(\infty\right)=0 \tag{11}$$

where prime indicates the differentiation is made with respect to similarity space variable η.

Also the local heat transfer rate is given by

$$q_w=-k_e\left.\left(\frac{\partial T}{\partial y}\right)\right|_{y=0}=-\left(k+k_d\right)\left.\left(\frac{\partial T}{\partial y}\right)\right|_{y=0}$$

where k_e is the effective thermal conductivity of the porous medium which is the sum of the molecular thermal conductivity k and the dispersion thermal conductivity k_d. The modified Nusselt number is defined as

$$\frac{Nu_x}{\left(Ra_x\right)^{\frac{1}{2}}}=-\left[1+\gamma Ra_d f'\left(0\right)\right]\theta'\left(0\right)$$

and also one can define a dimensionless Sherwood number as

$$\frac{Sh_x}{\left(Ra_x\right)^{\frac{1}{2}}}=-\left[1+\zeta\,Ra_d f'\left(0\right)\right]\phi'\left(0\right)$$

VOL. 11, NO. 23, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

RESULTS AND DISCUSSIONS

The dimensionless equations given by (8), (9) and (10) together with the boundary conditions given by (11) are solved numerically by means of Runge-Kutta fourth order method coupled with shooting technique. The effects of magnetic parameter (M) and buoyancy ratio (N) on the fluid velocity, temperature and concentration profiles are illustrated in Figures 1-3, respectively with respect to the following set of parameters: R = 0.4, E = 0.5, P = 0.73, Le = 0.5, Ra/Pe = 1. From Figure-1 it is noticed that increase in the magnetic parameter M reduces the velocity both in presence and absence of double dispersion. Also it is observed that increase in the buoyancy ratio enhances the fluid velocity. From Figure-2 it can be found that temperature is increased with the increase in the magnetic parameter adjacent to the wall and as we move away from the wall this will get reversed. Also from the same figure we found that increase in the buoyancy ratio enhances the temperature in both presence and absence of magnetism. Figure-3 depicts the fact that increase in the magnetic parameter enhances the concentration of the fluid.

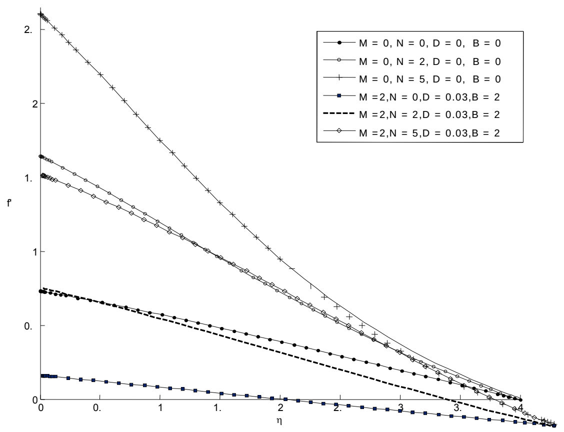

Figure-1. Variation of dimensionless velocity *f'* with similarity variable *η (R = 0.4, E = 0.5, P = 0.73, Le = 0.5, Ra/Pe = 1).*

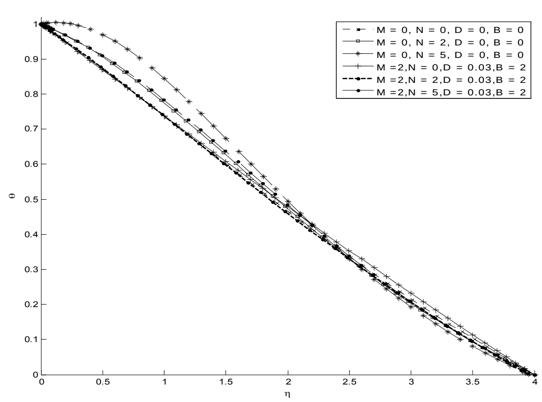

Figure-2. Variation of dimensionless temperature θ with similarity variable *η (R = 0.4, E = 0.5, P = 0.73, Le = 0.5, Ra/Pe = 1).*

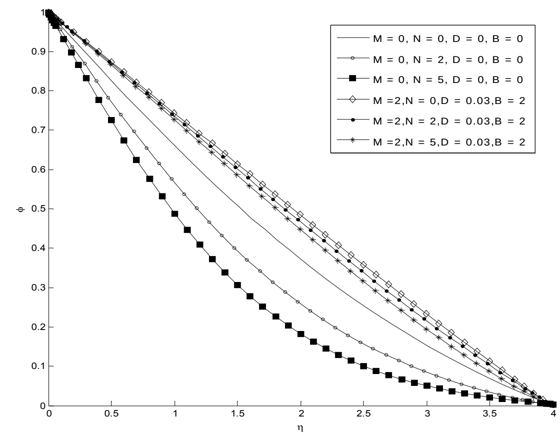

Figure-3. Variation of dimensionless concentration ϕ with similarity variable *η (R = 0.4, E = 0.5, P = 0.73, Le = 0.5, Ra/Pe = 1).*

The effects of mixed convection on velocity and temperature and concentration profiles are shown in Figures 4-6 respectively with respect to the following set of parameters: R = 0.4, E = 0.5, P = 0.73, N= 2, Le = 0.5. From Figure-4 we notice that velocity increases with an increase in the mixed convection parameter Ra/Pe.

