VOL. 11, NO. 24, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

COMPUTER SIMULATION OF FREE CONVECTIVE MHD STOKES PROBLEM FOR A VERTICAL PLATE THROUGH POROUS MEDIUM

Harouna Naroua and Bachir Moussa Idi
Département de Mathématiques et Informatique, Faculté des Sciences et Techniques, Université Abdou Moumouni, Niamey, Niger
E-Mail: hnaroua@yahoo.com

ABSTRACT

The problem of the MHD free convection flow past an impulsively started vertical porous plate through a porous medium, taking into account the heat due to viscous dissipation, is investigated and simulated. A generic computer program using the Galerkin finite element method is employed to obtain solutions for velocity and temperature fields. The energy equation, the momentum equation, and the parameters entering into the description of the flow, are transformed into an interpretable code. The influences of the dimensionless parameters entering into the description of the problem are investigated.

Keywords: computer simulation, generic computer tool, finite element method, stokes problem.

INTRODUCTION

Porous media are very widely used to insulate a heated body in order to maintain its temperature. They are considered to be useful in diminishing the natural free convection which would otherwise occur intensively on the vertical heated surface. To make the heat insulation of the surface more effective, it is necessary to study the free convection flow through a porous medium. The flow past an impulsively started infinite plate was studied by a number of authors such as [1-5]. On the other hand, flows through porous media are very much prevalent in nature and therefore the study of flows through media has become of practical interest in many scientific and engineering applications. Various authors have made important contributions in the area of fluid flows such as [6-13].

Fagbade *et al.* [14] presented an application of the spectral homotopy analysis method in order to solve a problem of darcy-forcheimer mixed convection flow in a porous medium in the presence of magnetic field, viscous dissipation and thermopherisis. Their analysis was aimed at studying the effects of chemical reaction, magnetic field, viscous dissipation and thermophoresis on mixed convection boundary layer flow of an incompressible, electrically conducting fluid past a heated vertical permeable flat plate embedded in a uniform porous medium. Dhar *et al*. [15] applied the group-theorytic approach to solve the problem of the unsteady MHD mixed convective flow past on a moving curved surface. They reduced the number of independent variables by two, and the obtained ordinary differential equations were solved numerically using the shooting method. Duan *et al*. [16] considered a two-dimensional symmetric space-fractional diffusion equation in which the space fractional derivatives are defined in Riesz potential sense. They solved the diffusion equation by using Crank-Nicolson technique in time and Galerkin finite element method in space. They also proved the stability and convergence of their schemes. Naroua *et al* [5] presented a formulation coupling the classical finite element method with a stepwise Lagrange polynomial in order to compute the solution of a fluid flow problem. Instead of using line segments within elements as used by the classical method, they used polynomials of degree two over couples of elements.

In this paper, we are proposing to study a specific non-linear fluid flow problem and compute its solution by a generic software tool using finite elements. We first investigate the interaction of the free convection with hydromagnetic flow past an impulsively started vertical plate through a porous medium by taking into account the heat due to viscous dissipation. Next, we make use of a generic finite element method to compute the numerical values of the flow fields.

MATHEMATICAL FORMULATION OF THE PROBLEM

We make an investigation of the MHD free convective flow past an impulsively started vertical infinite porous plate. The $x^/$-axis is taken along the plate in the vertical upward direction and $y^/$-axis is taken normal to the plate. The magnetic field is applied in the direction of $y^/$-axis. Initially, the temperatures of the plate and the fluid are assumed to be the same. At time $t^/>0$, the plate starts moving impulsively in its own plane with velocity U and its temperature is instantaneously rised or lowered to $T_\infty^/$ which is maintained constant later on. A uniform magnetic field B_0 is assumed to be applied transversely to the direction of the flow. Since the plate is infinite in length, all variables are functions of $y^/$ and $t^/$ only. The flow occurs with a low Mach number and hence the density of the fluid can be taken as constant. The induced magnetic field is neglected since the magnetic Reynolds number is very small [17]. Then under the usual Boussinesq approximation, the problem is governed by the following set of equations:

$$\begin{cases} \dfrac{\partial v'}{\partial y'} = 0 & (1) \\ \dfrac{\partial u'}{\partial t'} + v' \dfrac{\partial u'}{\partial y'} = g\beta\left(T' - T'_\infty\right) + \nu \dfrac{\partial^2 u'}{\partial y'^2} - \dfrac{\sigma}{\rho} B_0^2 u' - \dfrac{\nu}{K'} u' & (2) \\ \dfrac{\partial T'}{\partial t'} + v' \dfrac{\partial T'}{\partial y'} = \dfrac{k}{\rho C_p} \dfrac{\partial^2 T'}{\partial y'^2} + \dfrac{\nu}{C_p}\left(\dfrac{\partial u'}{\partial y'}\right)^2 & (3) \end{cases}$$

