THE EIGHT-POINT ALGORITHM IS NOT IN NEED OF DEFENSE

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ABSTRACT

In stereo vision, the fundamental matrix F encapsulates the epipolar geometric information which relates corresponding points on two views of a scene. The eight-point algorithm is a frequently cited method for calculating the fundamental matrix. Some researchers criticized the performance of such algorithm as it is extremely susceptible to noise and hence virtually useless for most purposes. Such criticism prompted Richard Hartley to defend the algorithm. He asserted that preceding the matrix calculation with normalization of the coordinates of the matched points ensures a high performance of the algorithm. This paper presents an analysis showing that the raised question about the performance of the eight-point algorithm lies in the way by which the fundamental matrix equation is derived rather than in the eight-point algorithm itself. It demonstrates that F calculated in the projection space is different of F defined in the Euclidean space as a one-to-one correspondence.

Keywords: stereo vision, essential matrix, fundamental matrix, defense, eight-point algorithm.

INTRODUCTION

The main goal of stereo vision is to acquire a 3D structure of a rigid scene using two images obtained from two different standpoints. The fundamental matrix encapsulates the epipolar geometric information that relates corresponding points on two images of the scene. The epipolar geometry; which can be depicted as in Figure-1 is described as follows:

A world point $M = (X, Y, Z)$ is defined in a world coordinate system and imaged by two pinhole cameras placed at two different positions C_l and C_r. Points C_l and C_r constitute the origins of the two cameras coordinate systems.

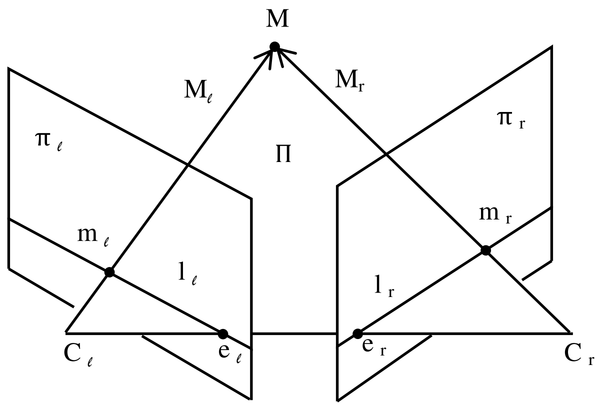

Figure-1. Epipolar geometry defined for a single 3D point that is projected onto left and right camera planes.

The point $m_l = (x_l, y_l)$ is the retinal image of point M acquired by the left camera; it belongs to the left camera plane π_l and is defined in the left camera coordinate system. Similarly, the point $m_r = (x_r, y_r)$ is the retinal image of the point M captured by the right camera; it belongs to the right camera plane π_r and it is defined in the right camera coordinate system. The points m_l and m_r are called corresponding points.

The relationship between points m_l and m_r through the essential matrix is expressed as $m_r^T E m_l = 0$ [9]. The relationship between the fundamental matrix and the essential matrix is $F = K_r E K_l$, where K_l and K_r are the calibration matrices of the left and right cameras, respectively [7].

The essential matrix encapsulates the epipolar geometry of the imaging configuration when the cameras are calibrated. In the case of uncalibrated cameras it has become customary to refer to the matrix as the fundamental matrix [6]. The analysis of this article is mainly about solving the equation $m_r^T E m_l = 0$ for E or $m_r^T F m_l = 0$ for F, we refer to both matrices as the same in this article.

The eight-point algorithm is one of the most used methods to calculate the fundamental matrix. Its performance has been criticized by some researchers and defended by others.

In this paper we show that the problem is in the development of the fundamental matrix equation rather than with the eight-point algorithm.

The paper is organized as follows: Section 2 presents the defense of the eight-point algorithm as described in [6]. Section 3 presents the derivation of the essential matrix equation as introduced to the computer vision community in [9]. Section 4 discusses the outcome of projecting the equation $M_r^T F M_l = 0$ onto a projective space. In section 5 we consider solving the equation $m_r^T F m_l = 0$ in place of $M_r^T F M_l = 0$ from an algebraic point of view. Section 6 is devoted to experimental results. Finally, the paper concludes in section 7.

