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VIBRATION ANALYSIS OF A CANTILEVER BEAM FOR OBLIQUE CRACKS

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ABSTRACT

Due to limited fatigue strength, the fatigue cracks ours in the material under service conditions. Cracks are also found inside the material due to poor manufacturing processes. Single sided cracks are produced in the material as a result of fluctuating loads. Crack generally may be of two types, transverse cracks and oblique cracks. The magnitude and orientation of the manufacturing defect decides the origin of either transverse cracks or oblique cracks in the beam. Hence it is very essential to study the effect of top side and bottom side oblique cracks on the beam. Out of two cracks i.e. top side or bottom side cracks, one crack will be comparatively more critical, hence it requires much attention. Crack get propagated in the material due to the action of fatigue load and at the end, it gives catastrophic failure. Understanding the dynamics of the cracked beam is of most importance because various vibration parameters like natural frequency, resonant amplitude of uncracked and cracked cases of a beam used as a basic criteria in the crack detection by vibration methods. In this study, most practical spring steel material (EN 47) is considered for the cantilever beam. ANSYS software used to find the natural frequency and zero frequency deflection of cracked cases of beams. Stiffness of defective beams is calculated by a conventional formula (Load / deflection). In this study, it is found that the value of stiffness and natural frequencies for top side cracked cases are comparatively on lower side than bottom side cracked cases when crack angle equal to 20^0. It is also found that up to 10^0 crack angle, the algebraic sum of stiffness of top side cracked cases is equal to the algebraic sum of stiffness of bottom side cracked cases. This condition is true also for natural frequency. It is also observed that, when crack angle is 20^0, then presence of top side crack and bottom side crack of the same configuration in the cantilever beam is a function of natural frequency, when cantilever beam is of a square cross section.

Keywords: natural frequency, ansys, rectangular shape crack, stiffness, crack angle, crack depth.

1. INTRODUCTION

Dynamic uniqueness of cracked and intact materials is very dissimilar; it means that the reliability of both the materials is not same. Due to this, the vibration analysis of a cracked beam or shafts is one of the most severe problems in various machinery. The investigation of cracked beams for various vibration parameters are very much needed because of its majority of practical applications like turbine blades, cantilever bridges, automobile propeller shaft. Measurement of natural frequencies, resonant amplitude, vibration modes is used to determine the location and size of the crack. Cracks are produced at the highly stressed region of the structures or beam due to fatigue loads. Less fatigue strength of the material is the main cause to produce the defect like cracks. Presence of cracks decreases the service life of structures or any machine parts. Cracks are likely to nucleate and cultivate in the tensile stress area of the beam. The main end result of these cracks is to alter the vibration characteristics of the beam like natural frequency, damping factor and stiffness of the beam. Thus the detection of any crack like small size crack or large size crack is very important to ensure safety of the structure. Chondros *et al.* [1] have been performed studies based on structural health monitoring for a very long times and almost all the concepts in a crack detection regard have been well established from mathematical theory. Radhakishnan [2] has been studied the resonance response of a cracked cantilever rectangular beam based on fracture mechanics quantities like strain energy release rate, stress intensity factor and compliance. The spring stiffness and the fundamental natural frequency decrease with increase in crack length. Parthi and Behera [3] have investigated the wave forms of different modes of cracked shaft, using stress intensity factor. Ostachowicz and Krawczuk [4] modeled the beam by triangular disk finite elements and studied the effects of crack parameters on the vibrational behavior of the structure. Sadettin Orhan [5] conducted the number of experiments on edge cracked cantilever beam to see the effect of crack on vibration parameters. Nandwana and Maiti [6] proposed a method for the detection of the location and size of a crack in a stepped cantilever beam based on the measurement of natural frequencies. In the proposed method, the crack is represented as a rotational spring and method involves obtaining the plots of its stiffness with the crack location for any three natural modes through the characteristics equation. The point of intersection of the three curves gives the crack location. Standard relation between stiffness and crack size is used to compute the size of crack. Moradi and Kargozarfard [7] present an inverse procedure to identify multiple cracks in beams using evolutionary algorithm. By considering the crack detection procedure as an optimization problem an objective function can be constructed based on the change of the eigen frequencies and some strain energy parameters. Lee [8] proposed a simple method to identify multiple cracks in a beam using the vibration amplitudes. The cracks are modeled as a massless rotational springs and the forward problem is solved using the finite element method. The inverse problem is solved iteratively for the crack location

