



ANALYZE OF PILOT REUSE WITH ACHIEVABLE SUM RATE FOR MASSIVE MIMO CELLULAR UPLINK

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ABSTRACT

The last ten years have seen important developments of massive multi-input multi-output (MIMO) in wireless communication. Massive MIMO has currently been presented in the 5G wireless standards. The number of terminals is increasing with additional appliances. At the same time, high transmission sum rates and communication reliability are required. Moreover, the multi-cell MMSE scheme, which includes an uplink MMSE and MRT decoders. Furthermore, this paper focuses how the MMSE activities all obtainable pilots for interference suppression. Specifically, this paper investigates the spectral efficiency of the massive MIMO, pilot contamination, which MMSE exploits all available pilots for interference suppression, and estimated locally at every BS, to actively suppress both intra-cell and inter-cell interference. Consequently, the average sum rate is proportional with SINR, using the linear scheme all of MMSE, ZF and optimal MMSE, while the sum rate is reverse proportional with linear decoding MRT. Then, when the number of base stations increases, the linear schemes MMSE, ZF and optimal MMSE have more convergence, while when the number of BS decreases the linear decoding schemes only have convergence except for MRT. However, at high SNR a higher number of antennas achieve better than a low number of antennas.

Keywords: massive MIMO, sum rate, pilot contamination, MMSE, MRT, SE.

INTRODUCTION

A massive multiple input multiple output (MIMO) system, which exploits a huge number of antennas array at the base station (BS) to assist ten or hundred users equipment, suffers from pilot contamination due to inter-cell interference. From the uplink of cellular networks, obtaining channel state information at the base stations (BSs) requires reverse link pilot signaling. Due to the limited number of orthogonal pilot sequences by channel coherence, it is more efficient to reuse the pilot across cells in order to achieve high spectral efficiency (SE). In massive MIMO networks, the system may continuously schedules as many users as possible in order to achieve higher SE (Zhu, X. *et al.*, 2016). However, the number of pilot sequences limits the number of active users (UEs) per cell. Therefore, this paper focuses on the ergodic data rate of massive MIMO with an average sum rate using the linear schemes minimum mean square error (MMSE), maximum ratio transmission (MRT) and zero forcing (ZF). This paper also focuses on the maximum achievable sum rate with relation to the increase number of BS antennas array of massive MIMO wireless systems exploiting linear decoding MMSE, MRT and ZF receivers.

In massive MIMO systems, several antennas array are exploited at the base station in order to provide high sum rate as well as an enhanced quality of service (QoS) for the mobile terminals. In a multi-cell, where every cell is operational with tens or more of antenna array, it is important to have the knowledge of current channel state information (CSI) parameter at the BS in order to achieve high channel estimation quality. Consequently, massive MIMO requires treating pilot contamination and inter-cell interference, therefore the most effective way to obtain CSI is through reciprocity by using uplink training of pilots. The main limitation in a

multi-cell situation is the apportionment of pilot signals to the mobile terminals. It changes the system operation, as the CSI is beyond supported the reuse allocation (Larsson, E.G. *et al.*, 2014).

In this paper, we establish the analytic attainable sum rates for both upper and lower bounds capacity of massive MIMO systems with SNR using linear decoding MMSE receivers and maximum ratio transmission (MRT). The results are analysed based on the MRT and ZF algebraic manipulations, which essentially relate the MMSE and MRT achievable sum rate to the number of BS antennas, M . The massive MIMO mutual information with optimal receivers are dependent on the increased number of base station antennas. Consequently, the massive multi-user-MIMO regime is chosen as a favorable technology for the 5G of cellular networks. In order to leverage the massive MIMO capability, the system may require accurate CSI at the BS and/or the users (Atzeni, I. *et al.*, 2015), (Larsson, E.G. *et al.*, 2014).

The BS is equipped with many antenna elements, M , to serve K multiple users in the same time-frequency resource when massive-MIMO using the reverse link. Where, we suppose that $M > K-1$, in Figure-1. Moreover, the BS uses a linear scheme to send the signal before transmitting to all users. This involves realization of channel state information at the BS. The time-division duplex (TDD), act so that the channels on the reverse link and forward link are equivalent because the reciprocity. Additionally, in small multi-user-MIMO regimes where the increase of elements antenna array in BS is discreetly small, usually, the BS is able to acquire an estimation of CSI via feedback signal in the frequency-division duplex (FDD) (Tan, W. *et al.*, 2015). Exactly, every user evaluations the channels dependent on the forward link and also it provides back the channel estimation to the BS



across the reverse link where accurate CSI is required at the BS. However, in massive multi user-MIMO, when the number of BS antennas increased, the channel estimation becomes challenging in FDD due to the number of desired forward link resources. Therefore, the pilots reuse scheme will be suited to accommodate the huge numbers of antennas array elements at BS. In addition, the bandwidth requirement for CSI feedback becomes very large. The BS is able to get CSI in open-region clearly from the reverse link training. To process the pilot contamination, we can use the pilot reuse transmission directly, which is proportional to the number of mobile terminals, in order to suppression inter-cell-interference, which is extremely less than the number of BS antennas (Rusek, F. *et al.*, 2013), (Zhu, X. *et al.*, 2016).

