VOL. 11, NO. 24, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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THE ENTANGLEMENT DYNAMICS INFIVE AND SIX-QUBITS SYSTEMS WITH HEISENBERG XX MODEL INTERACTION IN THE PRESENCE OF AN ALTERNATING MAGNETIC FIELD

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ABSTRACT

In this study the time evolution of entanglement between the terminal components of five or six-qubits spin chain under the effect of time-dependent magnetic fields is investigated. The interaction between the components of considered spin chain can be express with Heisenberg XX model. The most important result of this study is that if we want to have entanglement with high domain and low vibrations in an appropriate period of time; first, the magnetic field domain should be low, second, the magnetic field periodicity should be low and third we have to give enough time to the system to increase entanglement domain of the system.

Keywords: dynamics of entanglement, magnetic field, spin chain, concurrence.

INTRODUCTION

The study of quantum entanglement in dense matter systems such as spin chains, is taken into consideration [1-2]. The amount of entanglement between two spins in a chain will be changed due to system's interaction with the environment or internal factors that lead to its strengthening or weakening. Among the most important internal and external interactions, we can point to magnetic field and Dzyaloshinskii–Moriya (DM), respectively [3-4]. In addition to the tendency to constant behaviour of entanglement in multiple systems, the dynamic performance has attracted a lot of attention. One of the most important aspects of entanglement dynamic investigation is the spread of entanglement through multiple system. The spread speed of entanglement depends on various parameters and conditions such as system initial state, impurities in the system, coupling coefficient among system constructs and the external magnetic field [5-7].

Entanglement creation among various parts of a multiple system is compared to entanglement transmission through it, is further investigated. Entanglement creation among terminal spins of a spin-1/2 XY chain has been investigated [8]. The time evolution of entanglement in a one-dimensional spin system in the presence of various forms of external magnetic fields is investigated. It is noted that the interaction between the components of these systems is in the form of XY Hamiltonian [9-10]. Also, the time evolution of entanglement for similar systems, considering time-dependent coupling coefficient among neighbouring spins, is investigated [11].

To estimate entanglement between two certain parts of a multiple system, first, we should introduce Hamiltonian and the initial state of the system. Then, for the Hamiltonian of system that is not necessarily time-independent, we obtain the initial state of system as answer of Schrödinger equation [8]. This answer should be the same as the initial state in the initial time. Then, by dyadic multiply of system state, the density operator of system is obtained that is needed to estimate sub-systems' states. Because of the purpose of this study is to estimate entanglements among terminal qubits of the chain; through partial tracing on qubits between them, we convert system's density operator matrix into sub-system's density operator matrix that is consisting of terminal components. Because the system components are spinal, we use concurrence measurer to estimate entanglement between two spinal components [12-15]. We computeconcurrence for different values of system parameters and analyse entanglement by drawing targeted diagrams.

The considered system in this study is a five or six-qubits chain and one of the models can describe this chain spin is Heisenberg XX model.

Hamiltonian for this chain is as follows:

$$\vec{H} = \frac{J}{2}\sum_{i=1}^{N-1}(\vec{S}_i^x.\vec{S}_{i+1}^x + \vec{S}_i^y.\vec{S}_{i+1}^y) + \frac{\vec{B}(t)}{2}\sum_{i=1}^{N}\vec{S}_i \quad (1)$$

This chain includes spin-spin interaction. $\vec{B}(t)$ is the time-dependent external magnetic field and its direction can be variable.

Concurrence

In this study, concurrence measurer will be used to estimate entanglement. This measure is introduced by Wootters for two-component systems (pure and mixed) as follows [12]:

$$C(\hat{\rho}) = Max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right) \quad (2)$$

Where $\lambda_i\left(i = 1,\ 2,\ 3,\ 4\right)$ are eigenvalues of the following matrix that are arranged in descending order.

$$R = \hat{\rho}\left(\hat{\sigma}_y \otimes \hat{\sigma}_y\right)\ \hat{\rho}^*\left(\hat{\sigma}_y \otimes \hat{\sigma}_y\right) \quad (3)$$

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ARPN Journal of Engineering and Applied Sciences

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$\hat{\sigma}_y$ is the y component of Pauli matrix, $\hat{\rho}$ is two-qubits system density matrix, and $\hat{\rho}^*$ is the complex conjugate of the density matrix. $C(\hat{\rho})$ has values between 0 and 1 where for the state with maximum entanglement is 1 and for product states is 0.