VOL. 12, NO. 4, FEBRUARY 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

and sizes using the Newton-Raphson method and the singular value decomposition method. Yayli m o [9] proposed a finite element procedure for computations of natural frequencies based on strain gradient elasticity theory. The results reveal that for the microbeam with varying cross section comparable to its material length scale parameter, the effect of strain gradient is significant. They observed that the frequencies of microbeam can be controlled by choosing proper values of depths. Yayli m o [10] is studied the free axial vibration response of carbon nanotubes (CNTs) with arbitrary boundary conditions based on non- local elasticity theory. A unified analytical method has been developed, which can be used for a nanorod with any types of boundary conditions. It is suggested that by controlling the spring parameters and natural frequencies, the structures of nanotubes can be produced for nanosized devices. Wahab and Roeck [11] studied the application of the change in modal curvatures to detect damage in a prestressed concrete bridge.

Existence of cracks in beam either in transverse direction or in other direction may be possible and it depends upon the geometry of the manufacturing defects. Manufacturing defect may be corrosion or corrosion fatigue in the beam. Such problems are significant and require attention. In the present study, the vibrant behavior of a cracked cantilever beam is studied. Effects of crack angle and crack depth on the vibration parameters are deeply investigated by FEA and experimental method. So far the effect of crack location, crack depth on natural frequencies, damping factors of a cantilever beam is studied. None of the researcher is studied the effect of crack depth, crack angle on the stiffness of the beam. Cantilever beam is not investigated for the top side and bottom side oblique cracks. It is required to check whether top side or bottom side cracks of the same configuration is a function of natural frequency or not even though beam is of square cross section. Special attention is given to crack angle parameter, means on oblique cracks. Oblique cracks (10^0 and 20^0 cracks) are taken on the beam from top side and bottom side to study the effect of crack angle on the stiffness and natural frequency of the beam. This investigation helps to understand the dynamics of cracked cantilever beams, because in the inverse problem, natural frequency is used as an input in the detection of crack by vibration methods. In the design of structures also natural frequency have a great significance. The frequency based approach offers the advantage that it can be easily measured from a single accessible point on a component. This also makes it suitable even for inaccessible components.

2. MATERIALS AND METHODS

A. EXPERIMENTAL STUDY

The aim of experimentation is to monitor the change in natural frequency of a cantilever beam due to presence of oblique edge cracks. The instruments used for experimental analysis are accelerometer, Dewe FRF and related accessories, as shown in Figure-1. Specimens of EN 47 material are used to study the effects of cracks on vibration parameters. EN 47 materials is tested in ELCA Lab, Pune, India, for material properties like young's modulus and density. The material properties and specimen geometric properties are shown in Table-1. Poisson's ratio (μ) is assumed as 0.3. Wire EDM process is used to produce oblique cracks on the specimens.

Figure-1. Experimental set-up.

The beam is clamped at one end by a fixture and other end is free. An accelerometer of piezo electric type is mounted on the beam is used to measure the acceleration of the vibrating body. A vibration of transverse waves [12] is comparatively on higher side than longitudinal waves. Again pure bending mode is obtained at the first natural frequency only in the crosswise direction. Due to this reason, simply the vibration in the transverse direction is considered in this study.

Table-1. Material properties and geometric properties of a specimen.

Property	EN 47
Density (kg/m^3)	7800
Modulus of Elasticity (N/m^2)	1.95*10^{11}
Length of specimen (m)	0.360
Cross section of specimen (m^2)	0.02*0.02

B. CRACK CONFIGURATIONS

In this study, 37 specimens are considered. Out of 37 specimens, one specimen is crack free specimen, and 36 specimens have a single oblique crack. Two cases are considered for cracked specimens, i.e. case 1 and case 2.