The initial and boundary conditions are:

$$\begin{cases} u' = 0, \ T' = T'_\infty \ \ at \ \ t = 0 \\ u' = U, \ T' = T'_w \ \ at \ \ y = 0 \\ u' \to 0, \ T' \to T'_\infty \ \ as \ \ y \to \infty \end{cases} \qquad (4)$$

The equation of continuity gives

$$v' = -v'_0 \qquad (5)$$

where v_0 is the constant suction velocity. The symbols u', v' stand for the velocity components in the x' and y' directions, t' the time, g the acceleration due to gravity, β the coefficient of volume expansion, T'_∞ the temperature at infinity, T'_w the fluid temperature near the plate. ρ, C_p, k and υ are respectively the fluid density, the specific heat, the thermal conductivity and the kinematic viscosity. σ, B_0 and K' are the electrical conductivity of the fluid, the external magnetic field and the permeability of the medium respectively. We introduce now the following non-dimensional quantities:

$$\begin{cases} y = y'\dfrac{U}{\nu}, \quad t = t'\dfrac{U^2}{\nu}, \quad u = \dfrac{u'}{U}, \quad v = \dfrac{v_0}{U}, \quad \Pr = \mu\dfrac{C_p}{k}, \\ \mathrm{Gr} = \nu g\beta\dfrac{T'_w - T'_\infty}{\mathrm{U}^3}, \quad \mathrm{T} = \dfrac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \mathrm{M}^2 = \dfrac{\nu\sigma}{\rho \mathrm{U}^2}\mathrm{B}_0^2, \\ K = \dfrac{U^2 K'}{\nu^2}, \quad Ec = \dfrac{U^2}{C_p}\left(T'_w - T'_\infty\right) \end{cases} \qquad (6)$$

where Pr, Ec, Gr, M and K are the Prandtl number, the Eckert number, the Grashof number, the magnetic parameter and the permeability parameter respectively.

In view of equation (6), equations (2) and (3) reduce to the following non-dimensional form:

$$\begin{cases} \dfrac{\partial u}{\partial t} - v\dfrac{\partial u}{\partial y} = GrT + \dfrac{\partial^2 u}{\partial y^2} - \left(M^2 + \dfrac{1}{K}\right)u & (7) \\ \dfrac{\partial T}{\partial t} - v\dfrac{\partial T}{\partial y} = \dfrac{1}{\Pr}\dfrac{\partial^2 T}{\partial y^2} + Ec\left(\dfrac{\partial u}{\partial y}\right)^2 & (8) \end{cases}$$

The initial and boundary conditions are given as follows:

$$\begin{cases} u = 0, \quad T = 0 \quad for \ y \ge 0, t = 0, \\ u = 1, \quad T = 1 \quad for \ y = 0, t > 0, \\ u \to 0, \quad T \to 0 \quad for \ y \to \infty, t > 0 \end{cases} \qquad (9)$$

SOLUTION OF THE PROBLEM

The above system of equations (7) and (8) with initial and boundary conditions (9) has been solved numerically by a generic computer program based on the finite element method in step 1 and step 2.

Step 1: Energy equation finite element solution

We solve equation (8) with the help of initial and boundary conditions (9). Constructing the quasi-variational equivalent of equation (8), we obtain:

$$0 = \int_\Omega \phi_i \left(\frac{\partial T}{\partial t} - v\frac{\partial T}{\partial y} - \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} - Ec\left(\frac{\partial u}{\partial y}\right)^2 \right) dy \qquad (10)$$

where ϕ_i denotes the test function and Ω is the region of the flow.

Consider an N elements mesh and a two parameter (semi discrete) Galerkin approximation of the form [18]:

$$T(y,t) = \sum_{j=1}^{N} C_j(t)\phi_j(y) \qquad (11)$$

with

$$\begin{cases} \phi_1^{(i)}(y) = \dfrac{(y_{i+1} - y)}{(y_{i+1} - y_i)} \\ \phi_2^{(i)}(y) = \dfrac{(y - y_i)}{(y_{i+1} - y_i)} \end{cases} \qquad (12)$$

where i = 1,2,3,…,N and y_i and y_{i+1} are respectively the lower and upper coordinates of the element i.

