A LITERATURE SURVEY ON CHALLENGES IN STRESS BASED TOPOLOGY OPTIMIZATION

Hailu Shimels Gebremedhen, Dereje Engida Woldemichael and Fakhruldin M Hashim
Mechanical Engineering Department, Universiti Teknologi Petronas, Bandar Seri Iskandar, Tronoh, Perak, Malaysia
E-Mail: hshimels278@gmail.com

ABSTRACT

Topology optimization is an optimization technique used to find optimal material distribution with in a given design domain under applied loads and boundary conditions. Most of the developments in structural topology optimization have been formulated and solved for minimizing compliance. The absence of considering the displacement and stress constraints in the formulation and solution of topology optimization problems may lead the unfeasible optimal solutions where stress and displacement constraints are crucial criteria for the design considered. To include these two crucial elements in the optimization process some efforts have been devoted to formulate and solve the optimization problem by including stress constraints. Though considering the stress constraint in the optimization model is closer to the engineering point of view it facing three main challenges associated with the constraints. This paper aims to explore and discuss the three challenges in stress based topology optimization along with the proposed solutions.

Keywords: topology optimization, singularity, stress constraints, local nature of stress constraint, high nonlinearity of stress constraints.

INTRODUCTION

Topology optimization is a mathematical approach which seeks optimal material distribution within a given design domain to sustain applied load under specific boundary conditions. It includes determination of connectivity, geometries of cavities and location of voids in the design domain considered. Topology optimization has a great implication in the conceptual design stage where a lot of modifications are made and 80% of the cost of a given product/design is determined [3]. The changes in the design at the conceptual design affect the performance and manufacturability of the final structure. Topology optimization has significant control of these issues through optimal material distribution at the conceptual design stage of a structure.

Unlike other types of structural optimization methods [4-8], in topology optimization approach the number and location of holes/voids shapes and solid elements are not known prior to the optimization process. This gives the designer more freedom than that of other optimization methods which will let the designer to distribute the material optimally within the design domain. The definition of any topology optimization includes selection of design variables and formulation of objective and constraint functions.

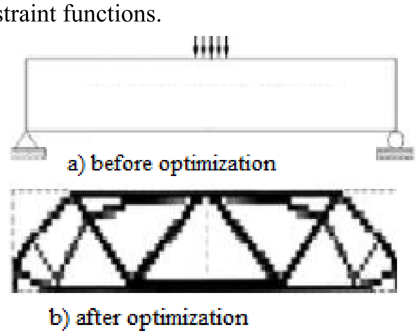

Figure-1. Topology optimization for simply supported beam [6].

Figure-1 (a) shows a design domain of a simply supported beam under applied load and boundary conditions. Figure-1(b) shows optimal materials distribution within the design domain shown in Figure-1(a) where all the elements represented with black colour are solid and others to be void elements.

Different approaches and algorithms have been suggested for formulating and solving an optimization problem, respectively. Optimality criteria methods (OC)[11], Method of Moving Asymptotes (MMA) [13], Sequential Quadratic and Linear Programming (SQP and SLP) [15] , Particle Swarm Optimization (PSO) are among algorithms used for solving an optimization problem. Homogenization Method [17, 18], Evolutionary Structural Optimization (ESO) [19-22], Bi-directional structural optimization (BESO) [23-25], Solid Isotropic Material with Penalization (SIMP)[26, 27] and Level Set Method[28, 29] are among the methods used to formulate topology optimization problems. Among the problem formulation approaches the SIMP approach is the common due to its conceptual simplicity and high computational efficiency[30].

Most of the researches in the area of structural topology optimization are focused on formulating and solving compliance minimization problems [31-38]. Though this approach is easy to formulate and becomes more popular, it faces some challenges including variation of results with the amount of material distrusted, un-able to consider stress and displacement in the optimization process which may let the results to be unfeasible in the real world application [30].

A formulation of structural optimization which takes stress into consideration has been developed and solved [14, 39-41]. Though this way of formulating and solving an optimization problem is closer to the engineering point of view it has some major challenges associated with the stress constraints, namely local nature of stress constraints, singularity phenomenon associated

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with void materials and high nonlinear dependence of stress constraint.