Case 1: In this case 18 specimens are considered and cracks are taken on the each specimen from top side. Case 1 is divided into two sub cases. In the first sub case 9 specimens are considered. 10^0 cracks are taken on the

ARPN Journal of Engineering and Applied Sciences

beam at 120 mm, 180 mm and 240 mm location and at each location crack depth varied from 4 mm to 12 mm by an interval of 4 mm. Second sub case is similar to first sub case; the only difference is that instead of 10^0 cracks, 20^0 cracks are taken on the beam.

Case 2: This case is similar to case 1; the only difference is that cracks are taken on the beam from the bottom side.

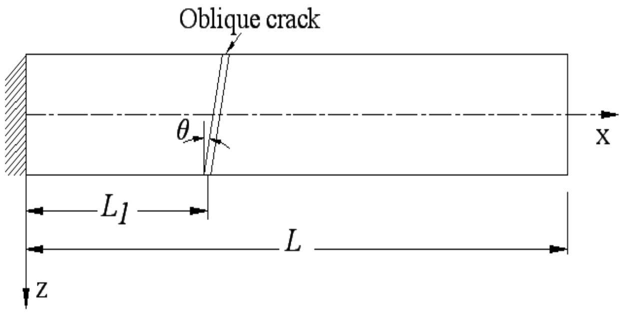

Figure-2. Schematic diagram of a cracked cantilever beam.

C. FINITE ELEMENT MODELING AND ANALYSIS

ANSYS [13-14] finite element program is used to determine natural frequencies of the undamaged as well as cracked beams. For this purpose, rectangle area is created. This area is extruded in the third direction to get the 3 D model. Then at the required location, small rectangular area of crack of 0.5 mm width and required depth is created and extruded. Then small volume of crack is subtracted from large volume of cantilever beam to obtain cracked three dimensional models. The width of crack is kept constant throughout its depth in this study. A 20 node structural solid element (solid 186) is selected for modelling the beam because of some special features like stress stiffening, large strain, and large deflection. Finite element boundary conditions are applied on the beam to constrain all degrees of freedom of the extreme left hand end of the beam. Static and modal analyses are carried out on each specimen to get zero frequency deflection and natural frequency. In static analysis 100 N loads is applied at the end of a cantilever beam.

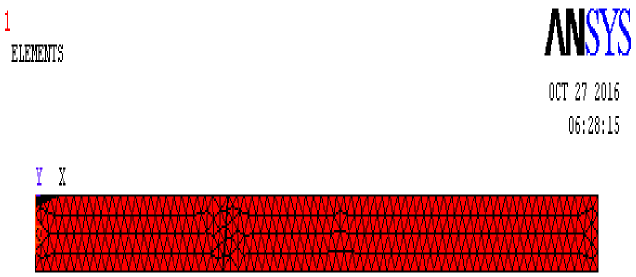

Figure-3. Finite element modeling of a cracked beam.

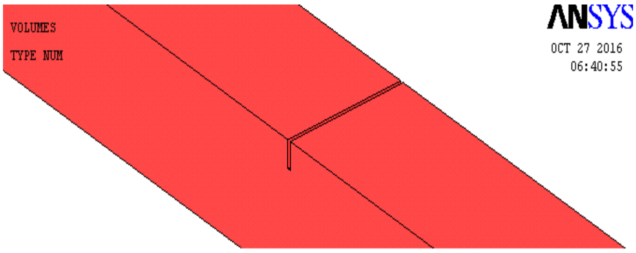

Figure-4. Crack zone details of a FEA model.

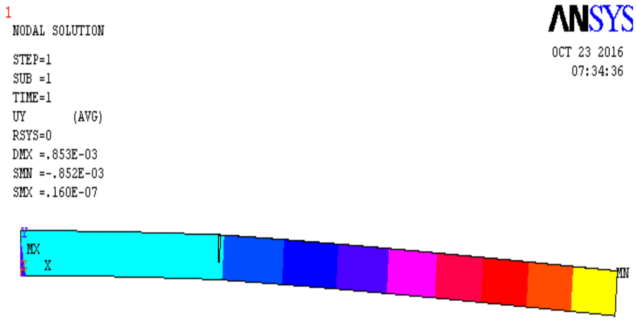

Figure-5. Zero frequency deflection plot; EN 47 TS 10^0 oblique crack; crack details: L_1/L= 0.333; a/H= 0.6.