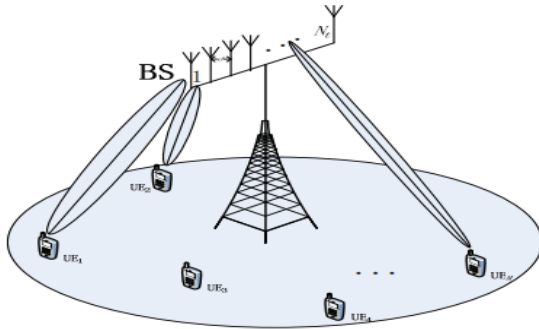


Figure-1. Multi-user-MIMO system where the BS has M transmitter antennas, and the number of active terminal (Larsson, E.G. *et al.*, 2014).

In contrast, conventional massive multi user-MIMO use a huge number of antennas at the BS; a hundred or thousands of antennas concurrently serve tens of terminal in the same frequency resource. The purpose of using a huge number of antennas in massive MIMO is to provide high performance and improving the sum rate, reliability, and power efficiency, for the reverse link. Coherent combining will make the system able to get a very high array gain, which permits a significant reduction in the transmitting power of each user. The users in a hexagonal cell are classified into two parts according to the scale fading. First, the center cluster users will only suffer from modest pilot contamination. Secondly, a group of users called the edge group may suffer from severe pilot contamination. The users at edge cell of a massive MIMO system suffer from severe pilot contamination as shown in Figure-2, which gives the poor QoS. It is required to develop better QoS for the edge users at edge cell and therefore we use the pilot reuse (PR) scheme.

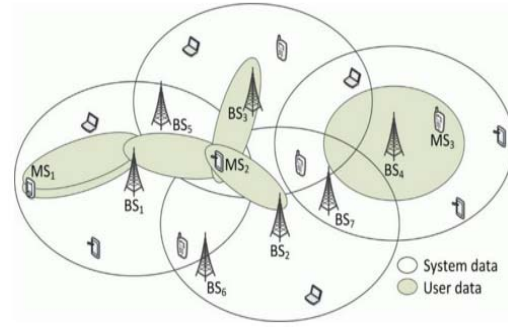


Figure-2. Massive MIMO for adjacent-cell users (Zhu, X., *et al.* 2016).

SYSTEM MODEL OF THE REVERSE LINK FOR MASSIVE MIMO

This paper analyse the massive MIMO system in a set of cells represented by L hexagonal cells with K uniform-randomly located users per cell. The BS in every cell $l \in L$ is prepared with M antennas and provides K single-antenna terminals in the uplink; in this setting, M is fixed however the K parameter is set adaptively as suggested in (Yin, H. *et al.* 2015). The forward link CSIs are estimated at every BS by reverse link pilot assuming channel exchange in TDD process. The channel model, where $h_{(l,k)} \in C^N$ symbolizes the channel response between BS j_{th} and active terminal k_{th} in cell l_{th} , with random direct $\theta_{lk} \in R^2$, through its coherence block involving of T_{th} channel uses. The received signal at base station (BS) j in a coherence symbol block, represented by Z_j as in Equation. (1).

$$Z_j = \sqrt{Q_{ul}} \sum_{l=1}^L \sum_{k=1}^K h_{jlk} x_{jlk} + \sum_{l=1, l \neq j}^L \sqrt{Q_{ul}} h_{ljk} x_{ljk} + n_j \quad (1)$$

Where, $h_{jl} = [h_{j1}, \dots, h_{jK}] \in C^{M \times K}$ is the channel transmitted from terminal k in cell l to BS j , x_{lk} is the transmitted signal, in UE k in cell l , Q_{ul} is the average SNR. The signals transmitted from active user k in cell l are assigned the power $Q_{ul} = \rho / d(\theta)_{lk}$, by $\rho > 0$. Each pilot signal is chosen $v \in C^M$, and a fixed pilot book $v = [v_1, \dots, v_B]$ where:

$$v_{b1}^H v_{b2} = \begin{cases} B & b_1 = b_2 \\ 0 & b_1 \neq b_2 \end{cases} \quad (2)$$

The assumption is that the active users UE at location θ_{lk} use transmit power of $\rho / d(\theta)_{lk}$ per



symbol, where ρ is the transmitted power and $d(\theta)_{lk}$ is the channel attenuation parameter to the serving BS. The resulting average SNR at every antenna of the providing BS is ρ/σ^2 where σ^2 is the noise variance per symbol, and the typical SINR also becomes the identical for all UEs in a cell since the uplink interference that affects a UE is independent of its own position. The ρ parameter is chosen so that all UEs in the cells comply with their amplifier power limitation. The transmitted power is calculated by:

$$Q_{ul} = \rho / d(\theta)_{lk} \quad (3)$$

In addition, the pilot signals, B , form an orthogonal basis, where β parameter is a pilot reuse signal, the transmitted pilot signal by k terminal in cell l , U_{ilk} , where $ilk \in \{1, 2, \dots, B\}$ is the index of the pilot sequence used by the UEs k in cell. Pilot reuse which supports the relation between the orthogonal pilot (B) and a number of active users (K) by $B = \beta K$, where $\beta \geq 1$ is called the pilot reuse, which is able to suppress inter-cell interference if the pilots are allocated carefully in the network.