DEFENSE OF THE EIGHT-POINT ALGORITHM

The eight-point algorithm is a frequently cited method for calculating the fundamental matrix through solving the equation $m_r^T F m_l = 0$ for eight or more corresponding points. It has the advantage of simplicity of implementation. If eight point matches are known, then the matrix F is obtained by solving a set of linear equations. If the number of points' matches is greater than eight, F is estimated through solving a linear least squares minimization problem.

Some researchers criticized the performance of such algorithm as it is extremely susceptible to noise and hence virtually useless for most purposes. Such criticism prompted Richard Hartley [6] to defend the algorithm by showing that by preceding the matrix calculation with a very simple normalization of the coordinates of the matched points; results are obtained comparable with the best iterative existing algorithms.

Normalization procedure

The normalization procedure consists of translating the coordinates in each image (by a different translation for each image) and scaling them in the following manner:

a) "The points are translated so that their centroid is at the origin.
b) The points are then scaled so that the average distance from the origin is equal to 2.
c) This transformation is applied to each of the two images independently [6]."

Purpose of normalization

Given eight or more pairs of corresponding points, the equation $m_r^T F m_l = 0$ produces a set of equations of the form $Af = 0$, where A is written in terms of the known homogeneous coordinates of $m_l = (x_l, y_l, 1)$ and $m_r = (x_r, y_r, 1)$ as

$$A = \begin{bmatrix} x_l x_r & x_l y_r & x_l \\ y_l x_r & y_l y_r & y_l \\ x_r & y_r & 1 \end{bmatrix} \text{ and } f = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & 1 \end{bmatrix}$$

The purpose of the normalization transformations is that the matrix $\bar{A}$, constructed from the normalized image coordinates, in general has a better condition number than A has before normalization. This means that the solution $\bar{f}$ is more well-defined as a solution of the homogeneous equation $\bar{A}\bar{f} = 0$ than f calculated from $Af = 0$ [6].

Performance evaluation procedure

The general evaluation procedure in [6] consists of the following points:

a) Matching points were computed by automatic techniques, and outliers were detected and removed.
b) The fundamental matrix was computed using a subset of all points.
c) In the case of algorithms, such as the eight-point algorithm, that do not automatically enforce the singularity constraint (i.e. the constraint that $det\, F = 0$) this constraint was enforced a posteriori by finding the nearest singular matrix to the computed fundamental matrix.
d) For each point m_l, the corresponding epipolar line Fm_l was computed and distance the line Fm_l from the matching point m_r was calculated. This was done in both directions (that is, starting from points m_l in the first image and also from m_r in the second image). The average distance of the epipolar line from the corresponding point was computed, and used as a measure of quality of the computed Fundamental matrix. This evaluation was carried out using all matched points, except outliers, and not just the ones that were used to compute F.

Performance of the 8-point algorithm before and after normalization

For the purpose of comparing the performance of the eight-point algorithm before and after normalization, Hartley [6] considered a set of images. He concluded that the effect of normalization is not so great in the case of images with matched points known with extreme accuracy, whereas, in the case of images were matches are less accurate the advantage of normalization is dramatic.

DEVELOPMENT OF THE ESSENTIAL MATRIX EQUATION

The essential matrix was introduced to the computer vision community by Longuet-Higgins through his article published in *Nature* in 1981 [9] as follows:

A world point is defined in the left camera's coordinate system by the vector $M_l = (X_l, Y_l, Z_l)$ and in the right camera's coordinate system by $M_r = (X_r, Y_r, Z_r)$.

