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PERIODIC BINARY SIGNALS WITH ZERO CROSS CORRELATION BASED ON WALSH SEQUENCES

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ABSTRACT

This study considers periodic binary signals coded using Walsh orthogonal functions. For periodic signals, it is shown that groups of signals that do not have mutual frequency components exist for each Walsh system with size $N = 2^n$ (n = 1, 2, 3, . . .). These groups of periodic signals have zero cross correlation (CC) or zero multiple access interference (MAI). Ensembles of periodic signals with zero MAI can be created from these groups of signals. The maximum number of binary signals in these ensembles is defined by the number of groups and is equal to n +1. Signals with zero MAI are orthogonal in the time domain when a time shift is present between them and can be separated without tight synchronization. Examples of separating filters are presented. Applying this principle to radio signals enables the creation of n+1 independent radio frequency signals. The results of this study can be used in asynchronous CDMA communication, telemetric networks (e.g. Wireless Body Area Networks (WBANs), Wireless Sensor Networks (WSNs)), and optical systems.

Keywords: binary signals, CDMA, cross correlation, cyclic orthogonal walsh-hadamard codes (COWHC), multiple access interference (MAI).

INTRODUCTION

Correlation properties of Walsh-Hadamard (WH) sequences have been considered in numerous publications [1]–[9]. Zero cross correlation (CC) (i.e., zero MAI) of a complete set of sequences of WH systems (i.e., matrices) with size $N=2^n$ (n = 1, 2, 3, . . .) exist only when the time shifts between all N sequences equal zero. The authors of previous studies noted the “poor” CC properties of a complete set of WH sequences when time shifts were present between sequences.

Previous studies have used numerical analysis methods related to the correlation properties of coded signals. In this study, we use the frequency approach (i.e., the frequency method of analysis) which utilizes the frequency characteristics of Walsh functions [12]–[17]. WH sequences corresponding to periodic signals with zero MAI are also known as Cyclically Orthogonal subsets (i.e., groups) of Walsh functions [3] or Cyclic Orthogonal WH Codes (COWHC) [4], [8] or signals with ideal cross-correlation (ICC) properties [14].

Two previous papers [4], [8] stated that WH systems of size $N=2^n$ (n = 1, 2, 3, . . .) have n+1 sequences that correspond to periodic binary signals with zero MAI. These two papers obtained this result based on computer simulations of the CC properties of WH functions. However, those conclusions were drawn without any analytical (i.e., mathematical) proof and only addressed n+1 functions, not n+1 groups of functions. Reference [14] demonstrated, that each WH system of periodic signals of size $N=2^n$ (n = 1, 2, 3, . . .) have n+1 groups of sequences which have ICC properties or zero MAI. Because reference [14] is a scientific magazine with low circulation (that is published by the Russian Institute of Radionavigation and Time (RIRT)), these results were overlooked by the scientific community. The only mathematical proof regarding the existence of n+1 cyclically orthogonal subsets (groups) in Walsh systems of size $N=2^n$ (n =1, 2, 3, . . .) was published in reference [3]. Reference [3] provided a mathematical proof using numerical methods, whereas reference [14] used the frequency approach (i.e., frequency method).

As shown in this study, the frequency approach [14] provides a clear physical interpretation and explanation of WH sequence cyclic orthogonality [3], [4], [8]. In addition, the frequency approach demonstrates that it is possible to create n+1 independent radio signals using periodic WH functions with zero MAI for binary phase modulation of radio signals.

FREQUENCY CHARACTERISTICS OF WH FUNCTIONS

Binary coded signals are used in modern communication [1]-[5], [7], telemetric [6], [8] and optical systems [9]. As a rule, these signals have the shape of rectangular pulses with time duration equal to T. These positive and negative unit amplitude rectangular pulses are binary coded and represented as “+” or “-” symbols based on the polarity of signals or phase (i.e. 0 or 180 degrees) of the pulses for radio signals (Figure-1a and Figure-1b respectively).

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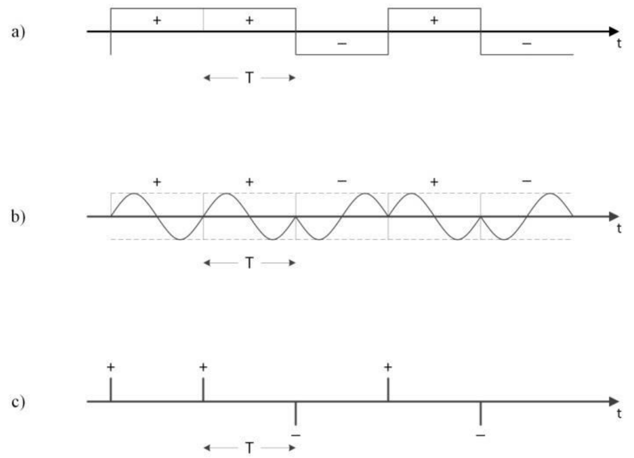

Figure-1. Periodic binary signals.