The base station is capable of obtaining perfect CSI from the reverse link pilots, which is the imperfect CSI. We signify the linear receive combining matrix by: $A = [a_1 \ a_2 \ \dots \ a_K] \in C^{M \times K}$ where the column a_k is allocated to the k_{th} UE. Based on the MRT, ZF, and MMSE techniques for reverse link detection, this provides:

$$A = \begin{cases} H & MR \\ H(H^H H)^{-1} & ZF \\ (HQ^{ul} H^H + \sigma^2 I_M)^{-1} H & MMSE \end{cases} \quad (4)$$

where $H = [h_1 \ h_2 \ \dots \ h_K]$, which contain all user channels, σ^2 is variance noise and I_M identity matrix.

FRACTIONAL PILOT REUSE

In order to achieve higher SE, the same carrier frequencies need to be reused in the neighboring cells by following specific reuse patterns. To obtain the channel state information, the number of available pilot sequences is seen as finite and limited by the channel behavior. This is because their duration cannot span larger than the coherence interval of the estimated channel. Consequently, mobile users in different cells need to reuse the same pilot sequences and this will result in corrupted channel estimation at each BS. This phenomenon is known as pilot contamination and considered as the main impairment that affects massive MIMO cellular network (Hoydis, J. *et al.*, 2011). In addition, the channel estimation for the reverse link is done at the BS by letting

all users send different pilot symbols. The time required for active user K to base station M the pilot reuse is independent of the number of antennas at BS.

Nevertheless, the performance of these multi-user, multiple antenna aided systems is highly depending on the accuracy of the CSI. The transmission signals in every frame reserved for the reverse link (UL) pilot reuse symbols. There is no forward link (DL) pilot reuse symbols and no response of channel state information, so the BS can process both reverse link and forward link signals using the reverse link channel reciprocity as shown in Figure-3a.

In Figure-3b the active users terminal of cell group 1 sent pilot signal through the period time T whereas the users terminal k in cell groups 2 and 3 receive a forward signal from the BS. The BS that receives pilots from cell group 1 will avoid infected pilots from cells 2 and 3. The interference merely originates from the forward link data transmitted by the base station. Moreover, the indication of pilot reuse introduced for employing orthogonal pilot subsets in adjacent cells, improving the number of required subgroups and the number of scheduled users per cell that increases the overall SE, for both the reverse and forward link (You, L., *et al.*, 2015).

The pilot reuse β structure at the BS determine the β pattern and assigns the available pilot signals to the UEs. Due to the slow-deferent of the long-term channel statistics, it is acceptable to exploit the statistical CSI at the BS to perform pilot scheduling. With the resulting of β pattern, the UEs transmit the respective assigned pilot signals periodically to enable the BS to achieve the estimated channel (Ngo, H.Q. *et al.*, 2013). The channel estimation performance might degrade because of the β parameter, thus it is acceptable to design the UL and DL data transmissions to be robust to the channel estimation error.

To get the pilot reuse β and fractional pilot reuse β_f , in addition, the number of arranged active users per cell K , that amplify the total SE in the cell, we will use the orthogonal pilot sequence (B), which given by:

$$B = K(\beta_f + (1 - \beta_f)\beta) \quad (5)$$

From equation (5) when the pilot reuse factor $\beta_f = 1$ the number of orthogonal pilots is $B=K$, on the other hand, when the pilot reuse factor is $\beta_f = 0$ the number of orthogonal pilots is $B = \beta K$. The interval pilot reuse factor is between, $\beta_f = [0,1]$.

The pilot contamination is the most serious problem in multi-cell TDD systems with very large number of BS antenna arrays. In reverse link training, every BS receives pilot symbols transmitted from the mobile users. Normally, the orthogonal pilot sequences are assigned to users in a cell so that the channel estimation for each user does not suffer interference from other users



in the same cell. Nevertheless, the same pilot sequences usage for users in other cells may contaminate the channel estimation. This phenomenon is called the pilot contamination which limits the achievable rate when the number of BS antennas, M , increased to infinity (Sakaguchi, K. *et al.*, 2015).