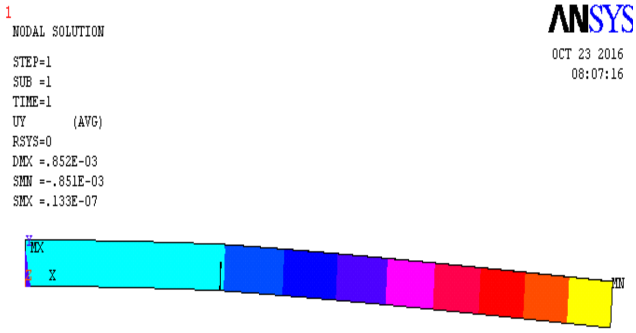

Figure-6. Zero frequency deflection plot; EN 47 BS 10^0 oblique crack; crack details: L_1/L= 0.333; a/H= 0.6.

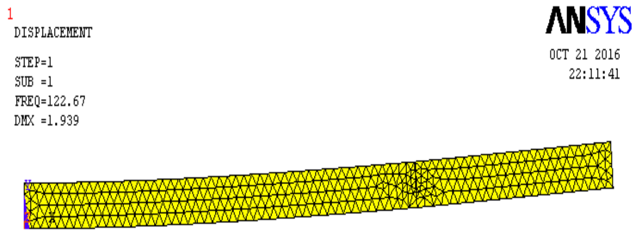

Figure-7. Natural frequency plot; EN 47 TS 10^0 oblique crack; crack details: L_1/L= 0.666; a/H= 0.6.

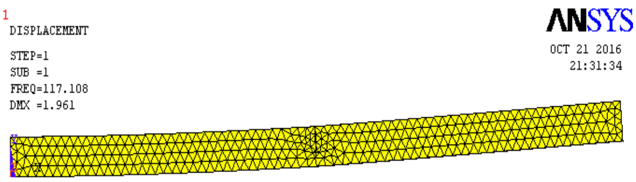

Figure-8. Natural frequency plot; EN 47 TS 20^0 oblique crack; crack details: L_1/L= 0.5; a/H= 0.6.

VOL. 12, NO. 4, FEBRUARY 2017 ISSN 1819-6608
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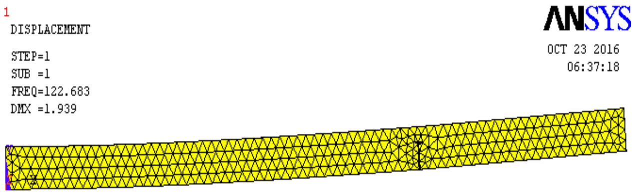

Figure-9. Natural frequency plot; EN 47 BS 10^0 oblique crack; crack details: L_1/L= 0.666; a/H= 0.6.

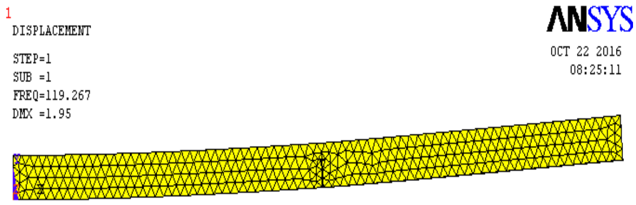

Figure-10. Natural frequency plot; EN 47 BS 20^0 oblique crack; crack details: L_1/L= 0.5; a/H= 0.6.