Longuet-Higgins [9] defined the image points m_l and m_r of the world point M in the coordinate systems of the two cameras as

$$\begin{cases} (x_l, y_l) = (X_l/Z_l\,, Y_l/Z_l) \\ (x_r, y_r) = (X_r/Z_r\,, Y_r/Z_r) \end{cases} \tag{1}$$

Given the translation vector of the right camera with respect to the left camera's coordinate system $t = [t_x \;\; t_y \;\; t_z]$ and given the rotation matrix of the right camera's coordinate system with respect to the left camera's coordinate system R, the relationship between the three-dimensional vectors representing the world point M may be expressed as

$$M_r = R(M_l - t) \tag{2}$$

The rotation R satisfies the relation

$$RR^T = R^T R = 1 \text{ and } det(R) = 1 \tag{3}$$

Longuet-Higgins [9] defines the essential matrix as

$$E = RS \tag{4}$$

where S is the skew-symmetric matrix

$$S = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \tag{5}$$

and the author adopted the length of the vector t as the unit of distance

$$t^2 = t_x^2 + t_y^2 + t_z^2 = 1 \tag{6}$$

Longuet-Higgins [9] then constructed the expression $M_r^T E M_l$ and used (2) to (6) to conclude $M_r^T E M_l = 0$. Dividing by $Z_l Z_r$ establishes the equation for the essential matrix that relates image points m_l and m_r

$$m_r^T E m_l = 0 \tag{7}$$

The essential matrix is defined when the used cameras are calibrated. Latter, it has been proved to be applicable when the cameras are not calibrated and only pixels information about the images are available, and it has been renamed the fundamental matrix F [4].

The definition of the fundamental (essential) matrix has been approached in a number of ways [4, 8, 9]. In all cases, the equation $m_r^T F m_l = 0$ is derived from the relationship between the vectors M_l and M_r representing a 3D point M in the two cameras coordinate systems.

THE FUNDAMENTAL MATRIX EQUATION IN THE EUCLIDEAN AND PROJECTIVE SPACES

In [2], the author discussed the relationship between the two equations $M_r^T F M_l = 0$ and $m_r^T F m_l = 0$. He considered a case represented by Figure-2 where two 3D points M and N lying on the same epipolar plane Π. The points m_l and m_r are the image points of M and the points n_l and n_r are the image points of N, on the two views represented by the planes π_l and π_r.

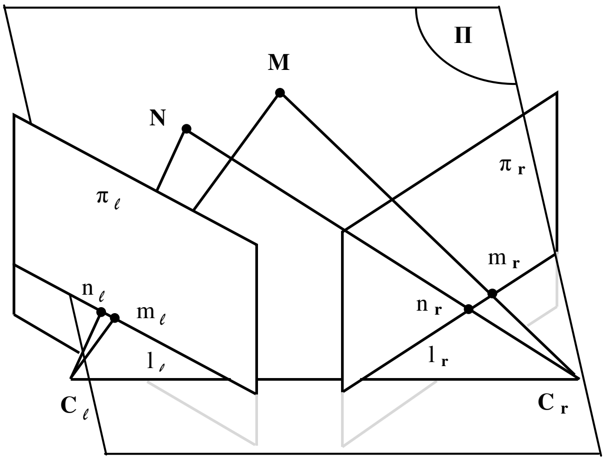

Figure-2. 3D points M and N lie on the same epipolar plane, their image points are projected on the same epipolar line on each camera plane.

He stated that Hartley and Zisserman [7] asserted: "The mapping from a point in one image to a corresponding epipolar line in the other image may be decomposed into two steps. In the first step, the point m_l is mapped to some point m_r in the other image lying on the epipolar line l_r. This point m_r is a potential match for the point m_l.

In the second step, the epipolar line l_r is obtained as the line joining the point m_r to the epipole e_r.

Step 1: Point transfer via a plane. Consider a plane Π in space not passing through either of the two camera centres. The ray through the first camera centre corresponding to the point m_l meets the plane Π in a point M. This point M is then projected to a point m_r in the second image. This procedure is known as transfer via the plane Π. Since M lies on the ray corresponding to m_l, the projected point m_r must lie on the epipolar line l_r corresponding to the image of this ray.