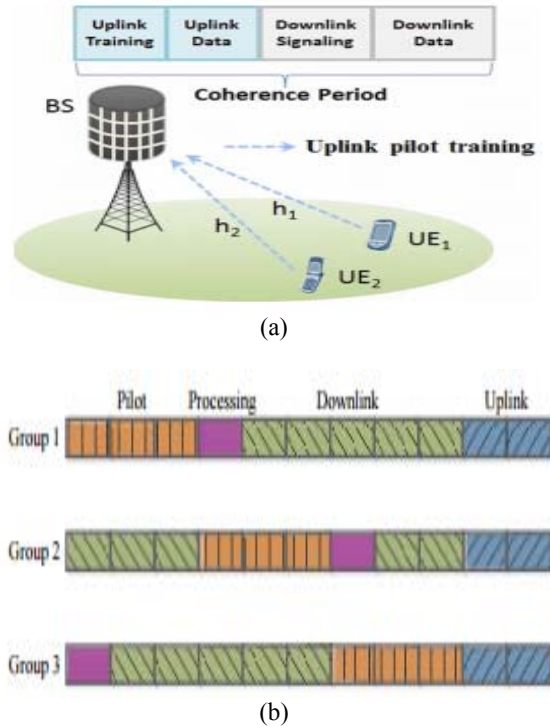


Figure-3. (a) Pilot contamination in reverse link, and (b) multiple users' frame structure with three groups (You, L., *et al.*, 2015).

Constantly, each BS chooses $K \leq B$ pilot symbols uniformly at random in each block, to avoid weighty pilot coordination. The average K/B of the cells reuse any given pilot symbol. We call $\beta = B/K \geq 1$ the pilot reuse factor such that $\beta k = B \leq S$. In this situation, the pilot symbol, sent by the typical UE and received at the typical BS, is interfered with by the subset of cells in which there is another UE reusing the same pilot symbol (Sohn, J.Y. *et al.*, 2015).

ACHIEVABLE SUM RATE

This section presents a thorough study on the impact of channel aging on the achievable sum rate of the system with linear decoding. Specifically, we have considered two standard linear techniques which are the MRT and MMSE. For both receivers, we have derived the closed-form lower bounds of the achievable sum rate with aged CSI, with the number of BS antennas. In addition, we study the impact of SNR with average sum-rate using linear schemes MMSE, MRT and ZF.

In massive MIMO system, the BS that has a large number of transmit antennas requires to achieve higher data rate using spatial multiplexing of users in the same time-frequency resource. Consider that the BS utilizes MRT or MMSE receivers from the lower bounds closed-form on the attainable sum rate for receivers with aged CSI. Therefore, in this paper we use close-fitting lower bounds on the attainable sum rate of the cellular networks in order to get strict sum rate bounds valid for arbitrary finite M and K , which provide an alternative perspective of quantifying the sum rate (Saxena, V. 2014). The average attainable data rate with perfect CSI with M antennas at the BS and a transmit power ρ per active user K orthogonal reverse link pilot symbols are transmitted over K channel with treating intra-cell interference and fast fading. To increase the achievable sum rate of the reverse link, we evaluate the closed forms of each system's performance. The achievable rates bound as follows:

$$\Gamma^U = \log_2 \left[1 + \frac{1}{E\left(\frac{I + \phi}{\Psi}\right)} \right] \leq E \log_2 \left[1 + \frac{\Psi}{1 + \phi} \right] \quad (6)$$

$$= \log_2 \left[1 + E\left(\frac{\Psi}{1 + \phi}\right) \right] \quad (7)$$

$$\Gamma^U = \log_2 [1 + \zeta_{jlk}] \quad (8)$$

Where Ψ , I , and ϕ are transmitted power, interference power, and noise power, and Γ represents the achievable sum rate.

The MRT decoding achieved by set $A = H$, the MMSE linear decoding is achieved by set $A = (HQ^H H^H + \sigma^2 I_M)^{-1} H$ is as given in (2) (Kong, C. *et al.*, 2015). This is depend on the corresponding channels model, a massive MIMO system with perfect CSI receiver and correspondent SINR ρ . Consequently, the SINR for the higher bound of vector regulation in the MMSE decoding is given by:

$$E\left[\frac{\Psi}{1 + \phi}\right] = E\left[\frac{Q_{jk} \left| h_k^T \frac{a_h}{\sqrt{k} \|a_h\|} \right|^2}{Q_{jk} \sum_{l=1, l \neq k}^K \left| h_k^T \frac{a_l}{\sqrt{k} \|a_l\|} \right|^2 + 1} \right] \quad (9)$$

$$= \left[\frac{Q_{jk}}{K \|a_k\|^2} \right] \quad (10)$$



$$= Q_{jk} \frac{(M-K+1)}{K} \quad (11)$$

Where Q_{jk} the power transmit is given by:

$$Q_{jk} = \frac{\rho d \sigma_H^2}{\sigma_v^2} \quad (12)$$

By using the equation upper sum rate in term of transmitted power and receive combining can be expressed as:

$$\Gamma_{mmse}^U = K \log_2 \left[1 + E \left(Q_{jk} \left| \frac{1}{\|A_h\|} \right|^2 \right) \right] \quad (13)$$