This study considers the CC properties of periodic binary signals coded with Walsh orthogonal functions. J.L Walsh introduced binary systems of orthogonal functions in 1923 [10]. These systems are used often in the theory of discrete signals and, in practice, were widely used in IS-95 (cdma One), cdma2000 networks under the name "variable-length orthogonal codes" [5], and modern wireless networks. Walsh systems only exist for $N = 2^n$ (n =1, 2, 3, . . .), where N is the size of the system and n is the set of positive integers.

Systems of discrete Walsh functions can be represented in different ways; one convenient way is in matrix form whereby Walsh orthogonal functions are represented using Hadamard matrices [11]:

$$H_N = \begin{bmatrix} H_{N/2} & H_{N/2} \\ H_{N/2} & -H_{N/2} \end{bmatrix} \quad (1)$$

where H_N is a Hadamard matrix of order (or size) $N = 2^n$ (n =1, 2, 3, . . .), and $H_1 = [\ +\ ]$ is the Hadamard elementary matrix for N = 1 (n = 0). Figure-2 presents samples of Hadamard matrices for N =2, N =4 and N = 8 based on Equation (1). The matrices are square matrices of size N x N, where the horizontal size (i.e., the number of columns) defines the length of sequences of "+" and "-" symbols, and the vertical size (i.e., the numbers of rows) defines the number of these sequences.

In Figure-2, k is the row number expressed in a decimal number, $(k)_2$ is the row number expressed in binary form, and G (G_1, G_2, . . .) is the group number of sequences corresponding to signals with ICC properties between the groups. In this presentation of Hadamard matrices (Figure-2), all sequences (i.e., rows) of "+" and "-" symbols are discrete orthogonal Walsh functions. The systems of orthogonal Walsh functions represented in the form of Hadamard matrices (1) (Figure-2) are usually called the system of Walsh functions in Hadamard ordering. In this study, these H_N matrices (Figure-2) are called WH matrices.

The primary properties of binary coded signals are defined by the properties of coding sequences which consist of sequences of "+" and "-" symbols. These sequences of "+" and "-" symbols can be represented in the time domain as sequences of short single pulses (e.g. unit Dirac delta functions or sampling pulses) (Figure-1c), the polarity (+/-) of which is defined by the sequences of "+" and "-" symbols (Figure-2). In this study, these sequences of short single pulses as functions of time are referred to as coding functions and are denoted as $f_{TN}(t)$ (Figure-1c). As is noted above, numerical analysis methods have been used in studies investigating the CC properties of coded signals [1]-[9]. This study employs the frequency approach (i.e., frequency method of analysis) which uses frequency characteristics of coding functions $f_{TN}(t)$ (Figure-1c) as functions of time [12]-[17]. In this study the terms "frequency spectrum" and "frequency characteristics" are used interchangeably for coding functions $f_{TN}(t)$ as functions of time (Figure-1c), and for sequences of "+" and "-" symbols which are discrete orthogonal Walsh functions (Figure-2).