The set of points m_l in the first image and the corresponding point m_r in the second image are projectively equivalent, since they are each projectively equivalent to the planar point set M. Thus, there is a 2D homography H_π mapping each m_l to m_r.

Step 2: Constructing the epipolar line. Given the point m_r, the epipolar line l_r passing through m_r and the epipole e_r can be written as $l_r = e_r \times m_r = [e_r]_\times m_r$. Since m_r may be written as $m_r = H_\pi m_l$, we have $l_r = [e_r]_\times H_\pi m_l = F m_l$ where we define $F = [e_r]_\times H_\pi$ as the fundamental matrix. The point m_r lies on the epipolar line $l_r = F m_l$, $m_r F m_l = 0$."

An epipolar line is the intersection of an epipolar plane with the image plane [7]. The points M and N in Figure-2 belong to the same epipolar plane Π. This epipolar plane intersects the left image plane in the epipolar line l_l and it intersects the right image plane in the epipolar line l_r.

It follows that the points n_r and m_r lie on the right epipolar line $l_r = F m_l$ which implies $m_r^T F m_l = 0$, $n_r^T F m_l = 0$. And the points n_l and m_l lie on the left epipolar line $l_l = F^T m_r$, which implies $n_l^T F^T m_r = 0$, and

$m_l^T F^T m_r = 0$. The two latter equations are equivalent to $m_r^T F n_l = 0$ and $m_r^T F m_l = 0$, respectively. Thus, we have $n_r^T F m_l = 0$ and $m_r^T F n_l = 0$ and neither m_l and n_r nor n_l and m_r are corresponding points.

The author [2] then stressed that a 3D point M is represented in the two cameras coordinate systems by exactly two vectors M_r and M_l, therefore F in $M_r^T F M_l = 0$ is a one-to-one correspondence between the pairs of vectors (M_l, M_r). However, the equation $m_r^T F m_l = 0$ holds in the following cases:

a) m_l and m_r are corresponding points,
b) m_l is a correspondent to more than a point m_r, and m_r is a correspondent to more than a point m_l; this is referred to as the occlusion phenomenon,
c) m_l and m_r are not corresponding points, simply they are images of two 3D points lying on the same epipolar plane.

Point 2 asserts that F calculated from $m_r^T F m_l = 0$ is not injective and consequently not bijective, i.e. not a one-to-one correspondence, as is the case of F in $M_r^T F M_l = 0$.

Point 3 affirms that the equation $m_r^T F m_l = 0$ not only holds for corresponding points which are images of the same 3D point M, but also for points that are not correspondents. These two facts affirm that F defined by $m_r^T F m_l = 0$ is different of F defined by $M_r^T F M_l = 0$.

ANALYSIS OF THE DERIVATION OF THE ESSENTIAL MATRIX EQUATION

The equation $M_r E M_l = 0$ holds for any 3D point M which is defined by a distinct vector $M_l = (X_l, Y_l, Z_l)$ in the left camera coordinate system and a distinct vector $M_r = (X_r, Y_r, Z_r)$ in the right camera coordinate system, i.e., no other 3D point shares these vectors with M.

Up to this level, the matrix E is a one-to-one correspondence between the vectors M_l and M_r. In deriving the essential matrix equation, Longuet-Higgins [9] divided the equation $M_r E M_l = 0$ by $Z_r Z_l$ therefore replacing a relationship between the distinct vectors representing the 3D point in the Euclidean space by a relationship between two homogeneous image points in the projective space. Consequently, the distinctiveness property is lost as each projective (homogeneous) point $[x, y, 1]$ is considered to be equal to an equivalence class of 3D points that belong to the 3D line passing through the Cartesian point (X, Y, Z) and the origin $(0,0,0)$; the relationship between the projective and Cartesian points is $[x, y, 1] = [X/Z, Y/Z, Z/Z]$ [3].