Where A_h the uplink linear receive combining matrix by $A = [a_1 a_2 \dots, a_k] \in C^{M \times K}$

$$\Gamma_{mmse}^U \geq \Gamma_{mmse}^U = K \log_2 \left[1 + \frac{1}{E \left(\frac{1}{\xi_{jlk}} \right)} \right] \quad (14)$$

In the linear decoding MMSE scheme can deal interference suppression anti signal power efficiency. The linear decoding MMSE is given by:

$$A_{mmse} = (HQ^u H^H + \sigma^2 I_M)^{-1} H \quad (15)$$

The MRT decoding is achieved by setting:

$$A_{mrt} = H \quad (16)$$

The ZF decoding is achieved by setting:

$$A_{zf} = (HH^H)^{-1} H \quad (17)$$

Accordingly, the signal transmitted from UEs to BS can be expressed by parameters Q , M , and K . To calculate SINR ζ_{jlk} and then gets the achievable sum rate using the linear decoding scheme MMSE:

$$\Gamma_{mmse}^U = K \log_2 \left(1 + E \left(Q_{jk} \frac{1}{(HQ^u H^H + \sigma^2 I_M)^{-1} H} \right) \right) \quad (18)$$

Where, equation (17) represents the effect of coherence channel estimation in term of power transmit.

$$\Gamma_{mmse}^U = K \log_2 \left(1 + Q_{jk} \frac{(M-K+1)}{K} \right) \quad (19)$$

From the equation (18), the achievable sum rate depend on the number of base station M , number of active users K , and transmit power from the cell j to any position of active users K . For MRT, assuming, $M > K$ the following approximation SINR ζ_{jlk} :

$$E \left[\frac{\Psi}{1 + \phi} \right] = E \left[\frac{Q_{jk} \left| h_k^T \frac{a_h}{\sqrt{k} \|a_h\|} \right|^2}{Q_{jk} \sum_{l=1, l \neq k}^K \left| h_k^T \frac{a_l}{\sqrt{k} \|a_l\|} \right|^2 + 1} \right] \quad (20)$$

$$\zeta_{mrt} = E \left[\left(Q_{jk} \left| \frac{1}{\|A_{MRT}\|} \right|^2 \right) \right] \quad (21)$$

Similarly, to achieve sum rate using linear decoding MRT, we can obtained by $A_{mrt} = H$, as

$$\Gamma_{mrt}^U = E \left[\log_2 \left(1 + Q_{jk} \left| h_k^T a_k \right|^2 \right) \right] \quad (22)$$

$$\Gamma_{mrt}^U = \log_2 \left(1 + Q_{jk} \left| h_k^T a_k \right|^2 \right) \quad (23)$$

$$= K \log_2 \left(1 + E \left(Q_{jk} \frac{1}{H} \right) \right) \quad (24)$$

$$\Gamma_{mrt}^U = K \log_2 \left(1 + \frac{Q_{jk} (M+1)}{Q_{jk} (M-1) + K} \right) \quad (25)$$

For ZF, assuming $M > K$ the following ergodic sum rate per user is attainable. Similarly, from the equation (20), we can simplify the SINR ζ_{zf} using linear decoding in term ZF, as

$$\zeta_{zf} = E \left[\left(Q_{jk} \left| \frac{1}{\|A_{MRC}\|} \right|^2 \right) \right] \quad (26)$$

The upper bound sum rate in term SINR using linear decoding zero forcing express as



$$\Gamma_{zf}^U = K \log_2 \left(1 + E \left(Q_{jk} \frac{1}{H(H^H H)^{-1}} \right) \right) \quad (27)$$

$$\Gamma_{zf} = K \log_2 (1 + Q_{jk} (M - K)) \quad (28)$$

ACHIEVABLE SPECTRAL EFFICIENCY

To achieve and enhance SE, it requires all active users sent pilot signals, which is orthogonal to others active users inside this cells. Consequently, the channel estimation evaluated between active users (UEs) and all base station, M , using the received pilot symbols, from the corresponding terminal, K . To make BS able to identify data signals, from any UEs require applied scheme a linear receive combining vector, a_{jk} , which determine signals that connect from BS j with K_{th} UEs, to reject interference from other UEs depends on the scheme linear vector which amplifies the signal, from the received signal Ψ_j , which can be defined as:

$$\Psi_j = \sqrt{Q_{jk}} \sum_{k=1}^K h_{jlk} x_{lk} + n_j \quad (29)$$

$$\hat{x}_{jk} = \sqrt{Q_{jk}} \sum_{k=1}^K a_{jk}^H x_{lk} + a_{jk}^H \sum_{(L,M) \neq (J,K)} \sqrt{Q_{lk}} h_{jlm} x_{lm} + a_{jk}^H n_j \quad (30)$$

Where, \hat{x}_{jk} is the transmitted signal from cell j to any random position active users UE in the same cell. If we assume that every filter output is decoded individually, then the achievable sum rate is given by:

$$\Gamma^{mmse} = \sum_{k=1}^K E_{SINR} [\log_2 [1 + \zeta_{jk}]] \quad (31)$$

Where Γ^{mmse} , represents the achievable sum rate.