It can be also seen from fig.5 that increase in the mixed convection parameter enhances the temperature of the fluid near the wall. Also from fig.6 we notice that increase in the mixed convection parameter reduces the concentration profile. This is observed both in presence and absence of the magnetic parameter. Figure-7 shows that the variation in velocity profile is much significant in non MHD flows, this is due to the absence of dragging forces.

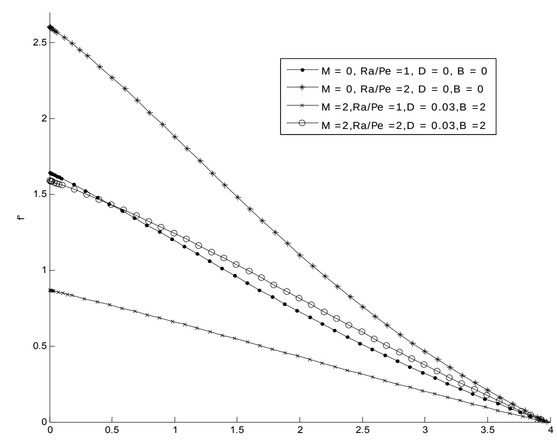

Figure-4. Variation of dimensionless velocity f' with similarity space variable *η (R = 0.4, N = 2, E = 0.5, P = 0.73, Le = 0.5).*

VOL. 11, NO. 23, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

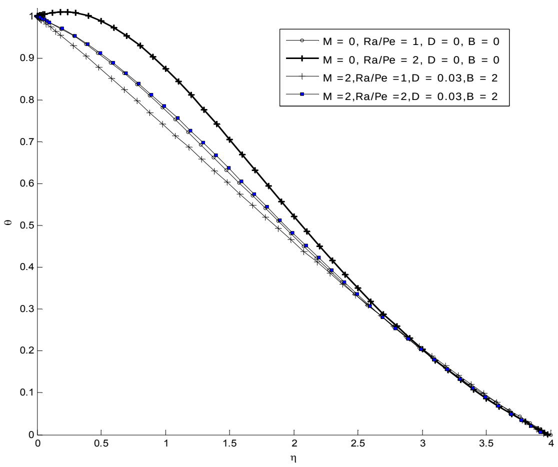

Figure-5. Variation of dimensionless temperature θ with similarity space variable η *(R = 0.4, N = 2, E = 0.5, P = 0.73, Le = 0.5).*

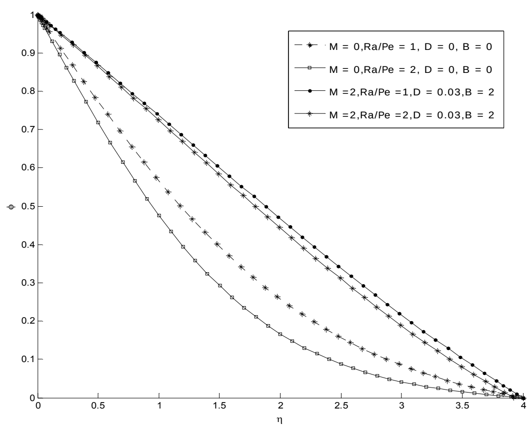

Figure-6. Concentration ϕ with similarity variable *η (R = 0.4,N = 2, E = 0.5, P = 0.73,Le = 0.5).*

Figure-8 depicts that increase in Lewis number Le reduces the concentration of the fluid. This change is more in the absence of magnetic parameter. From Figure-9 we observe that increase in the Radiation parameter R does not produce any significant change in the velocity profile. The variation of the heat and mass transfer coefficients with the mixed convection parameter (Ra/Pe) for various magnetic parameter M, buoyancy ratio N, thermal dispersion D and solutal dispersion coefficient B. It can be observed from the Figure-10 that increase in magnetic parameter reduces the heat transfer rate. In case of opposing buoyancy and double dispersion, the effect of mixed convection on Nusselt number is negligible for a magnetic fluid flow whereas in non- magnetic flows, the Nusselt number slightly decreases with the increase in Ra/Pe. Increase in buoyancy brings a considerable change in the Nusselt number. It reduces the heat transfer coefficient on increase of Ra/Pe. Also it is found from the Figure-11 that Sherwood number increases with the increase in the buoyancy ratio. With increase in the mixed convection parameter, we observe that the mass transfer rate increases.