In other words, a number of 3D vectors representing different world points could be projected onto the same homogeneous point. And many 3D points are projected on the same epipolar lines as they belong to the same epipolar plane.

Algebraically, solving the projection equation $m_r^T E m_l = 0$ for E instead of $M_r^T E M_l = 0$ results in the following:

a) Two points M and N occluded on right camera plane, the equation $M_r^T E M_l = 0$ is replaced by $m^T E m_l = 0$ and the equation $N_r^T E N_l = 0$ is replaced by $m^T E n_l = 0$, where two points from the left camera plane are related to a single point m from the right camera plane.
b) Two points M and N occluded on left camera plane, $M_r^T E M_l = 0$ and $N_r^T E N_l = 0$ could be replaced by equations $m_r^T E m = 0$ and $n_r^T E m = 0$, where two points from the right camera plane are related to a single point m from the left camera plane.
c) Some $m_r^T E m_l = 0$ equations hold for non-corresponding points where the points m_r and m_l are images of two different 3D points lying on the same epipolar plane. This is a very serious consequence of replacing $M_r^T E M_l = 0$ by $m_r^T E m_l = 0$.

In terms of solving equations over different domains, equation (7) is derived to some extent in a similar way to relaxing an integer programming problem (IP) to a linear one (LP), i.e. solving the same problem in two different domains.

To illustrate this point more clearly, let us consider the following IP problem where the objective is to minimize a function that depends to a certain number of variables. These variables are subject to constraints. Some of these constraints require the variables to be integer (i.e. 0 or 1),

$$Z = min \sum_i c_i x_i \tag{8}$$

Subject to

$$\begin{cases} a_i x_i \le b_i, i = 1, \cdots, n \\ x_i = 0 \text{ } or \text{ } 1, i = 1, \ldots, n \end{cases}$$

By relaxing the integrality constraints on the variables x_i, the resulting problem will be

$$Z = min \sum_i c_i x_i \tag{9}$$

Subject to

$$\begin{cases} a_i x_i \le b_i, i = 1, \cdots, n \\ x_i \ge 0, i = 1, \ldots, n \\ x_i \le 1, i = 1, \ldots, n \end{cases}$$

Problem (9) is a linear programming problem which can be solved easily by the Simplex method. And, the obtained objective function value is even better (less) than the optimal solution value of problem (8). However, real-valued solutions of (9) violate the integrity constraint of problem (8), $x_i = 0 \text{ } or \text{ } 1, \ i = 1, \ldots, n$.

To conclude, solving a problem in two different domains may have a solution in the one which is not a solution in the other. And this is what happened to the essential matrix equation. First the equation $aEb = 0$ is defined in the Euclidean space where a, b are 3D vectors having three variable components (X, Y, Z), and then it is solved in a projective space where a, b are defined by two variable components $[x, y, 1]$, the third is constant and equal to 1 for all projective points.

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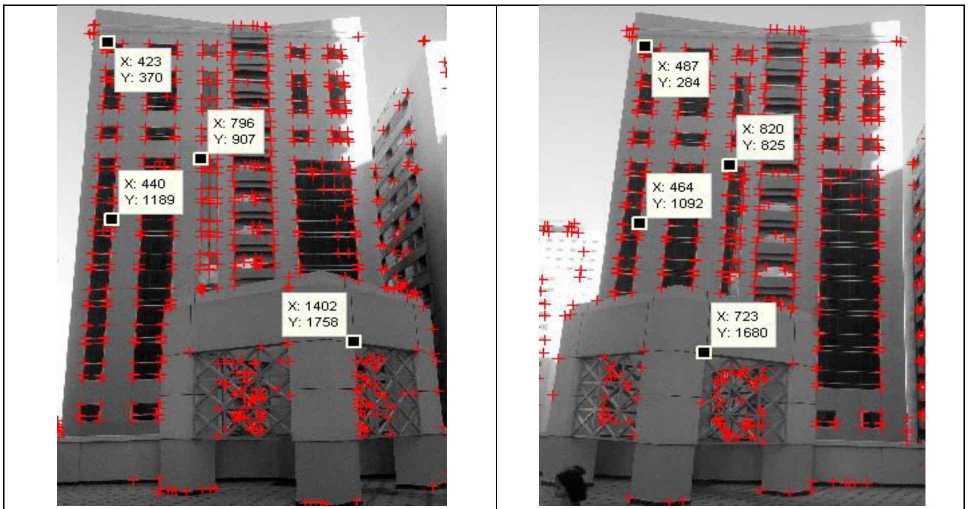
EXPERIMENTAL RESULTS