In general, the exact distribution of SINR ζ_{jk} does not occur to be available in closed-form, other than for the explicit cases of independent and identically distributed and Rayleigh fading. With the MMSE achievable sum rate obtained in this form, the mandatory prospects are the equivalent as those essential for the estimation of the ergodic sum rate for shared information with desirable receivers (Lim, Y.G. *et al.*, 2015), (Bjornson, E. *et al.*, 2014), where cell l provides K_l UEs for each $l \in L$. The minimum average SE achieved in cell j is

$$SE_j = \left(1 - \frac{k\beta}{S} \right) \log(1 + \zeta_{jk}) [bit/s/Hz/cell] \quad (32)$$

Which, is an aggregate of the average SEs of the K_j UEs in that cell. From equation (3), we can express a

transmitted power in term SINR ζ_{jlk} and active users UE at position θ_{lk} , the SINR of the K_{th} active users (UEs) in cell j is:

$$\zeta_{jk}^{mmse} = \frac{\rho/d(z)_k |E_h \{a_{jk}^H \hat{h}_{jlk}\}|^2}{\sum_{m=1}^k Q_{jk} \rho/d(\theta)_{lk} E_h \{a_{jk}^H \hat{h}_{jlm}\}|^2 - \rho/d(\theta)_{lk} |E_h \{a_{jk}^H \hat{h}_{jlm}\}|^2 + \sigma^2 E_h \{a_{jk}\|^2} \quad (33)$$

Where, the performance $E_h \{a_{jk}^H \hat{h}_{jlm}\}$ is with acknowledge to the channel recognitions and pilot allocations. The pilot signal at BS the average received power depends on transmit power and path loss of the edge mobile terminal, to get the better SINR ζ_{jk} in this case if the terminal located nearest to the BS. Mostly this terminal has a lower path and hence a higher signal power at the BS. Then path loss increases when the terminals are placed at the cell edge and have lower signal power, while transmitting at their maximum power. This makes them prone to interference from non-orthogonal pilot transmissions in neighbouring cells (McKay, M.R. *et al.*, 2010). The channel estimation used linear decoding in the reverse link, where increasing data favourite at active users (UE) when several channel antennas are used to improve (SE) of an illogical active user in the cell.

$$\zeta_{jk}^{mmse} = \frac{Q_{jk} |E_h \{a_{jk}^H \hat{h}_{jlk}\}|^2}{\sum_{m=1}^k Q_{jm} E_h \{a_{jk}^H \hat{h}_{jlm}\}|^2 - Q_{jk} |E_h \{a_{jk}^H \hat{h}_{jlm}\}|^2 + \sigma^2 E_h \{a_{jk}\|^2} \quad (34)$$

To evaluation channel estimation, all the orthogonal pilot B available estimated directions in \hat{h}_{jlm} , j are utilized in the MU-MMSE detector so that BS j can also actively suppress parts of inter-cell interference when $\beta > K$ (Bethanabhotla, D. *et al.*, 2015). Therefore, the detector can actually maximize the SINR. Moreover, equation (28) shows that the reverse link SINR ζ takes the form of a generalized Rayleigh quotient (Li, X. *et al.*, 2015). Therefore, a new multi-cell MMSE (M-MMSE) detector can be derived to maximize this SINR ζ for given channel estimates:

$$a_{jk}^{mmse} = \left(\sum_{l=L}^k Q_{jk} \hat{h}_{jk} \hat{h}_{jk}^H + \sum_{l=L}^k Q_{jk} a_{lk} + \sigma^2 I_M \right)^{-1} h_{jlk} \quad (35)$$

Where, $E_h \{a_{jk}^H \hat{h}_{jlk}\}$ effective channel is the desired signal, and a_{lk} , linear receive combining vector, based on channel state information (CSI).

$$\zeta_{jk}^{mmse} = \frac{B}{(\sum_{l \in B} \alpha_{jl}^2 \frac{K}{M} + \frac{\sigma^2}{M\rho}) (\sum_{l=b}^K \alpha_{jl}^2 v_{jlk}^H v_{jlm} + \frac{\sigma^2}{\rho}) + (\sum_{l=b}^K \alpha_{jl}^2 \frac{\alpha_{jl}^2 - \alpha_{jk}^2}{M}) (v_{jlk}^H v_{jlm} - B)} \quad (36)$$



The spectral efficiency used the propagation parameters and pilot allocation, α_{jL}^1 and α_{jL}^2 . Consequently, it can define the average ratio between channel variance to base station in cell j , and channel variance to base station in active users in cell, as shown in the following equation:

$$\alpha_{jL}^w = E_{jlm} \left(\frac{d_j(jlm)}{d_l(ilm)} \right)^w, w=1,2 \quad (37)$$