Investigating the effect of alternative magnetic field on entanglement of five-qubits spin chain

According to the process that mentioned above, we want to obtain entanglement between terminal components of a five-qubits spin chain under sinusoidal alternating magnetic field with interaction in accordance with Heisenberg XX model that will have Hamiltonian as equation (4) and in initial state, all spins are up.

$$\vec{H} = \frac{J}{2}\sum_{i=1}^{N-1}(\vec{S}_i^x.\vec{S}_{i+1}^x + \vec{S}_i^y.\vec{S}_{i+1}^y) + \frac{B_x \operatorname{Sin}(\frac{2\pi t}{T})}{2}\sum_{i=1}^{N}\vec{S}_i \quad (4)$$

$$\left|\phi^{\uparrow}\right\rangle = \left|\uparrow\uparrow\uparrow\uparrow\uparrow\right\rangle = \left|00000\right\rangle$$

It is show that for such a system, in the absence of magnetic field and through applying the field along z, the entanglement will be zero.

First, we consider a state where the magnetic field is in x direction with low and constant domain, because the more the magnetic field, the more the curve irregularities causes. We have $B_x = 0.05$ and $J = 1$ and the time interval for t is between 0 and 440.

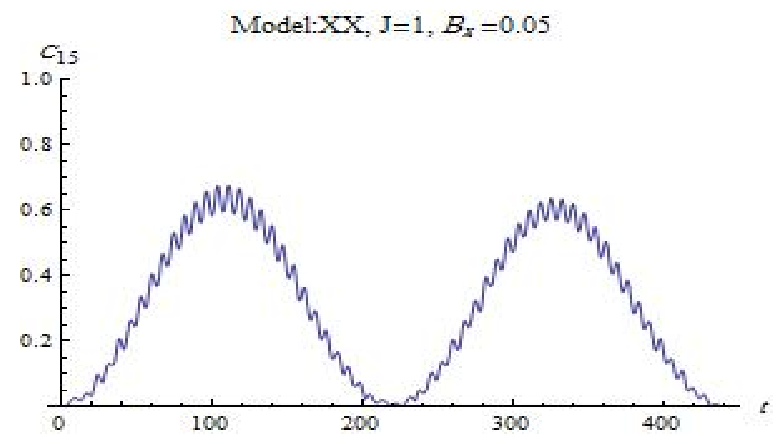

Figure-1. The time evolution of C_{15} for five-qubits chain.

Figure-1 shows that entanglement versus the field along x has two main peaks between 0 and 440 which are rough.

Now, with applying sinusoidal field we want to decrease these rippling. In equation (4) with applying sinusoidal field with different periodicities, we encounter necessary graphs in Figures (2) and (3). In low periodicities, the entanglement domain is low but with the increase of the periodicity, entanglement increases, too. Also, the total period of entanglement is changed.