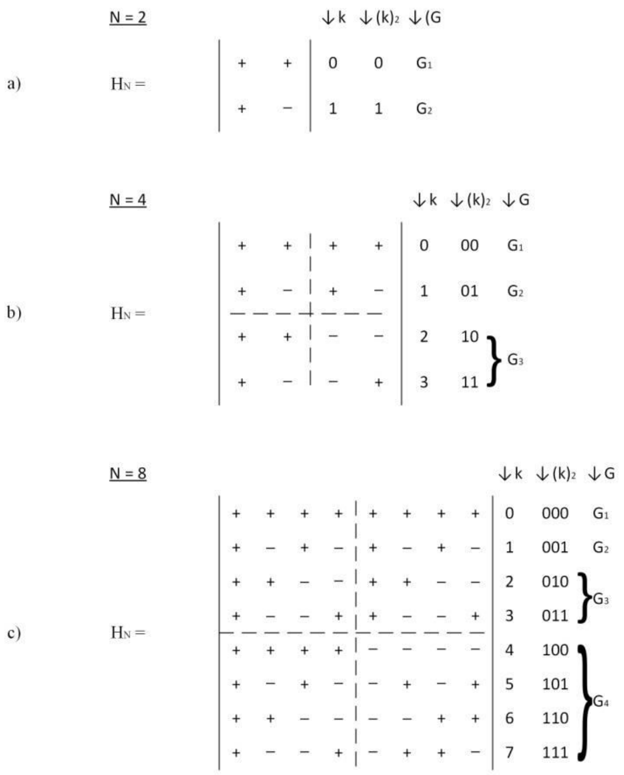

Figure-2. Walsh-Hadamard matrices.

References [12]-[13] demonstrated that the amplitude frequency spectrums of coding functions $f_{TN}(t)$ that correspond to sequences of WH matrices (Figure-2) can be represented as:

$$S_{TN}{}^{k}(\omega) = N \left| \prod_{m\,=\,1,2..n} \begin{pmatrix} cos \\ sin \end{pmatrix} 2^{n-m}\,\omega T/2 \right)_{(k)_2} \right| \quad (2)$$

where $|\prod(\ ..... \ )|$ is the module of multiplication. The absolute value of $\prod(\ . \ . \ . \ )$ is used because amplitude

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frequency characteristics can only have positive values. The multiplication $\prod(\ .\ .\ .\ )_{(k)2}$ contains sequences of multipliers that correspond to values of k in binary form, where cos (. . .) functions correspond to 0 and sin (. . .) functions correspond to 1. The total number of multipliers in Equation (2) is equal to n, because m = 1, 2, 3, . . ., n.In the general case, for $N=2^n$ (n =1, 2, 3, . . .), $S_{TN}{}^{k}(\omega)$ (2) can be represented as the matrix column in Equation (3):

$$S_{TN}{}^{k}(\omega) = N\begin{bmatrix} |\cos N\omega T/4 \ \ldots \ldots \ \cos \omega T/2| \\ |\cos N\omega T/4 \ \ldots \ldots \ \sin \omega T/2| \\ \vdots \\ \vdots \\ |\cos N\omega T/4 \ \ldots \ldots \ \cos \omega T/2| \\ |\cos N\omega T/4 \ \ldots \ldots \ \sin \omega T/2| \\ ------------ \\ |\sin N\omega T/4 \ \ldots \ldots \ \cos \omega T/2| \\ |\sin N\omega T/4 \ \ldots \ldots \ \sin \omega T/2| \\ \vdots \\ \vdots \\ |\sin N\omega T/4 \ \ldots \ldots \ \cos \omega T/2| \\ |\sin N\omega T/4 \ \ldots \ldots \ \sin \omega T/2| \end{bmatrix} \begin{array}{cc} \downarrow k & \downarrow (k)_2 \\ 0 & 00..00 \\ 1 & 00..01 \\ \vdots & ..... \\ \vdots & ..... \\ N/2\text{-}2 & 01..10 \\ N/2\text{-}1 & 01..11 \\ & \\ N/2 & 10..00 \\ N/2+1 & 10..01 \\ \vdots & ..... \\ \vdots & ..... \\ N-2 & 11..10 \\ N-1 & 11..11 \end{array} \qquad (3)$$

In the N =8 (n =3) case, from Equation (2) $S_{TN}{}^{k}(\omega)$ we obtain Equation (4):

$$S_{TN}{}^{k}(\omega) = N\begin{bmatrix} |\cos 2\omega T \cos \omega T \cos {}^{\omega T}/_{2}| \\ |\cos 2\omega T \cos \omega T \sin {}^{\omega T}/_{2}| \\ |\cos 2\omega T \sin \omega T \cos {}^{\omega T}/_{2}| \\ |\cos 2\omega T \sin \omega T \sin {}^{\omega T}/_{2}| \\ |\sin 2\omega T \cos \omega T \cos {}^{\omega T}/_{2}| \\ |\sin 2\omega T \cos \omega T \sin {}^{\omega T}/_{2}| \\ |\sin 2\omega T \sin \omega T \cos {}^{\omega T}/_{2}| \\ |\sin 2\omega T \sin \omega T \sin {}^{\omega T}/_{2}| \end{bmatrix} \begin{array}{cc} \downarrow k & \downarrow (k)_2 \\ 0 & 000 \\ 1 & 001 \\ 2 & 010 \\ 3 & 011 \\ 4 & 100 \\ 5 & 101 \\ 6 & 110 \\ 7 & 111 \end{array} \qquad (4)$$