Data and programs

In this paper, experiments were conducted on real images. The images consist of an apartment-building imaged from two different positions by the same camera. In the middle between the camera and the apartment-building there exists a separate small playhouse as shown in Figure-3. The points on the images were detected and used as input for the program we used to calculate the fundamental matrix.



Left image | Right image

Figure-3. Two images of a scene captured from two different positions.

The program we used is coded in MATLAB and having the following components:

a) A corner detection algorithm developed by Harris [5] to detect points of the two images.
b) A procedure in MATLAB to input the points detected by Harris algorithm. The points are represented by a matrix of four columns: the first two columns contain the coordinates of the right image points and the last two contain the coordinates of the left image points.
c) A MATLAB Toolbox [11] that contains a number of methods for estimating the fundamental matrix using the eight-point algorithm.

In total, 282 pairs of points were collected from the two images. Before used to estimate the fundamental matrix, these points have been cleaned in a sense that points in one view without images in the other view are removed. The robust estimation method functions are selected for the estimation process between other available methods as they considered performing better than the others:

a) M-Estimator using least squares,
b) M-Estimator using least squares with an Eigen analysis,
c) M-Estimator proposed by Torr,
d) Least Median of Squares (LMedS) using least squares,
e) Least Median of Squares using least squares with an Eigen analysis,
f) RANSAC
g) MLESAC implemented by Torr
h) MAPSAC implemented by Torr [11].

In the literature, the average distance between image points and epipolar lines is used as a measure in evaluating the performance of the eight-point algorithm [1, 6].

"For each point m_l, the corresponding epipolar line Fm_l was computed and distance the line Fm_l from the matching point m_r was calculated. This was done in both directions (that is, starting from points m_l in the first image and also from m_r in the second image). The *average distance* of the epipolar line from the corresponding point was computed, and used as a measure of quality of the computed fundamental matrix. This evaluation was carried out using all matched points, except outliers, and not just the ones that were used to compute F [6]."

Besides the above mentioned measure, we opt to use a measure that directly evaluates how much the matrix F satisfies the equation $m_r^T F m_l = 0$ for any pair of corresponding points (m_l, m_r). For such purpose, three arbitrarily pairs of corresponding points were selected from the main building and one point from the playhouse as shown in Figure-3 and in Table-1.

First we used the 282 pairs of corresponding points from the main building to estimate the matrix F. Once F is available we calculated the expression $m_r^T F m_l$ for the four pairs of points of Table-1, Point 4 is from the playhouse.

Table-1. Corresponding points to evaluate $m_r F m_l$

#	$\boldsymbol{m_r}(\mathbf{x}, \mathbf{y})$	$\boldsymbol{m_l}(\mathbf{x}, \mathbf{y})$
1	(487,284)	(423,370)
2	(820,825)	(796,907)
3	(464,1092)	(440,1189)
4	(723,1680)	(1402,1758)

RESULTS AND COMMENTS

Regarding normalization, the program is run in three different modes; without normalization, with normalizing the points' coordinates in the interval $[-1 \quad 1]$, and with a normalization proposed by Hartley in [6]. The values reported in Tables 2-4 are the values of the expression $m_r F m_l$ for each of the points 1, 2, 3, and 4 of Table 1 along with the outliers detected by the methods we used. As indicated by their captions, Table 2 contains the values of $m_r F m_l$ generated by the program without normalization, Table 3 with normalizing the points in the interval $[-1 \quad 1]$, and Table 4 with the normalization proposed by Hartley.