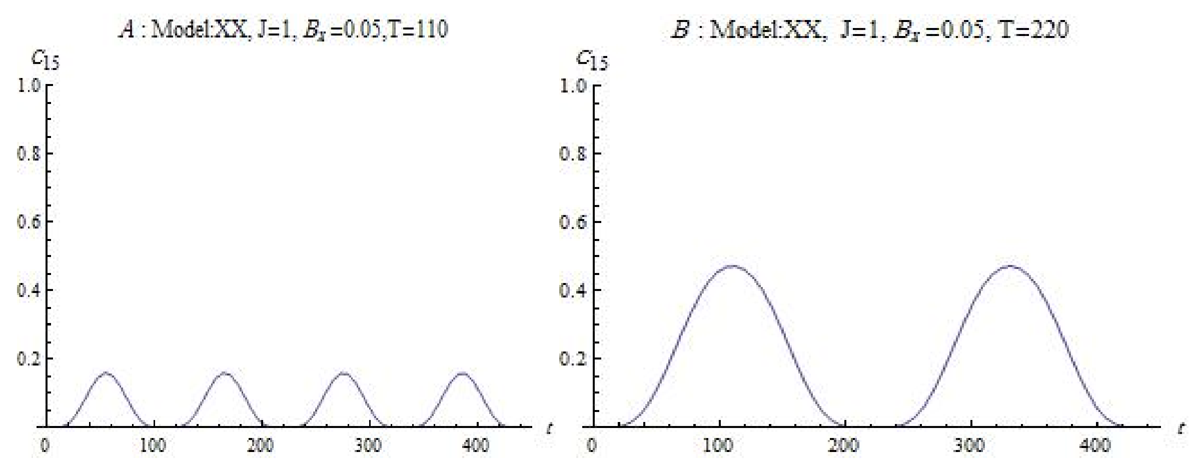

Figure-2. The time evolution of C_{15} for $T = 110$ and $T = 220$ for five-qubits chain.

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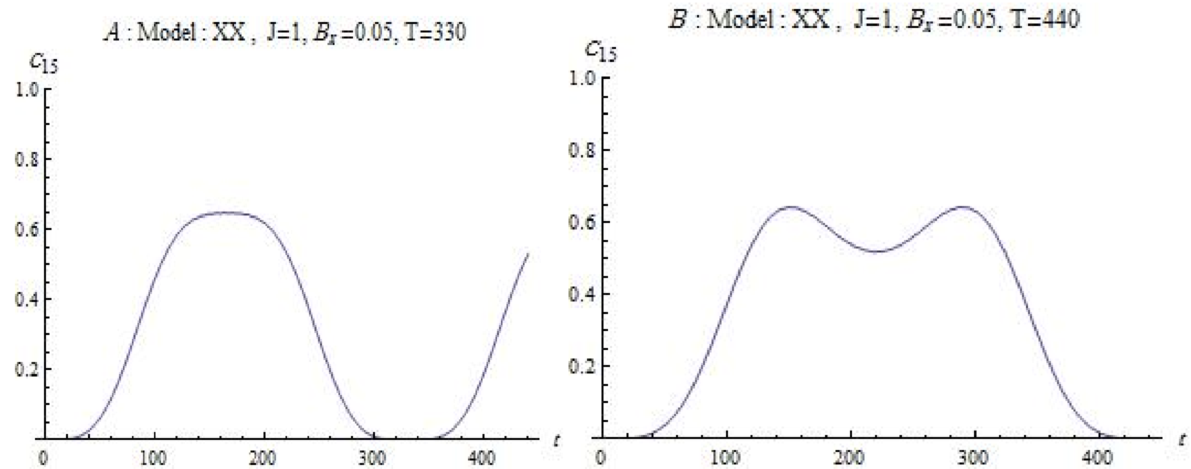

Figure-3. The time evolution of C_{15} for $T = 330$ and $T = 440$ for five-qubits chain.

The main feature of this graph is flatness compared to the constant field state. Of course, in peaks, there is a little fluctuation that is relatively small.

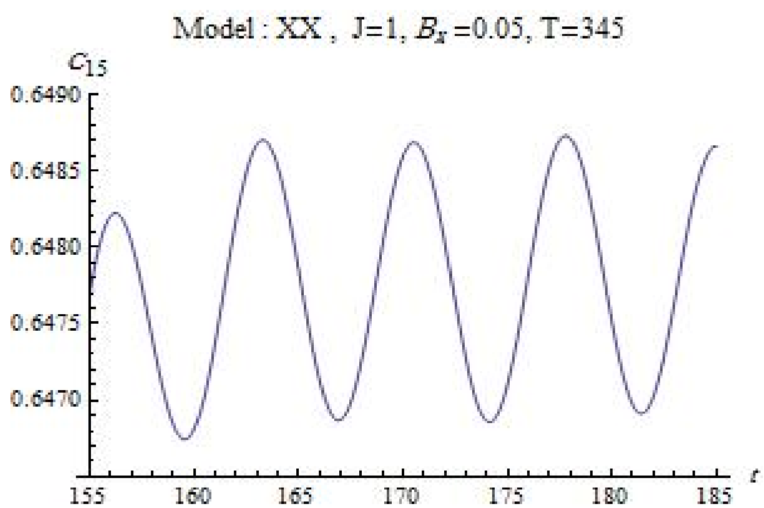

Figure-4. The time evolution of C_{15} for $T = 345$ for five-qubits chain.