The sequences of cos (…) and sin (…) inside of matrices (3) and (4), based on Equation (2), correspond to the matrix rows numbers in the binary $(k)_2$. For example in matrix (4), the sequence cos (…) * cos (…) * cos (…) corresponds to $(k)_2$= 000, the sequence cos (…) * cos (…) * sin (…) corresponds to $(k)_2$= 001, the sequence sin (…)* cos (…)* cos (…) corresponds to $(k)_2$= 100, and so on. Figure-3 shows the amplitude characteristics (dotted lines) of spectrum $S_{TN=8}{}^{k}(\omega)/N$ (4) for all sequences of WH matrix with size of N=8, i.e. sequences with k=0, 1, 2, 3, 4, 5, 6, 7 (Figure-2).

The amplitude frequency characteristics in Equation (2) are periodic functions in the frequency domain, with a period equal to ω = [0, 2π/T) (Figure-3).

The frequency spectrum of coded rectangle pulses (Figure-1a) is a multiplication of the coding functions $f_{TN}(t)$ spectrum (2) and the spectrum of a rectangle pulse with time duration T, because coded rectangle pulses are the convolution of coding functions $f_{TN}(t)$ and a rectangle pulse with time duration T. Furthermore, the frequency interval ω = [0, 2π/T) is equal to the width of the primary lobe of the frequency spectrum of a rectangle pulse with a time duration T.

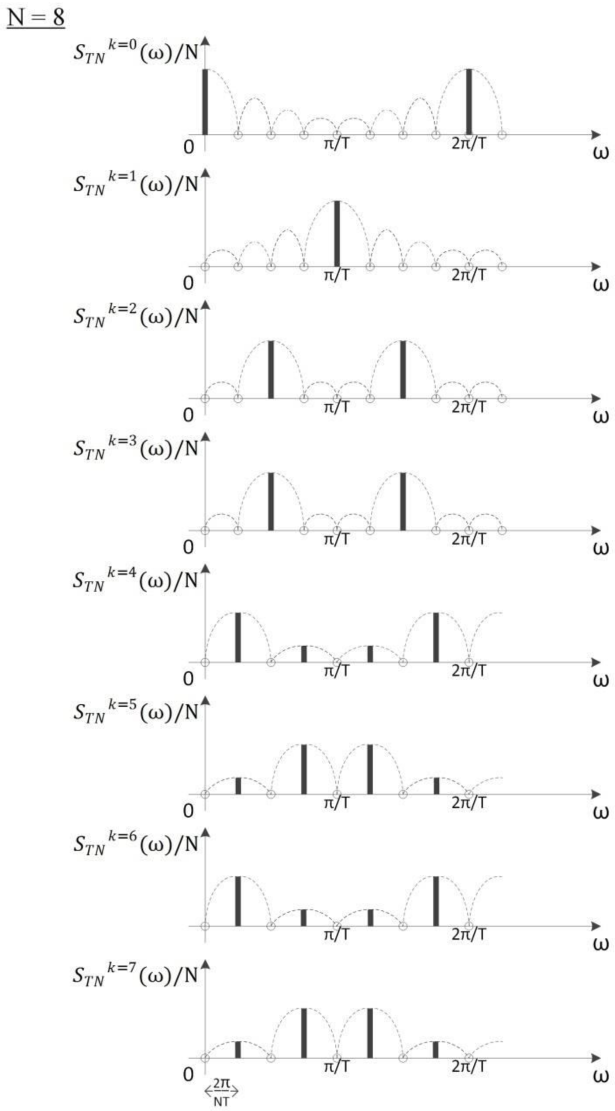

Figure-3. Amplitude frequency spectrum.

When k is represented in binary as $(k)_2$, whichcorresponds to a combination of sin(…) and cos(…) in multiplication, Equation (2) can be easily used to calculate the frequency characteristics of WH functions of any size $N=2^n$ (n=1, 2, 3,...).

The frequency characteristics of Walsh functions were considered in references [15]-[17]. Certain formulas [15]-[17] are so similar, but not identical, to Equation (2), which represents k in binary as $(k)_2$. Furthermore, those previous studies did not discuss the existence of WH functions with ICC properties or zero MAI applied to periodic signals.

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Additionally, the amplitude frequency spectrum of any discrete sequence can be calculated using Equation (2).This calculation should be performed using the discrete Walsh-Hadamard Transform (WHT) [18]. After using the WHT, any original discrete sequence can be represented as a composition of WH functions. Subsequently the amplitude frequency spectrum of the original discrete sequence can be calculated using the frequency spectrum of WH functions (2).