This paper aims to discuss about the three challenges in stress based topology optimization and the solutions that has been suggested so far to alleviate these challenges.

The rest of the paper is organized as follows: the first section discusses about the local nature of stress constraints with the proposed solutions and limitations. The second section discusses about the singularity phenomenon with the most common solution proposed for it. The third discusses high nonlinear behavior of stress constraints and the solution proposed along with their limitations.

LOCAL NATURE OF STRESS CONSTAINTS

In topology optimization of a structure under stress constraints, every element in the design domain is expected to be free from stress failure. For a discrete finite element model of stress based topology optimization problem, the number of stress constraints will be very large as one need to impose local stress constraints on all elements. Due to this large number of the stress constraints, the computational time and cost is expensive to be solved by existing algorithms [14, 15, 42].

To alleviate this problem, different scholars suggested alternate solutions. Among the proposed solutions, the recent and most prominent one's are global constraint approach [42], block aggregation approach [15], enhanced aggregation [14] and the clustered approach [43].

In the global approach, the whole design domain is defined by a single inequality constraint [9, 42]. Though this approach has a significant advantage on reducing the computational time it fails to control the global optimum and local stress measures [15].The failure in controlling the global optimum will lead the optimization problem to have unstable convergence and trapped in local optima which will vary the final output of an optimization result.

The failure in controlling the local stress measures will limit the control on the stress level of elements within a design domain. Thus it will let the final optimized layout to have elements with violated stress level which may not be feasible for real world application.

In the case of block aggregation approach, other than defining the whole design domain with a single inequality constraint, the constraints are grouped in different blocks [15]. Each block in the design domain are created by just grouping elements with correlative indexes in the finite element mesh. A general Kreisselmeier-Steinhauser (K-S) function is used for the formulation of the global function in each block with the design domain. Though block aggregation is able to retain the advantages of the global approach and mitigate the computational cost to some extent, still the computational cost is highly dependent on the number of blocks and number of elements in each block. It is difficult to determine the number of blocks for better representation of design domain without losing control on the local stress measures.

In both approaches it is difficult to determine the appropriate value for the aggregation parameter and left open for the end user. This makes the final result to be highly dependent on the skill and experience of the end user which will limit the reliability and implementation of the final results.

An enhanced aggregation approach combines active set and global constraint method[14]. In the active set method only those potentially critical stress constraints are taken into consideration in each iteration step [10]. In this method all constraints are divided into optimal active constraint set and inactive constraint set based on the stress state of the elements. A general K-S function is used to identify active and inactive constraints within the design domain of a given problem and original K-S function is used for aggregating those constraints in the active and inactive constraint set respectively .The optimization will be handled with the constraints which are active at the optimum. The method enables the user to use those elements removed during the iteration. Even if this method is able to overcome some the problems in block aggregation still the number of elements to define the block and number of blocks to represent a given design domain is dependent on the end user.

In the case of clustered approach, which is somewhat similar to block aggregation, a moderate number of stress constraints are used and several stress evaluation points are clustered into each constraints [43]. The problem is formulated using nested formulation where the equilibrium equation is not used as a constraint as in the simultaneous formulation used by other approaches [9, 14, 15]. The displacement vector is considered as a given function of the design variables and it is solved in the finite element analysis.

For clustering stress from multiple stress evaluation points into a single constraint a P-norm method is used. Those stress evaluation points which have similar stress level are clustered together. Even if this approach is able to reduce the computational cost which arises from the local nature stress constraints, still the problem is influenced by the way how the clusters are created and still it is difficult to determine the number of points to be clustered together.

In all aggregation techniques achieving certain level of smoothness of the aggregation function for preventing numerical instabilities and accuracy of approximating the local stress levels are the two conflicting requirements. The other major problem is it is impossible to know the best value of the aggregation parameter prior to the design and it is problem dependent.

SINGULARITY PHENOMENON

This problem was first identified during the designing of trusses under stress constraint. During the optimization process, a degenerate subspace of dimension less than n was found in n-dimensional feasible design space[44, 45] Further the globally optimum design is often an element of the degenerate subspaces.