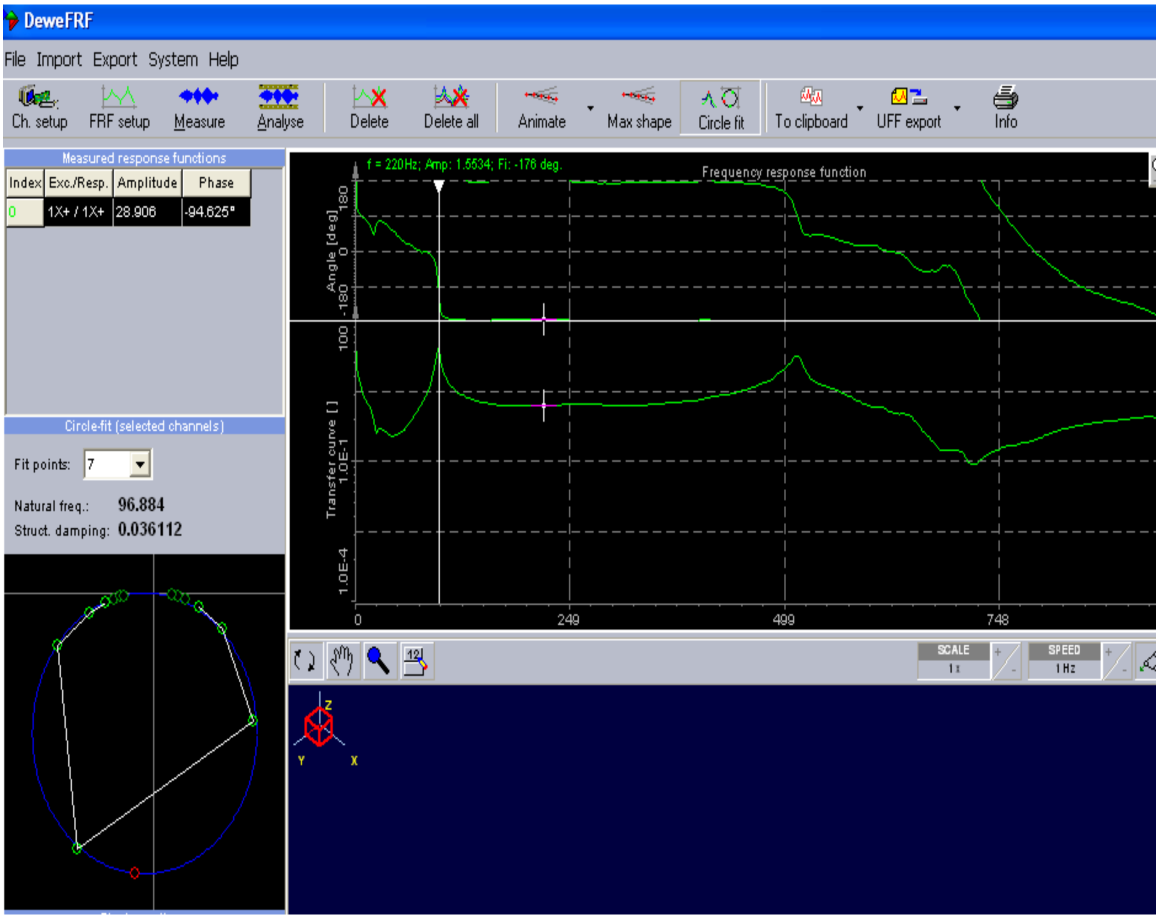

Figure-11. Natural frequency plot; EN 47 TS 20^0 oblique crack; crack details: L_1/L= 0.5; a/H= 0.6.