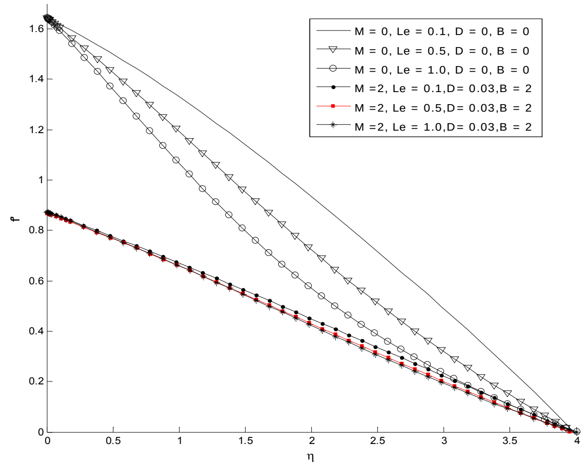

Figure-7. Variation of dimensionless velocity *f'* with similarity space variable η *(R = 0.4, N = 2, E = 0.5, P =0.73, Ra/Pe = 1).*

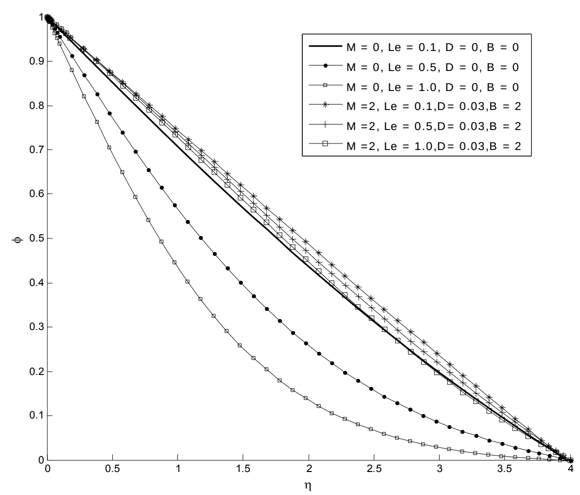

Figure-8. Variation of dimensionless concentration *ϕ* with similarity space variable η *(R =0.4,N =2, E = 0.5, P =0.73, Ra/Pe = 1).*

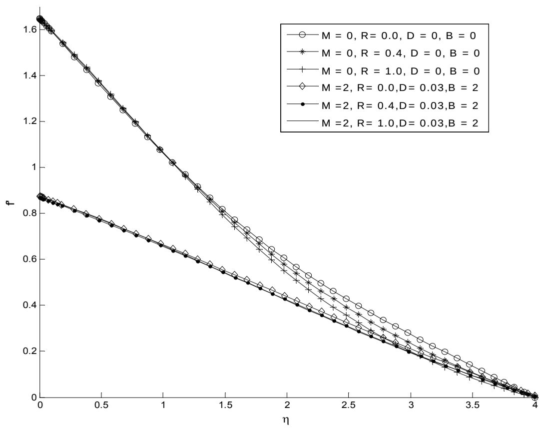

Figure-9. Variation of dimensionless velocity *f'* with similarity space variable η *(Le =1.0, N = 2, E = 0.5, P = 0.73, Ra/Pe = 1).*

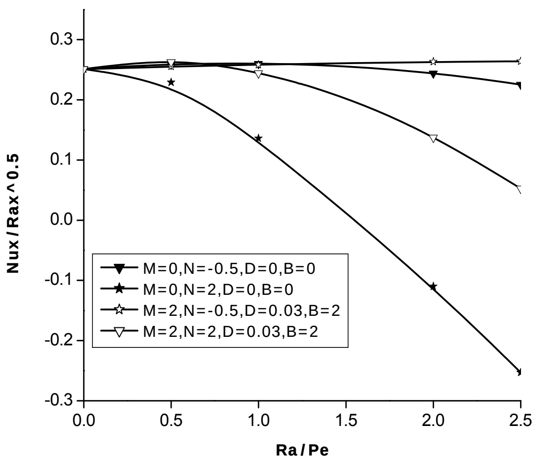

Figure-10. Heat transfer coefficient as a function of *Ra/Pe* when *R=0.4, Le=0.5, P = 0.73, E =0.5.*