Using equations (11-12), equation (10) reduces to:

$$[A]\left\{\frac{\partial C}{\partial t}\right\} + [B]\{C\} = \{P\} \qquad (13)$$

where

$$\begin{cases} A_{ij} = \int_\Omega \phi_i \phi_j \, dy \\ B_{ij} = \int_\Omega \left(\dfrac{1}{\Pr}\dfrac{\partial \phi_i}{\partial y}.\dfrac{\partial \phi_j}{\partial y} - v\phi_i \dfrac{\partial \phi_j}{\partial y} \right) dy \\ P_i = \int_\Omega \left(Ec\,\phi_i \left(\dfrac{\partial u}{\partial y}\right)^2 \right) dy \end{cases} \qquad (14)$$

Using the Θ - family of approximation developed by Reddy [18], equation (13) reduces to:

$$\left[\hat{A}\right]\{C\}_{n+1} = \left[\hat{B}\right]\{C\}_n + \left\{\hat{P}\right\} \qquad (15)$$

where

$$\begin{cases} \left[\hat{A}\right] = [A] + \Theta\Delta t[B] \\ \left[\hat{B}\right] = [A] - (1-\Theta)\Delta t[B] \\ \left[\hat{P}\right] = \Delta t\left[\Theta\{P\}_{n+1} + (1-\Theta)\{P\}_n\right] \end{cases} \quad (16)$$

The initial value C_0 is obtained by the Galerkin method from a 64 elements mesh and is given by:

$$C_0 \;=\; [1,0,0,.......................,0]^T \quad (17)$$

For t > 0,

$$\{C\}_{n+1} = \left[\hat{A}^{-1}\right][B]\{C\}_n + \left[\hat{A}^{-1}\right]\left\{\hat{P}\right\} \quad (18)$$

Step 2: Momentum equation finite element solution

We solve equation (7) with the help of initial and boundary conditions (9). Constructing the quasi-variational statement of equation (7), we obtain:

$$0 = \int_{\Omega}\psi_i\left(\frac{\partial u}{\partial t} - v\frac{\partial u}{\partial y} - GrT - \frac{\partial^2 u}{\partial y^2} + \left(M^2 + \frac{1}{K}\right)u\right)dy \quad (19)$$

where Ψ_i is the test function and Ω is the region of the flow.

Consider a two parameter (semi-discrete) Galerkin approximation of the form [18]:

$$u(y,t) \;=\; \sum_{j=1}^{N} d_j(t)\Psi_j(y) \quad (20)$$

With

$$\begin{cases} \Psi_1^{(i)}(y) = \dfrac{(y_{i+1} - y)}{(y_{i+1} - y_i)} \\ \Psi_2^{(i)}(y) = \dfrac{(y - y_i)}{(y_{i+1} - y_i)} \end{cases} \quad (21)$$

where i=1,2,3,…..,N and y_i and y_{i+1} are respectively the lower and upper coordinates of the element i. Using equations (20-21), equation (19) reduces to:

$$\left[\bar{A}\right]\left\{\frac{\partial d}{\partial t}\right\} + \left[\bar{B}\right]\{d\} = \left\{\bar{P}\right\} \quad (22)$$

where

$$\begin{cases} \bar{A}_{ij} = \int_{\Omega}\Psi_i\Psi_j\,dy \\ \bar{B}_{ij} = \int_{\Omega}\left(\dfrac{\partial\Psi_i}{\partial y}.\dfrac{\partial\Psi_j}{\partial y} - v\Psi_i\dfrac{\partial\Psi_j}{\partial y} + \left(M^2 + \dfrac{1}{K}\right)\Psi_i\Psi_j\right)dy \\ \bar{P}_i = \int_{\Omega}Gr\Psi_i T\,dy \end{cases} \quad (23)$$

Using the Θ - family of operators developed by Reddy [18], equation (22) takes the form:

$$\{d\}_{n+1} = \left[\hat{\bar{A}}^{-1}\right]\left[\hat{\bar{B}}\right]\{d\}_n + \left[\hat{\bar{A}}^{-1}\right]\left\{\hat{\bar{P}}\right\} \quad (24)$$

Where

$$d_0 = [1,0,0,..................0]^T \quad (25)$$

The numerical values of the temperature and velocity fields have been computed from equations (18) and (24). All input elements such as matrix and vector elements are transformed into postfix code which will be interpreted in the process of calculations.