Further the globally optimum design is often an element of the degenerate subspaces[46]. Nonlinear programming algorithms cannot identify these region and they converge to locally optimum designs. In case of topology optimization, it arises in density based topology optimization when materials with zero density reappear in the finite element matrices, which may prevent the algorithm to reach a feasible solution. This zero density gives rise to the singular stiffness matrix and the numerical breakdown of linear solvers in structural analysis. The stress constraints and the design variables are relaxed to eliminate the degenerate regions and thereby allow the nonlinear programming algorithms to find the global optimum. Different relaxation approaches/solutions have been suggested by different scholars for tackling this problem [30, 40, 47-49] which includes ε-relaxation[47], phase field relaxation[48] and qp relaxation[50, 51] Among the alternative solutions ε-relaxation [47] is the most common in stress based topology optimization.

In ε-relaxation approach, the internal force constraints of the structural member are relaxed and the shape of the feasible domain is modified [2, 30, 47]. The stress constraints are replaced by internal force constraint functions which will let discontinuity of the constraint function at the zero cross sectional area to be removed. The value of parameter epsilon determines to what extent the constraint can be relaxed [2, 10].

The approaches that has been suggested so far are able to alleviate the singularity phenomenon to some extent but there are two major challenges associated with the solutions.

The first challenge is the computational cost associated with the additional steps to find the elements with zero density and reconstruction of the system of equations.

The second is the physical inconsistency that weak material appears on the element containing essentially no material, which will make the manufacturing of optimal topology challenging.

The other difficulty is related to constraint relaxation techniques which are used to avoid singularity phenomenon within the design domain. Hence these techniques are based on smoothing the original constraint by which the design space is enlarged and gradient based techniques can access local minima. But this results in a highly non-convex design space which will let the optimization problem to converge locally.

The summary of literatures the ε-relaxation approach in stress based topology optimization is shown in Table-1.

Table-1. Summary of literatures for epsilon relaxation.

Author	Relaxed stress constraint	Problem statement
Pereira et.al [1]	$\rho(\frac{\sigma^{vm}}{\rho^{P}\sigma^{max}} - 1) \le \varepsilon(1-\rho)$	Minimizing volume subjected to local stress constraint
Guilherme and Fonsseca [2]	$\frac{\sigma^{vm}}{\sigma^{max}} - 1 \le \varepsilon^{P} - \varepsilon$	Minimizing volume subjected to global stress constraint
Paris et.al [9]	$\rho(\frac{\sigma^{vm}}{\rho^{P}\sigma^{max}} - 1) \le \varepsilon(1-\rho)$	Minimizing volume subjected to local stress constraint
Duysinx and Bendsoe [10]	$\rho(\frac{\sigma^{vm}}{\rho^{P}\sigma^{max}} - 1) \le \varepsilon$	Minimize volume subjected to local stress constraints
Duysinx and Sigmund [12]	$\rho(\frac{\sigma^{vm}}{\rho^{P}\sigma^{max}} - 1) \le \varepsilon(1-\rho)$	Minimizing volume subjected to global stress constraint
Yangjun et.al[14]	$0 < \rho_{min} \le \rho \le 1$	Minimize volume subjected to local stress constraints
Paris et.al [16]	$0 < \rho_{min} \le \rho \le 1$	Minimizing weight subjected to global stress constraint

NON-LINEARITY OF STRESS CONSTRAINTS

This is the third major challenge in stress based topology optimization. It is caused by the significant alteration of the stress level by small changes in the neighbouring regions which will create a numerical inconsistency in the constraint. The numerical inconsistency in the constraints will lead the optimization problem to have unstable convergence during the optimization process. Different researchers have been suggesting solutions for alleviating this problem [46, 52].

Imposing local stress measure [10] provides precise control over the local stress levels, but it needs a large computational time and cost. By means of the global stress measure the computational cost can be compromised but it will be difficult to control the local stress levels.