GROUPS OF PERIODIC ORTHOGONAL SIGNALS WITH ZERO MAI BETWEEN THE GROUPS

Let us consider periodic coded signals with a period of repetition equal to NT. The amplitude frequency spectrums of periodic coding functions on an interval of $\omega = [0, 2\pi/T)$ are discrete, with a frequency interval $\Delta\omega = 2\pi/NT$. The amplitude values in these discrete points are equal to S_{TN}^{k} ($\omega = \ell * \Delta\omega = \ell * 2\pi/NT$), where ℓ = 0, 1, 2, 3,..., N-1. Figure-3 shows the discrete amplitude spectrums $S_{TN}^{k}(\omega)$ (2) (unbroken vertical lines) of the periodic coding functions $f_{TN}(t)$, which correspond to the coding sequences of orthogonal Walsh functions with size N = 8 (i.e., $S_{TN=8}^{k}(\omega)$ (4)), for the different values of k = 0, 1, 2, ..., 7 (Figure-2c).

The following conclusions can be drawn by analyzing the discrete amplitude spectrums of periodic coding functions:

For any system of Walsh orthogonal functions with size $N = 2^n$, there exist some groups of periodic sequences which do not have common (or mutual) discrete frequency components. These groups of periodic sequences are orthogonal in the frequency domain, which indicate that these groups are also orthogonal in time domain if any time shift exists between them.

For instance, in the N=8 case, the first group, G_1, corresponds to the WH sequences (rows) with k=0 (Figure-2c). This group has only one sequence (i.e., one row). The periodic coded signal that corresponds to this sequence has only one frequency: ω=0 (Figure-3). The same results can be obtained from (2) for N=2 and N=4.

The second group, G_2, corresponds to the WH sequences (rows) for N=8 with k=1 (Figure-2c). This group has only one sequence (i.e., one row). The periodic coded signal that corresponds to this sequence has only one frequency: $\omega = \pi/T$ (Figure-3). This signal is orthogonal in frequency domain to periodic signals of group G_1. The same results can be obtained from (2) for N=2 and N=4.

The third group, G_3, corresponds to the WH sequences (i.e., rows) for N=8 with of k=2 and k=3 (Figure-2c). The periodic coded signals that correspond to these sequences have two discrete frequencies: $\omega = \pi/2T$ and $\omega = 3\pi/2T$ (Figure-3). Both of these signals are orthogonal in the frequency domain to the periodic signals of groups G_1 and G_2, but both of these signals (k=2 and k =3) have common frequency components and are not mutually orthogonal in the frequency domain. The same results can be obtained from (2) for N=2 and N=4.

The fourth group, G_4, corresponds to the WH sequences (i.e., rows) for N=8 with value of k = 4, 5, 6 and 7 (Figure-2c). The periodic coded signals that correspond to these sequences have four discrete frequencies: $\omega = \pi/4T$, $\omega = 3\pi/4T$, $\omega = 5\pi/4T$ and $\omega = 7\pi/4T$ (Figure-3). All of these four periodic signals are orthogonal to the periodic signals of groups G_1, G_2, and G_3 in the frequency domain, but all four of these signals have common frequency components (Figure-3) and are not mutually orthogonal in the frequency domain.

In the general case, for $N=2^n$, it is possible to show that the last group, the $(n + 1)^{th}$ group G_{n+1}, corresponds to the WH sequences (i.e., rows) with k =N/2, N/2 + 1, ... , N-1. This group applies to the lower half of the WH matrix (size of $N = 2^n$) and consists of the largest number (i.e., N/2) of sequences (3). The periodic signals that correspond to these sequences have N/2 discrete frequencies. All of these N/2 periodic signals are orthogonal to the signals of all the other groups (G_1, G_2, G_3, ..., and G_n) in the frequency domain. However, all of these N/2 signals have common frequency components and are not mutually orthogonal in the frequency domain.

This conclusion regarding groups of signals that are orthogonal in the frequency domain only applies to periodic signals.

ENSEMBLES OF PERIODIC SIGNALS WITH ZERO MAI

When combining signals into a composition, we refer to the composition of signals as an “ensemble” of signals, where only one signal in mutually orthogonal groups of signals in the frequency domain will be represented. The entire ensemble will consist of periodic signals with zero CC properties or zero MAI.