For specific T (about 345), an interesting phenomenon occurs (Figure-4). In an interval of time (155-185), entanglement is constant (low ripple). We have drawn this interval with high resolution in Figure-5 that indicates entanglement is practically constant in this interval.

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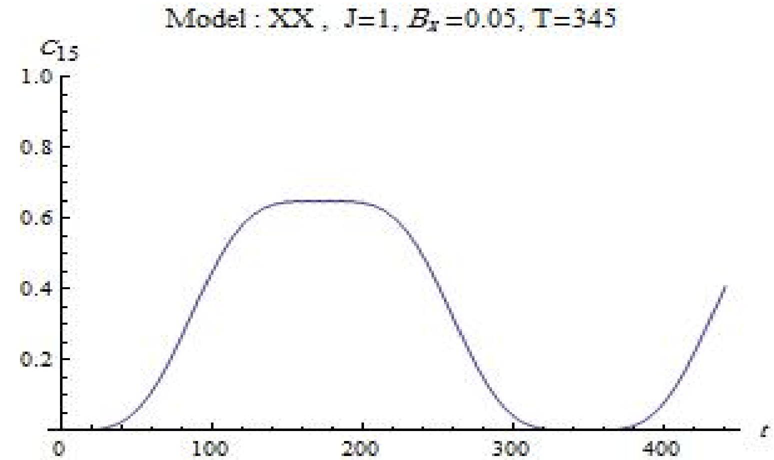

Figure-5. The time evolution of C_{15} for $T = 345$ for five-qubits chain.

Investigating the effect of sinusoidal alternative magnetic field on the system in x direction of six-qubits spinal chain

Investigating the effect of the uniform magnetic field or sinusoidal alternative on six-qubits spinal chain system and interaction according to Heisenberg XX model and in the initial state which all spins are up.

$$\left|\phi^{\wedge}\right\rangle = \left|\uparrow\uparrow\uparrow\uparrow\uparrow\right\rangle = \left|000000\right\rangle$$

According to equation (1), we apply a uniform magnetic field in x direction to six-qubits spinal chain system. As we expected, it led to entanglement in system. Now, we increase the magnetic field intensity. We have drawn entanglement graphs for different magnetic fields in Figure-6.