Table-2. No normalization.

Method	1	2	3	4	outliers
1	0.299	0.023	0.000	0.256	75/282
2	0.299	0.023	0.000	0.256	75/282
3	0.319	0.004	0.010	0.203	64/282
4	0.001	0.000	0.002	1.973	111/282
5	0.002	0.003	0.001	4.036	105/282
6	0.028	0.002	0.001	0.290	76/282
7	0.388	1.463	1.244	138.286	266/282
8	0.061	1.290	8.353	145.809	267/282

Table-3. Normalization in [-1 1].

Method	1	2	3	4	outliers
1	0.184	0.218	0.013	0.346	61/282
2	0.000	0.007	0.001	0.499	99/282
3	0.020	0.009	0.021	0.189	58/282
4	0.002	0.002	0.004	7.516	67/282
5	0.006	0.001	0.001	5.800	50/282
6	0.002	0.004	0.009	1.819	0/282
7	0.013	0.022	0.009	0.178	21/282
8	0.014	0.020	0.007	0.289	21/28

Table-4. Hartley normalization.

Method	1	2	3	4	outliers
1	0.151	0.184	0.020	0.299	39/282
2	0.000	0.007	0.001	0.498	99/282
3	0.021	0.010	0.018	0.255	63/282
4	0.003	0.007	0.004	1.392	83/282
5	0.003	0.002	0.002	1.187	84/282
6	0.006	0.006	0.008	1.190	0/282
7	0.014	0.020	0.009	0.319	31/282
8	0.018	0.021	0.013	0.154	42/282

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Whereas the slight difference of $m_r F m_l$ values as well as the difference in the number of outliers in Tables 2-4, the values for point 4 are very large when compared to the values for the other three points especially after normalization. Also the values for point 4 are far away from zero as they should be.

The depths of the points 1, 2 and 3 in Table 1 and the depths of points used to estimate F are in the same range, whereas the depth of point 4 is substantially different. This distinguishing feature of point 4 confirms that the value of the expression $m_r F m_l$ depends to the depth of the world points being projected onto the cameras planes.

Figures 4-6 show a similarity in the average distance of points from the epipolar lines in all normalization modes. Method number 1 performs badly in the three modes, methods 6 and 7 came in second category and the other methods perform well except the second method in the no normalization mode.

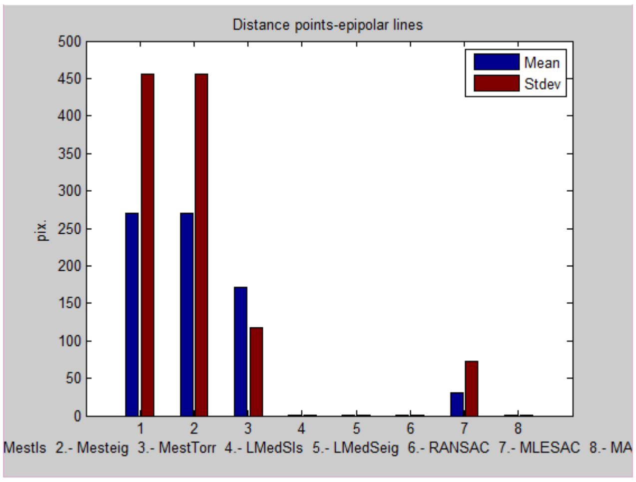

Figure-4. No normalization: the average distance of points from the epipolar lines.