In the case for N = 8 (n = 3), the ensemble of periodic signals can consist of any combination of signals, if only one signal is chosen from each group, corresponding to the following groups of WH matrix sequences: Group 1(G_1), Group 2 (G_2), Group 3 (G_3), and Group 4 (G_4) on (Figure-2c). For a WH matrix of size $N=2^n$, the maximum number of signals that can be used in an ensemble of signals with ICC is equal to the number of groups G, which is equal to n+1.

For a WH matrix of any size, it is possible to create certain ensembles with less than the maximum value of n+1 orthogonal signals with a wide permutation of choices from the groups G_1, G_2, . . . , G_{n+1} of signals. The choice depends of many real-world conditions such as the type of transmission channel, type of transmitting information, and type of possible disturbances (e.g., natural and/or organized), etc. These ensembles of signals with ICC properties can be used in asynchronous CDMA systems that are similar to Wireless Body Networks (WBANs) [8] and Wireless Sensor Networks (WSNs) [6].

The results of the practical implementation of these signals have been presented in reference [8]. The ensemble (i.e., unique set) of n+1 coded signals for N=8 (n=3), which correspond to WH functions with k number k=0, k=1, k=3, and k=5 (Figure-2c), were presented in [8] after some examinations. These coded signals for N=8 (k=0, k=1, k=3 and k=5) belong to different groups: G_1, G_2, G_3 and G_4 respectively (Figure-2c). In simulation

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experiments [8], is also possible to use the other ensembles of signals corresponding to k=0, 1, 2 and 6 (or k=7, or k=8) for N=8, which belong to groups G_1, G_2, G_3 and G_4 respectively (Figure-2c). All of these ensembles, like the ensemble shown in reference [8], consist of n+1 periodic signals with ICC properties coded by WH functions with N=8 (n=3).

SEPARATING FILTER EXAMPLES

A variety of filters can be used on the receiver side to separate signals with ICC properties. Figure-4 shows the simplest filters that can be used to separate periodic signals from an ensemble of signals for N=2 and N= 4. In Figure-4, blocks with R_T are shift registers at time T and operate similar to delay lines. The remaining blocks combine two signals (i.e., blocks containing "+" sum; blocks containing "-" difference) and multiply the resulting signals by a given factor, such as 1/2 and 1/4.

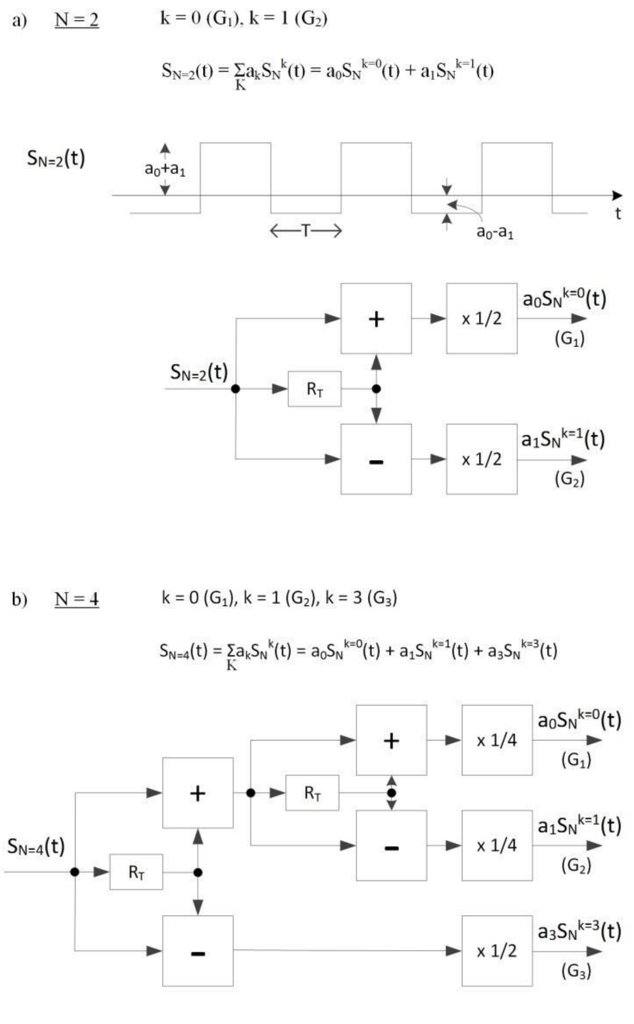

Figure-4. Samples of separating filters.