DISCUSSION OF RESULTS

In order to achieve a physical understanding of the problem and for the purpose of discussing the results, numerical calculations have been carried out for the velocity and temperature distributions. The results obtained are displayed in Figures 1-3. The method used is unconditionally stable and is independent of the time step Δt. The velocity profiles are examined for the cases Gr > 0 and Gr < 0. Gr > 0 (= +5) is used for the case when the flow is in the presence of cooling of the plate by free convection currents. Gr < 0 (= -5) is used for the case when the flow is in the presence of heating of the plate by free convection currents. Figures 1-2 show the velocity distribution for the two cases from which we observe that the velocity (u) decreases away from the plate.

From Figure 1, for the case when Gr>0 (in the presence of cooling of the plate by free convection currents), we observe that:

- The velocity (u) increases due to an increase in the Prandtl number (Pr), the permeability parameter (K) and the time (t);
- An increase in the Eckert number (Ec) leads to an insignificant change in the velocity profile (u).

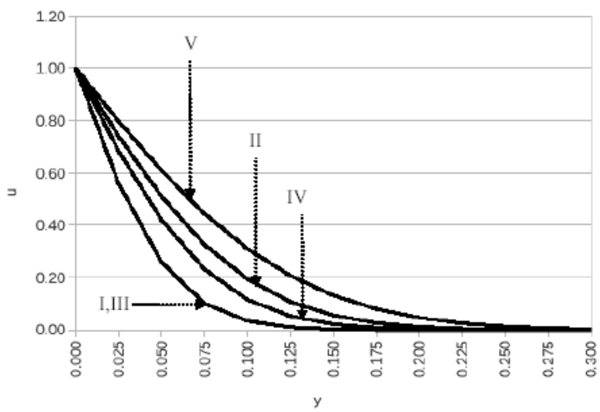
From Figure-2, for the case when Gr<0 (in the presence of heating of the plate by free convection currents), we observe that:

- The velocity (u) decreases due to an increase in the Prandtl number (Pr) and the permeability parameter (K);
- The velocity (u) increases with an increase in the time (t);
- There is an insignificant change in the velocity profile (u) due to an increase in the Eckert number (Ec).

VOL. 11, NO. 24, DECEMBER 2016 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

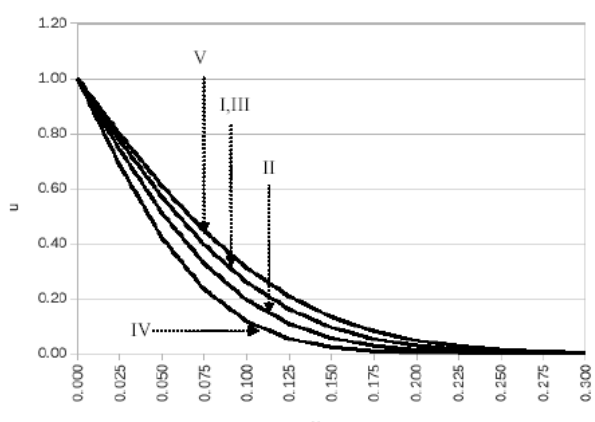
From Figure-3, we observe that:

- The temperature (T) decreases away from the plate. The decrease is greater for a Newtonian fluid than it is for a non-Newtonian fluid (T decreases with Pr);
- There is a rise in temperature profiles (T) due to an increase in the time (t);
- An increase in the Eckert number (Ec) leads to an insignificant change in the temperature profile (T).



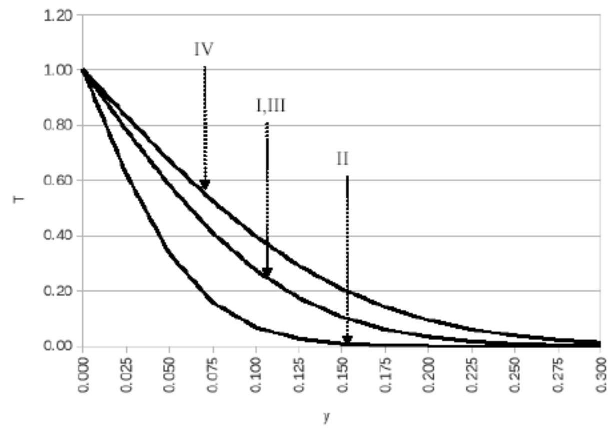
Series	Pr	Ec	K	t
I	0.83	0.01	0.5	0.01
II	3.00	0.01	0.5	0.01
III	0.83	0.05	0.5	0.01
IV	0.83	0.01	1.5	0.01
V	0.83	0.01	0.5	0.03

Figure-1. Velocity distribution for Gr = +5.