Where $\mathbf{v}_{ljk}^H \mathbf{v}_{ilm} = B$. orthogonal pilot signals form basis when $ljk = ilm$

$$\sum_{l=L}^K \sum_{m=1}^K \alpha_{jl}^w \mathbf{v}_{ljk}^H \mathbf{v}_{ilm} = \sum_{m=1}^K \alpha_{jL}^2 \sum_{m=1}^K \mathbf{v}_{ljk}^H \mathbf{v}_{ilm} = \sum_{l=L}^K \alpha_{jl}^w \quad (38)$$

$$\sum_{m=1}^K \mathbf{v}_{ljk}^H \mathbf{v}_{ilm} = \begin{cases} B & l = L_j \\ 0 & l \neq L_j \end{cases} \quad (39)$$

Where, the SE cell, l , serves K_j UEs for all $l \in L$. The average ergodic sum SEs is calculated with respect to different UEs positions, different pilot allocations within the cells, small-scale fading variations, and CSI estimation errors. It is the minimum SE which is indefinite for multi-cell scenarios with imperfect CSI. In order to calculate the closed-form SE expression for the MMSE, a lower bound on the average spectral efficiency achieved in cell j is defined as

$$SE_j^{MMSE} = \left(1 - \frac{k\beta}{S} \right) \log_2(1 + \xi_j^{MMSE}) [\text{bit/s/Hz/cell}] \quad (40)$$

The equation of the spectral efficiency using MRT is expressed as:

$$SE_j^{MRT} = \left(1 - \frac{k\beta}{S} \right) \log_2(1 + \xi_j^{MRT}) [\text{bit/s/Hz/cell}] \quad (41)$$

NUMERICAL ANALYSIS

Figure-4, illustrates maximizing the attainable sum rate of the reverse link massive-MIMO regime the number of base station antenna simulated uplink achievable rate in equation (13), with its corresponding analytical approximation linear receive combining vector in (14) and (18). When the number of BS antennas increase from $M=10^1$ to $M=10^2$, the achievable sum rate has the same values when using MMSE and MRT, while, when the number of base BS M increase to more than $M > 10^2$, the achievable sum rate increases and the MMSE become better than MRT. Consequently, a throughput scheme of MMSE is achieved, giving a high sum rate and coverage of many active users in a set of cells. We show in Figure-4, the achieved sum rates of several systems as the number of antennas increases. It can be seen that the

MMSE perform better than the MRT. Therefore, the increase in number of base station antennas reserved many active users and an achievable high sum rate. That means that two linear pre-coding schemes, MMSE and MRT, are able to mitigate pilot contamination and suppress inter-cell interference better than MRT.

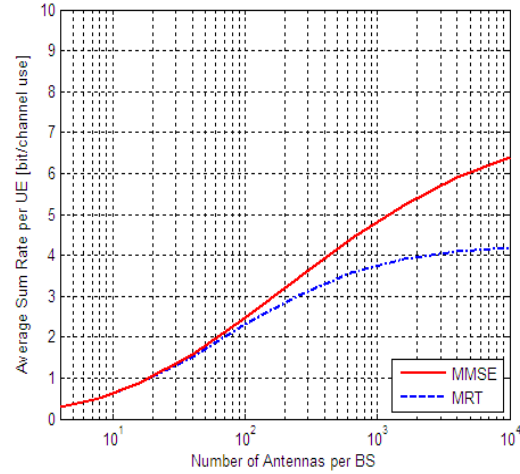


Figure-4. Attainable sum rate vs. number M of BS antennas.

Figure-5 displays how the spectral efficiency (SE) with number of antenna array BS, M , and the aimed from analytical lower and upper bounds for MRT and MMSE receivers with perfect CSI at SNR = 0 to 20 dB, when active user UE, $k=10$ users for CSI estimation from reverse link pilots, when coherence block length of a pilot sequences $S=K$. Obviously, the upper and lower bounds are very strict, especially at large number of M .

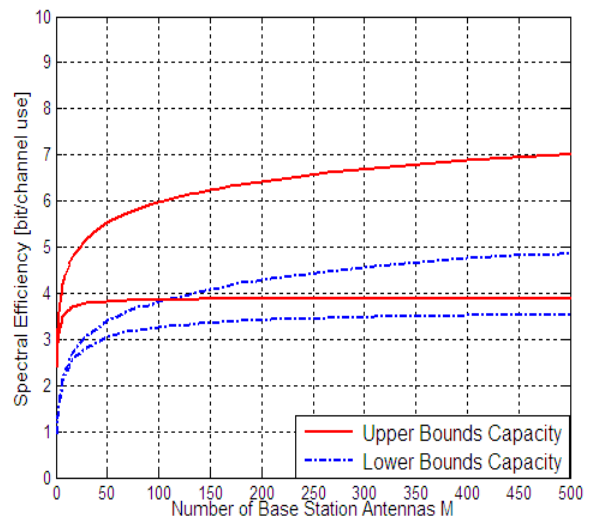


Figure-5. Spectral efficiency vs. the number M of BS antennas in terms of upper and lower capacity.