Input signals $S_N(t)$ are periodic rectangular pulses of duration T (Figure-1a, Figure-4a) with a period of repetition NT. These signals $S_N(t)$ form an ensemble of signals with ICC properties for N =2, and N = 4, and can be represented as the sum of signals $S_N(t) = \sum a_k S_N^k(t)$, which correspond to a different group G (G_1, G_2, G_3, . . .) of signals. For N =2, input signal $S_N(t)$ consists of 2 signals (Figure-2a), the first from Group G_1 with a value of k=0 and the second from Group G_2 with a value of k=1 (Figure-4a). For N =4, input signal $S_N(t)$ consists of 3 signals (Figure-2b) from groups G_1, G_2, and G_3 with k=0, k=1, and k=3 respectively (Figure-4b).

In Figure-4, the coefficients a_k are constant coefficients that correspond to the value of k. Figure-4 shows that signals $S_N^k(t)$ on the outputs filters are separated by different values of k. It is necessary to emphasize that the filters in Figure-4 are the simplest filters and only illustrate the procedure for separating periodic signals which belong to different groups G (G_1, G_2, G_3, . . .) of periodic signals with ICC properties or zero MAI. After separating these signals, additional filtering procedures may be used depending of the type of noise (i.e., distortions) that may appear during the signal transmission process. The separation of signals in the two filters (Figure-4) does not depend on the time shifts between signals.

BINARY PHASE MODULATION OF RADIO SIGNALS

Let us consider another interesting feature of periodic signals with ICC properties (or zero MAI) applying to radio signals. As noted in introduction of this study, WH functions can be used for phase modulation (i.e., binary phase modulation) of the frequency of radio signals (Figure-1b). In this case, there will be n+1 independent frequency channels after binary phase modulation.

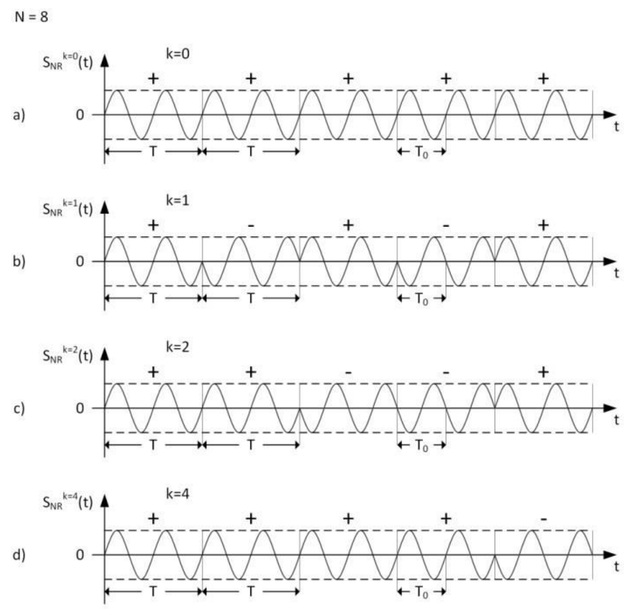

Figure-5. Radio signals $S_{NR}^k(t)$.

The use of periodic WH coding functions (N=8) that correspond to k=0 (Group 1, G_1) (Figure-2c) for the binary phase coding of a radio signal with a frequency ω_0 leads to the radio signal $S_{NR}^{k=0}(t)$ shown in Figure-5a with

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frequency spectrum $S_{TNR}^{k=0}(\omega)$ (Figure-6a, unbroken vertical line). This case illustrates the absence of phase coding. In Figure-6a, the amplitude frequency spectrum of a rectangular radio pulse with a frequency ω_0 and duration T is represented as dotted lines. The primary lobe width of this amplitude frequency spectrum is equal to $\Delta\omega = 4\pi/T$, where $\omega_0 >> \Delta\omega=4\pi/T$.

In this case (k=0), a periodic sequence of rectangle radio pulses with a duration of T is the same as a continuous (i.e., not modulated) radio signal with frequency $\omega_0 = 2\pi/T_0$ (Figure-5a). The frequency spectrums of these coded rectangle signals $S_{TN}^{k=0}(\omega)$ are represented on Figure-3 for N=8. Frequency spectrum of radio signals $S_{TNR}^{k=0}(\omega)$ (Figure-6a) corresponds to the frequency spectrum of $S_{TN}^{k=0}(\omega)$ (Figure-3) transferred to radio frequency range $\Delta\omega=4\pi/T$ with a central frequency ω_0. This frequency spectrum of radio signal $S_{TNR}^{k=0}(\omega)$ (Figure-6a) consists of only one frequency ω_0 which is typically called a carrier frequency. The central frequency $\omega_0= 2\pi/T_0$ is considerably larger than frequency range $\Delta\omega=4\pi/T$ (i.e., that $T >> T_0$).