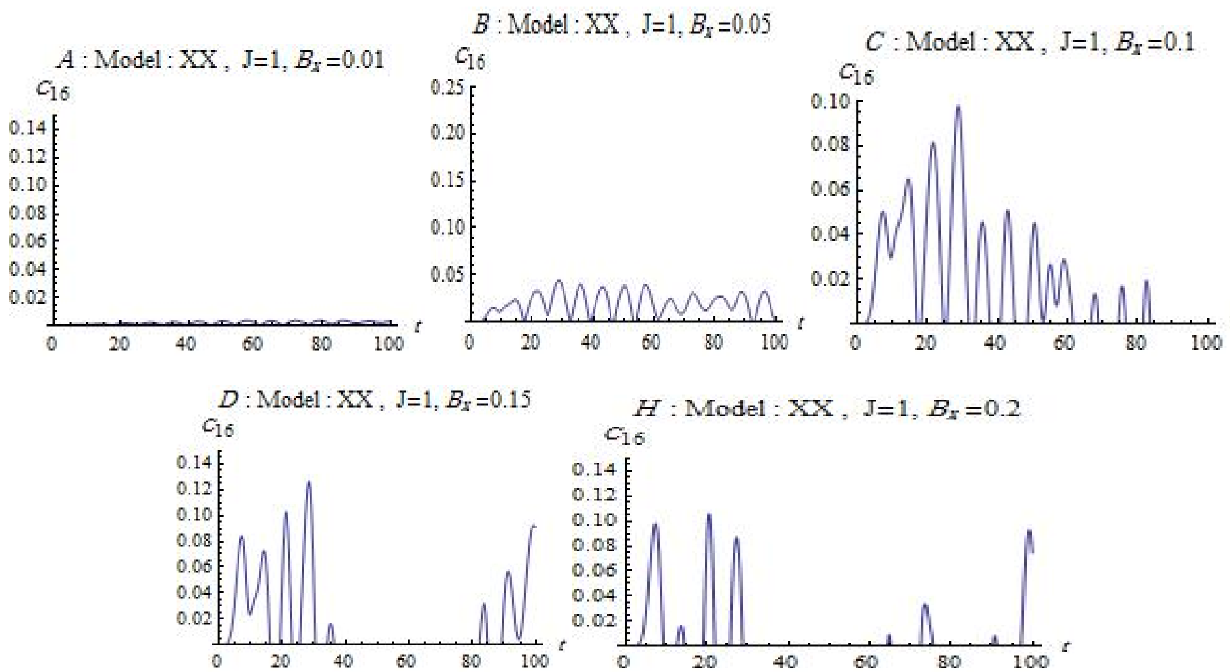

Figure-6. The time evolution of C_{15} for $T = 8$ for six-qubits chain.

As Figure-6 shows, when a uniform magnetic field applies to the total system in x direction, it creates entanglement in system. But, by increasing the intensity of the magnetic field, entanglement graphs become more irregular over time that is not desired for entanglement applications.

Now, we apply a sinusoidal magnetic field with relatively low domain $B_x = 0.05$ to the system according to equation (4). Then, we consider the different periodicity

VOL. 11, NO. 24, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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values, but assume the field as constant in x direction. The entanglement graph versus time for two terminal qubits has been shown in Figure-7.

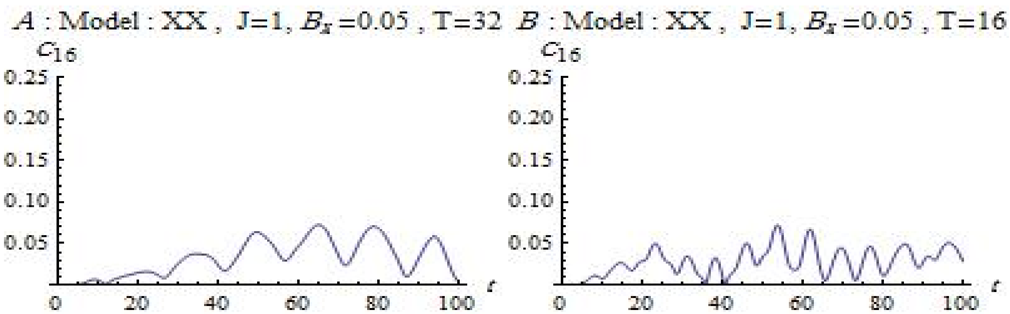

Figure-7. The time evolution of C_{16} for $T = 32$ and $T = 16$ for six-qubits chain.

It can be seen from Figure-7, when we apply a sinusoidal magnetic field with a relatively low domain $B_x = 0.05$ and various periods (T) to the whole system, different results will be obtained based on the values of periodicity. When T is large, the entanglement of system is relatively irregular.

We further decrease periodicity time and in various time intervals, we have drawn entanglement time evolution graph as Figure-8.