Figure-5 illustrates the achievable spectral efficiency affected with an increased number of base station antenna, if the SNRs are adequately high SNR = 20 dB, the majority of the multi antenna is achieved at moderately at low of number of antennas M ; only slight improvements can be achieved by adding antennas $M = 100$. So, with SE when using the pilot reuse factor and selecting active users per cell, we can say that the spectral efficiency is enhanced, at upper bound, because the SE require a low number of base station antennas, while at SNR = 0 the spectral efficiency in massive MIMO requires huge of number of antennas M . Therefore, the upper bound gives the high spectral efficiency better than lower bound at increased number of BS antenna in addition to the value of SNR

Figure-6 shows the attainable sum rate of every active users K (in the 1 km² area) of the number of antennas M . The simulation shows that the linear schemes both of optimal MMSE, MMSE, and ZF in both of low and high SNR are a convergence of the result, while this simulation result is different from MRT. Figure-6 shows that the linear decoding scheme using MRT gives the optimal values at low SNR. While giving a low result of sum rate of MRT at high SNR, this means that the MRT has lower efficiency in case a transport data rate at high SNR comparison with MMSE and ZF. However, the linear decoding scheme of ZF gives lower values of data rate at lower SNR. From Figure-6, it can be seen that the average sum rate is proportional with the SNR, using the linear scheme of MMSE, ZF and optimal MMSE. In addition, the sum rate is reverse proportional with MRT.

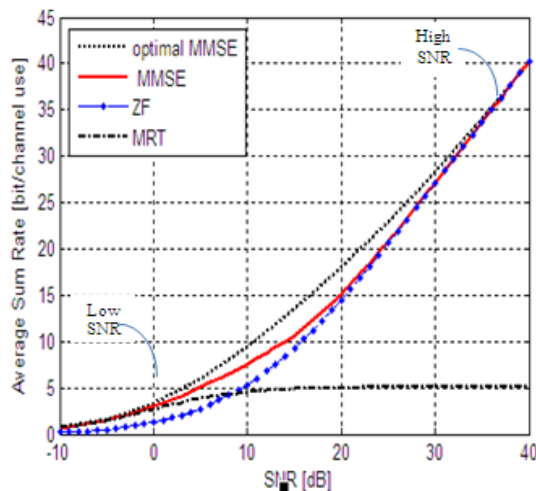


Figure-6. Average sum rate vs. signal-to-noise ratio (SNR), at number of base stations $M = 4$.

Figure-7 shows that the average sum rate both of MMSE, optimal MMSE, ZF and MRT are similar at low SNR and more convergence. While, these results also have more convergence at high SNR except for MRT. Then we can conclude that the number of base station $M = 4$, the linear schemes has more convergence with optimal MMSE, MMSE and ZF, in both low SNR and high SNR.

In contrast the number of base stations $M = 10$, the linear decoding only have convergence at low SNR, except for MRT.

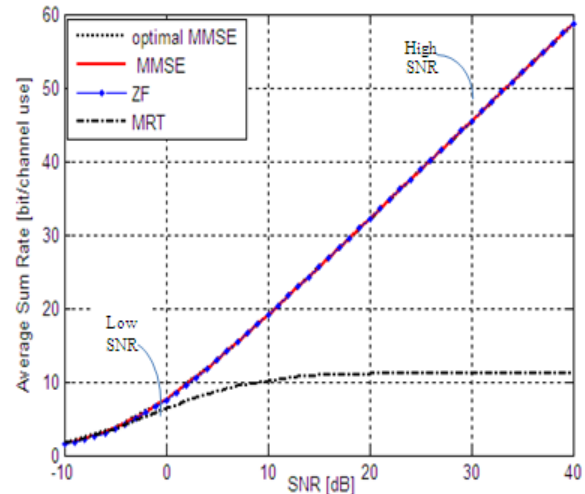


Figure-7. Average sum rate vs. average SNR, at number of base stations $M=10$.

CONCLUSIONS

In this paper, we considered a reverse link multiuser cellular system, where the BS is provided with a huge number of antennas array, which increases the data rate in addition to the enhanced SE. The channels are assumed to be developed by the base station through training symbols transmitted by the active users. With estimated channels at the base station, we derived sum rates that are achievable with MMSE and MRT. In addition, we have investigated the achievable sum rate of reverse link massive MIMO systems and study the effect of channel aging between BS and active users. In addition, we derived the lower bounds and upper bound of ergodic sum rate for both MRT and MMSE with channel prediction by increasing the number of base station antennas and active users. Moreover, the spectral efficiency is enhanced, which is robust in suppressing signal-to-interference noise ratio (SINR), however, this is not useful for inter-user interference suppression and multiplexing. We can say that the average sum rate is proportional with the SNR, using the linear scheme of MMSE, ZF and optimal MMSE. In addition, the sum rate reverse is proportional with MRT, also, this is the result when the number of base stations $M=10$, the linear scheme both of MMSE, ZF and optimal MMSE have more convergence except MRT, while when $M = 10$ the linear decoding schemes only have convergence except for MRT. However, at high SNR a larger number of antennas, M at BS is better than a small number of antennas.

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