Series	Pr	Ec	K	t
I	0.83	0.01	0.5	0.01
II	3.00	0.01	0.5	0.01
III	0.83	0.05	0.5	0.01
IV	0.83	0.01	1.5	0.01
V	0.83	0.01	0.5	0.03

Figure-2. Velocity distribution for Gr = -5.



Series	Pr	Ec	t
I	0.80	0.01	0.01
II	7.00	0.01	0.01
III	0.80	0.05	0.01
IV	0.80	0.01	0.03

Figure-3. Temperature distribution for Gr = ±5.

REFERENCES

[1] Soundalgekar V.M. and Takhar, H.S. 1980. Combined forced and free convection MHD flow past a semi-infinite vertical plate. Wärmeund Stoffübertragung. 14: 153-158.

[2] Raptis A. 1983. Unsteady free convective flow and mass transfer through a porous medium bounded by an infinite vertical limiting surface with constant suction and time-dependent temperature. Energy Research. 7: 385-389.

[3] Raptis A. and Perdikis C. 1983. Effects of mass transfer and free convection currents on the flow past an infinite porous plate in a rotating fluid. International Journal of Energy Research. 7: 391-395.

[4] Raptis A. and Massalas C.V. 1997. Magnetohydrodynamic flow past a plate by the presence of radiation. Journal of Magnetohydrodynamics and Plasma Research. 7(2/3): 121-128.

[5] Naroua H., Takhar H. S. and Slaouti A. 2006. Computational challenges in fluid flow problems: a MHD Stokes problem of convective flow from a vertical infinite plate in a rotating fluid. European Journal of Scientific Research. 13(1): 101-112.

[6] Raptis A. and Perdikis C. 2003. Thermal radiation of an optically thin gray gas. Int. J. Appl. Mech. Engng. 8(1): 131-134.

[7] Yusuf A., Aiyesimi Y. M., Jiya M. and Okedayo G. T. 2016. Analysis of Couette Flow of a Nanofluid in

an Inclined Channel with Soret and Dufour Effects. American Journal of Computational and Applied Mathematics. 6(2): 57-64.

[8] Daniel A.A. and Seini Y. I. 2016. MHD Boundary Layer Flow Past an Inclined Plate with Viscous Dissipation. American Journal of Computational and Applied Mathematics 6(4): 149-161.

[9] Medikare M., Joga S. and Chidem K. 2016. MHD Stagnation Point Flow of a Casson Fluid over a Nonlinearly Stretching Sheet with Viscous Dissipation. American Journal of Computational Mathematics. 6: 37-48.

[10] Aiyesimi Y. M., Jiya M., Olayiwola R. O. and Wachin A. A. 2016. Mathematical Analysis of Convective Flow of an Unsteady Magnetohydrodynamic (MHD) Third Grade Fluid in a Cylindrical Channel. American Journal of Computational and Applied Mathematics. 6(2): 103-108.

[11] Ghosh S. K. and Pop I. 2007. Thermal radiation of an optically thick gray gas in the presence of indirect natural convection. International Journal of Fluid Mechanics Research. 34(6): 515-520.

[12] Bestman A. R. and Adiepong S. K. 1988. Unsteady hydromagnetic free-convection flow with radiative heat transfer in a rotating fluid. Astrophys. Space Sci. 143: 73-80.

[13] Brewster M. Q. 1992. Thermal radiative transfer and properties. John Wiley and Sons, New York.

[14] Fagbade, A.I., Falodun, B.O. and Boneze, C.U. (2015) Influence of Magnetic Field, Viscous Dissipation and Thermophoresis on Darcy-Forcheimer Mixed Convection Flow in Fluid Saturated Porous Media. American Journal of Computational Mathematics, 5, 18-40.

[15] Dhar D., Alim M. and Andallah L. 2016. Group Method Analysis of MHD Mixed Convective Flow Past on a Moving Curved Surface with Suction. American Journal of Computational Mathematics. 6: 74-87.

[16] Duan, B.P., Zheng, Z.S. and Cao, W. (2015) Finite Element Method for a Kind of Two- Dimensional Space-Fractional Diffusion Equation with Its Implementation. American Journal of Computational Mathematics. 5: 135-157.

[17] Shercliff J. A. 1965. A text book of magnetohydrodynamics. Pergamon Press. Oxford.

[18] Reddy J.N. 1984. An Introduction to the finite element method. Mc Graw Hill book company. Singapore.