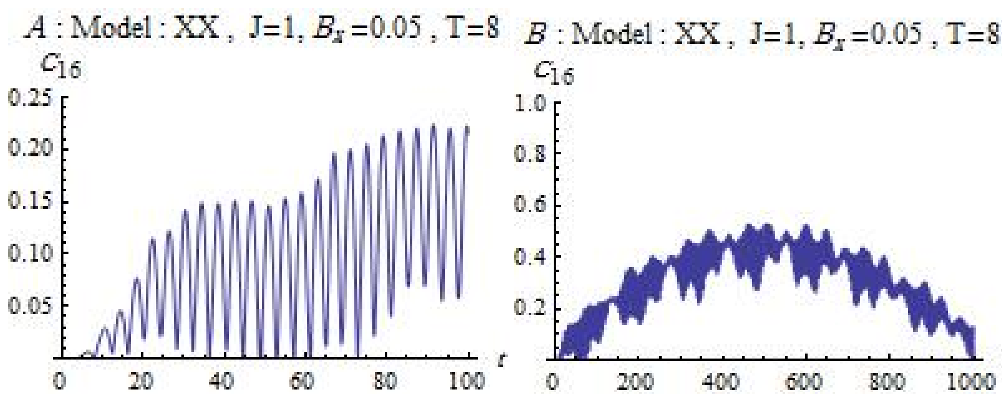

Figure-8. The time evolution C_{16} for $T = 8$ for six-qubits chain in various time intervals.

From Figure-8 it is clear that when T decreases, graph fluctuation intensively increases, but the whole graph will remain regular as a sinusoidal graph with appropriate domain. In Figure (9), we have drawn the time evolution of entanglement for lower periodicity.

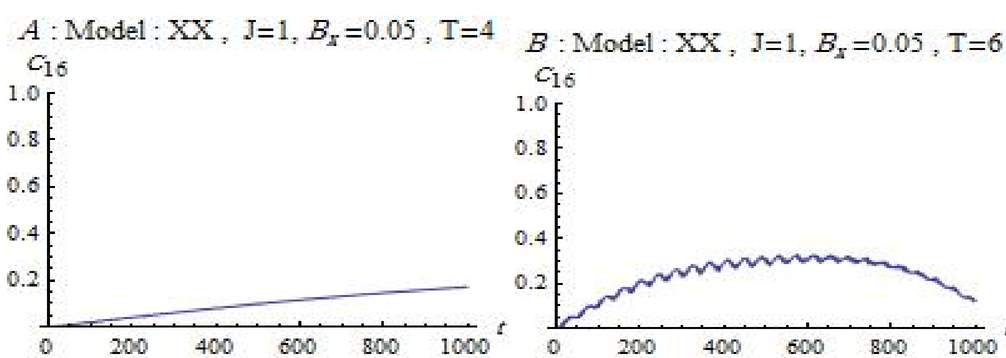

Figure-9. The time evolution of C_{16} for $T = 4$ and $T = 6$ for six-qubits chain.

Figure-9 shows an interesting result. If T is further decreased, the graph fluctuation decreases significantly, but the periodicity increases. If we want to have entanglement with acceptable domain and low fluctuation in a specific time interval, first, the magnetic field domain should be relatively low; second, this field should be alternative with low periodicity and; third, give enough time to systems to increase entanglement domain.

RESULTS

We investigated the time evolution of entanglement among terminal components of a five or six-qubits spin chain which are under the effect of time-dependent magnetic field and their interaction is described by Heisenberg XX model. The main purpose of this study is to show the importance and efficiency of applying variable wave with time in some of the cases.

The most important results are obtained as follows:

a) When a magnetic field is applied to the whole system is in z direction, there would be no entanglement over time.

b) When a uniform magnetic field is applied to the six-qubits system in x direction, in fact creates entanglement in the system but with the increase of the magnetic field intensity, the entanglement graphs become irregular over the time.

c) When a sinusoidal magnetic field with relatively low domain $B_x = 0.05$ and various periodicity is applied to the six-qubits system, various results will be obtained depend on the values of periodicity. When T is large, the entanglement will be relatively irregular.

d) If we want to have entanglement with acceptable domain with low ripple in a six-qubits system in appropriate interval of time, first, the magnetic field domain should be relatively low, and; second, this field should be alternative with low periodicity.

e) In a five-qubits system with the field along x axes, there are two peaks between 0 and 440 that are ripple.

f) In a five-qubits system, the entanglement is constant in specific intervals of time.

g) In a five qubits system, as the field size increases, entanglement irregularity increases, too. However, in small fields, the entanglement will be lower.

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