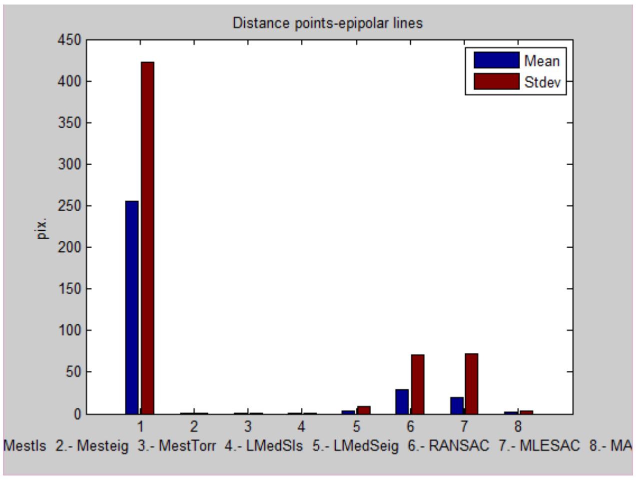

Figure-5. Normalization in [-1 1]: the average distance of points from the epipolar lines.

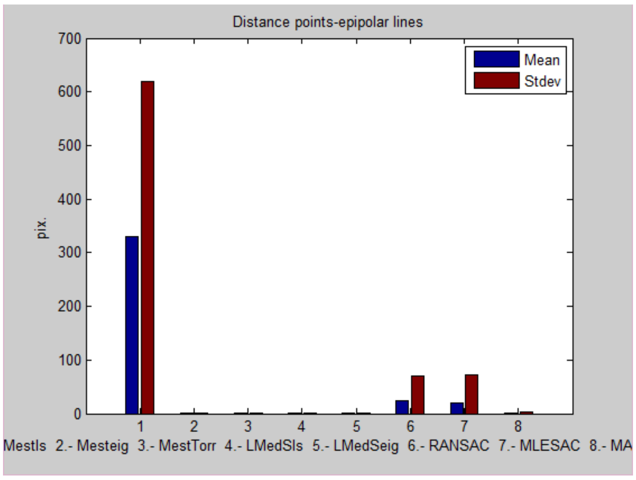

Figure-6. Hartley normalization: the average distance of points from the epipolar lines.

CONCLUSIONS

There is no doubt that the normalization procedure produced a considerable improvement on the quality of the fundamental matrix F. The matrix F calculated from normalized homogeneous points is more well-defined compared to F calculated from unnormalized image points. The empirical results in [6] indicates that the improvement was dramatic in the case where the image points' matches are less accurate.

Based on experimental results, researchers including Longuet-Higgins [10] himself have criticized the performance of the eight-point algorithm to an extent that some of them concluded that it is virtually useless for most purposes [6]. In reality, the problem lies in the way in which the equation $m_r^T F m_l = 0$ is derived rather than in the eight-point algorithm performance. The matrix F is defined primarily by the equation $M_r^T F M_l = 0$ as a one-to-one correspondence between the unique vectors M_l and M_r representing a 3D point M on the two cameras coordinate systems in the Euclidean space. However, when F is calculated from the equation $m_r^T F m_l = 0$, the "one-to-one correspondence" feature of F is lost. The new F is not injective as one point m_r which is the projection of 3D points lying on the same ray from the right camera center is related to all projections m_l of these 3D points onto the left camera plane through the equation $m_r^T F m_l = 0$. And similarly, a point m_l that is the projection of 3D points lying on the same ray from the left camera centre, is related to all projections m_r of these 3D points onto the right camera plane through the equation $m_r^T F m_l = 0$. Furthermore, in the projective space, the system of equations $m_r^T F m_l = 0$ holds for corresponding and non-corresponding points. It holds for an image point m_l of a 3D point M on the left view and an image point n_r of a 3D point N on the right view, simply because M and N lie on the same epipolar plane, i.e., $n_r^T F m_l = 0$. Consequently, F calculated from $m_r^T F m_l = 0$ is different from F calculated from $M_r^T F M_l = 0$. In general, the matrix F calculated from $m_r^T F m_l = 0$ does not satisfy $M_r^T F M_l = 0$. As a matter of fact, the occurrence of 3D points lying on the same epipolar plane increases for scenes containing objects at different depths which is certainly one of the features of 3D scenes.

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