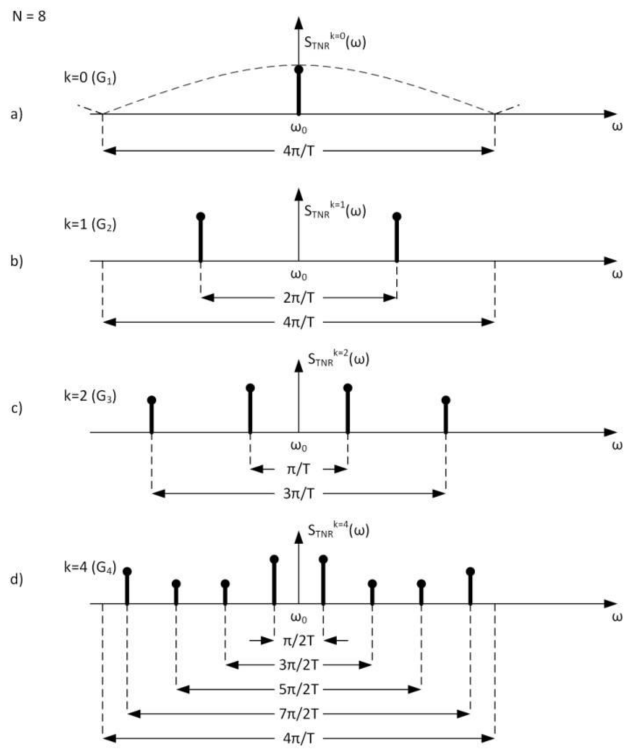

Figure-6. Frequency spectrum $S_{TNR}^{k}(\omega)$.

The use of periodic coding sequences (N=8) with k=1 (Group 2, G_2) (Figure-2c) for the binary phase coding of a radio signal with frequency ω_0 leads to the radio signal $S_{NR}^{k=1}(t)$ shown in Figure-5b with a frequency spectrum $S_{TNR}^{k=1}(\omega)$ (Figure-6b). This amplitude frequency spectrum $S_{TNR}^{k=1}(\omega)$ corresponds to the frequency spectrum of $S_{TN}^{k=1}(\omega)$ (Figure-3) that are transferred into the radio frequency range $\Delta\omega=4\pi/T$ with a central frequency ω_0. This spectrum $S_{TNR}^{k=1}(\omega)$ consists of two frequency components: $\omega = \omega_0 +/- \Delta\omega$, where $\Delta\omega = \pi/T$ (Figure-6b).

When k=2 (Group 3, G_3) and k=4 (Group 4, G_4) (N=8) (Figure-2c) there exist radio signals $S_{NR}^{k=2}(t)$ and $S_{NR}^{k=4}(t)$ with amplitude frequency spectra $S_{TNR}^{k=2}(\omega)$ and $S_{TNR}^{k=4}(\omega)$ respectively (Figs. 5c, 5d, 6c, and 6d).

Figure-6 shows that all of radio signals phase coded by periodic WH functions with ICC properties have discrete frequency spectra in the frequency range of $\Delta\omega = 4\pi/T$ with a central frequency ω_0. All of these radio signals have mutually exclusive frequencies; thus all of these radio signals are separated in the frequency domain and are also orthogonal in the time domain when any time shifts are present between them. Furthermore, all of these radio signals have zero MAI.

Thus, it is possible to create several independent radio signals in a frequency range $\Delta\omega=4\pi/T$ with a central frequency ω_0, where $\omega_0 >> \Delta\omega$. The number of these independent radio signals is equal to n+1.

CONCLUSIONS

In this study, we demonstrate that for orthogonal WH matrices of size $N = 2^n$ (n=1, 2, 3, . . .) n+1 groups of sequences that do not have mutual frequency components exist when applied to periodic signals. Thus, these groups of signals are always orthogonal in the time domain when any time shift is present between them, and these signals exhibit the zero MAI or ICC property. It is possible to create an ensemble of signals with zero MAI, where the maximum number of these signals in an ensemble is no larger than n+1. These signals can be separated on the receiver side without a tight synchronization between them. When applied to periodic radio signals it is possible to create n+1 independent radio signals using the processes described in this study.

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