The method proposed by Chau Le *et.al* [46] uses several regional stress measures to improve the local stress control. Other than using local stress definitions or using a single global constraint, in the proposed method regional constraints were defined for defined regions based on physical location, stress distribution. Even if setting several regional stress measures let the designer to control the local stress states still the computational cost and time will be higher.

CONCLUSIONS

Topology optimization which deals about an optimum material distribution with in a given design domain has been an active research area since the land mark paper of Bendsoe and kickuchi [18] in 1988.

Most of the research works in topology optimization so far are formulated and solved to minimize

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compliance of a structure. Though this approach is easy for implementation it faces some problems which may prevent the final results to be used in real world applications.

Formulation and solution of optimization problem considering stress as a design constraint has been also formulated and solved which is quite realistic and acceptable from engineering point of view. Though the stress based approach is much closer to the engineering point of view it has been facing challenges associated with the design variables and stress constraint.

The three main challenges, namely local nature of stress constraints, singularity phenomenon and high nonlinear dependency of stress constraints, in stress based topology optimization are discussed. Solutions that have been suggested by different researchers, as summarized in Table-2 are reviewed and the limitations of each proposed solution are discussed.

The following conclusion can be drawn regarding the limitations of proposed solutions from authors' perspective

1. Determination of the aggregation parameter while using aggregation techniques to reduce the number of constraints.
2. Determination of optimum number of elements in each blocks to define a given design domain using group of elements.
3. Balancing level of smoothness of the aggregation function and accuracy for approximating the local stress levels.
4. Computational cost associated with finding zero elements and reconstructing system of equation.
5. Treatment of thin material which does not represent any material which will make the manufacturing of output results.
6. Inclusion of relaxation to access singular regions within a design domain will increase the level of complexity of problems to be solved, which needs more computational time and cost.

Table-2. Summary of solutions for the three challenges in based topology optimization.

Title	Suggested solution
An enhanced aggregation method for topology optimization with local stress constraints[14]	A general and modified KS- function is used to aggregate the constraints as optimal active and inactive constraint sets based on the stress limits during the iteration. A reduction parameter is used to reduce the feasible design domain for conducting the removal and regeneration strategy to reach the true optimum.
Stress-based topology optimization for continua[46]	The P-norm stress measure is used for the formulation of the global function. For better approximation of the maximum stress global stress measure is used.
Stress constrained topology optimization[43]	A clustered approach is used by which a moderate number of stress constraints are used and several stress evaluation points are clustered into each constraint, in a way somewhat similar to the block aggregation in Par´ıs et al. stresses from several stress evaluation points are clustered using P- norm.
Block aggregation of stress constraints in topology optimization of structures[16]	Other than defining the whole domain by a single constraint which fails to secure the global optimum and local stress measures, the constraints are grouped in different blocks and each block is defined by an equality function. A general function is used for formulation of the global function for each block
On an alternative approach to stress constraints relaxation in topology optimization[50]	It chooses a suitable stress interpolation parameter differing from that of the stiffness penalty factor used in the original density based optimization problems
ε-relaxed approach in structural topology optimization [47]	The internal force constraint of the structure members is relaxed. The stress constraints are replaced by internal forces constraints by which the discontinuity of the constraint function at zero cross- sectional area is removed. The parameter epsilon determines the extent of relaxation. This was adapted to continuum structures subjected to stress constraints from truss problems by relaxing the design variable (the relative density of the elements in the analysis). The common value for epsilon is [0.001 -0.1]
Phase field relaxation of topology optimization with local stress constraints [48]	It starts by reformulating the optimization problem into linear and 0-1 type of problem (because most of the optimization problems are nonlinear and The need for the optimization is black and white (solid -void)). The constraints are relaxed and approximated using Cahn-Hilliard* type penalty in the objective functional, which yields convergence of minimizers to 0–1 designs as the penalty parameter decreases to zero
Stress-based topology optimization for continua[46]	Several regional stress measures are used to improve the local control of the stress constraints
Improvements in the treatment of stress constraints in structural topology optimization problems[52]	An improved SLP algorithm with quadratic line search is used. The optimization problem is linearly approximated and solved using simplex method and solved at each iteration

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