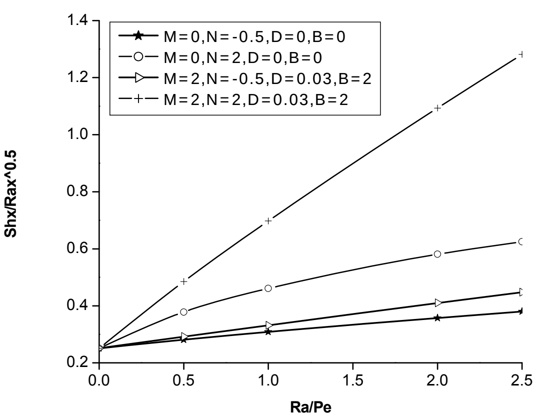

Figure-11. Mass transfer coefficient as a function of *Ra/Pe* when *R=0.4, Le=0.5, P = 0.73, E =0.*

ACKNOWLEDGMENTS

The authors are thankful to the Department of Science and Technology, Government of India for the financial sanction towards this work under FIST programme SR\FST\MSI-107\2015.

REFERENCES

[1] Cheng Ching-Yang. 2005. An integral approach for hydro-magnetic natural convection heat and mass transfer from vertical surfaces with power law variation in wall temperature and concentration in porous medium. Int. Commu. in Heat and Mass Transfer. 32: 204-213.

[2] Sarangi KC and Jose CB. 2005. Unsteady free convective MHD flow and Mass Transfer past a vertical porous plate with variable temperature. Bull. Cal. Math. Soc. 97: 137-146.

[3] EL-Kabeir SMM, El-Hakiem MA and Rashad AM. 2007. Natural Convection from a Permeable Sphere Embedded in a Variable Porosity Porous Medium Due to Thermal Dispersion, Nonlinear Analysis: Modelling and Control. 12: 345-357.

[4] Anuar Ishak, Roslinda Nazar, Norihan Arifin M and Ioan Pop. 2008. Dual Solutions in Mixed Convection Flow near a Stagnation Point on a Vertical Porous Plate, International Journal of Thermal Sciences. 47: 417-422.

[5] Bejan A and Khair KR. 1985. Heat and Mass Transfer by Natural Convection in a Porous Medium, Int. J. Heat Mass Transf. 28: 909-918.

[6] Murthy PVSN and Singh P. 1999. Heat and Mass Transfer by Natural Convection in a Non-Darcy Porous Medium, Acta Mech. 138: 243-254.

[7] Lai FC. 1991. Coupled Heat and Mass Transfer by Mixed Convection From a Vertical Plate in a Saturated Porous Medium, Int. Commun. Heat Mass Transfer. 18: 93-106.

[8] Mohammadein AA and El-Amin MF. 2000. Thermal Dispersion Radiation Effects on Non- Darcy Natural Convection in a Fluid Saturated Porous Medium. Transport Porous Media. 40: 153-163.

[9] Duwairi H and Duwairi RM. 2004. Thermal Radiation Effects on MHD-Rayleigh Flow with Constant Surface Heat Flux. Heat Mass Transfer. 41: 51-57.

[10] Pop SR, Grosan T and Pop I. 2004. Radiation Effects on the Flow near the Stagnation Point of a Stretching Sheet. Tech. Mech. 25: 100-106.

[11] Hayat T, Mustafa M and Pop I. 2010. Heat and Mass Transfer for Soret and Dufour's Effect on Mixed Convection Boundary Layer Flow over a Stretching Vertical Surface in a Porous Medium filled with a Visco-elastic Fluid, Commun. Nonlinear Sci. Numer. Simulat. 15: 1183-1196.

[12] Sparrow EM, Patankar SV, Ramadhyani S. 1977. Analysis of melting in the presence of natural convection in the melt region, ASME J. Heat Transfer. 99: 520-526.

[13] Kumari M, Nath G. 2006. Conjugate mixed convection transport from a moving vertical plate in a non-Newtonian fluid, Int. J. Therm. Sci. 45: 607-614.

[14] Degan G, Akowanou C, and Awanou NC. 2007. Transient natural convection of non-Newtonian fluids about a vertical surface embedded in an anisotropic porous medium, Int. J. Heat Mass Transfer. 50: 4629-4639.

[15] Poulikakos D and Spatz TL. 1988. Non-Newtonian natural convection at a melting front in a permeable solid matrix, Int. Commun. Heat Mass Transfer. 15: 593-603.

[16] Chamkha AJ. 1996. Non-Darcy hydro magnetic free convection from a cone and a wedge in porous media, Int. Comm. Heat Mass Transfer. 23: 875-887.

[17] Afify AA. 2007. Effects of variable viscosity on non-Darcy MHD free convection along a non-isothermal vertical surface in a thermally stratified porous medium. Appl. Math. Modell. 31: 1621-1634.

[18] Ali ME. 2007. The effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation, Int. J. Therm. Sci. 46: 157-163.