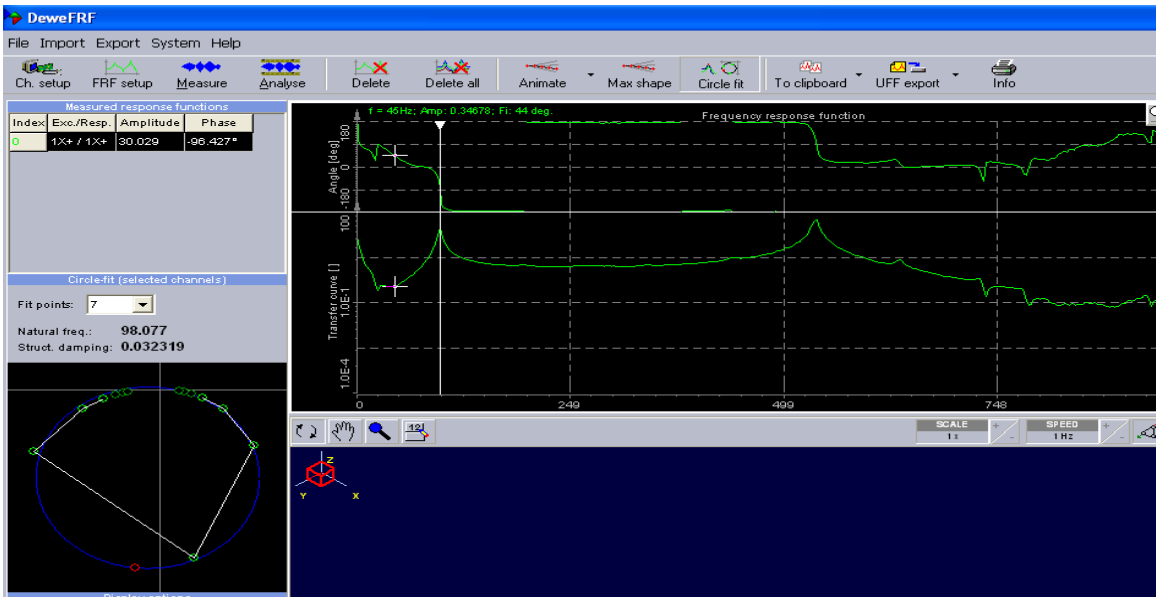

Figure-12. Natural frequency plot; EN 47 BS 20^0 oblique crack; crack details: $L_1/L= 0.5$; $a/H= 0.6$.

D. THEORY

The beam with an oblique edge crack is clamped at left end, free at other end; it has a uniform square cross-section. The Euler-Bernoulli beam model is assumed because length to width ratio of a beam is 18. The crack is assumed to be an open crack.

Governing equation of free vibration

The free bending vibration of an Euler -Bernoulli beam [14] is given by the following differential equation.

$$EI\frac{d^4y}{dx^4} - m\omega_i^2 y = 0 \quad (1)$$

where m is the mass of the beam per unit length (kg/m), ω_i is the natural frequency of the ith mode (rad/s), E is the modulus of elasticity (N/m^2) and, I is the area moment of inertia (m^4).

By defining $\lambda^4 = \omega_i^2 m/EI$ Equation. (1) is rearranged as a fourth- order differential equation as follow:

$$\frac{d^4y}{dx^4} - \lambda_i^4 y = 0 \quad (2)$$

The general solution for Equation. (2)

$$y = A\cos\lambda_i x + B\sin\lambda_i x + C\cosh\lambda_i x + D\sinh\lambda_i x, \quad (3)$$

where A, B, C, D are constants λ_i is a frequency parameter. As the bending vibration is studied, edge crack is modeled as a rotational spring with a lumped stiffness. The crack is assumed open. Based on this modeling, the beam is divided into two segments: the first and second segments are left and right-hand side of the crack, respectively. When this equation is solved by applying beam boundary conditions and compatibility relations, the natural frequency of the ith mode for uncracked Equation. (4) and cracked Equation. (5) beams is finally obtained.

$$\omega_{i0} = c_i\sqrt{\frac{EI}{mL^4}} \quad (4)$$

$$\omega_i = r_i c_i\sqrt{\frac{EI}{mL^4}} \quad (5)$$

where ω_{i0} is the ith mode frequency of the uncracked beam and c_i is known constant depending on the mode number and beam end conditions (for clamped-free beam, c_i is 3.516 and 22.034 for the first and second mode, respectively). ω_i is i_{th} mode frequency of the cracked beam. r_i is the ratio between the natural frequencies of the cracked and uncracked beam. L is the length of the beam.

3. RESULTS AND DISCUSSIONS

The zero frequency deflections and natural frequencies of cracked cases of beams are found by using ANSYS software as shown in Figures 5-10.

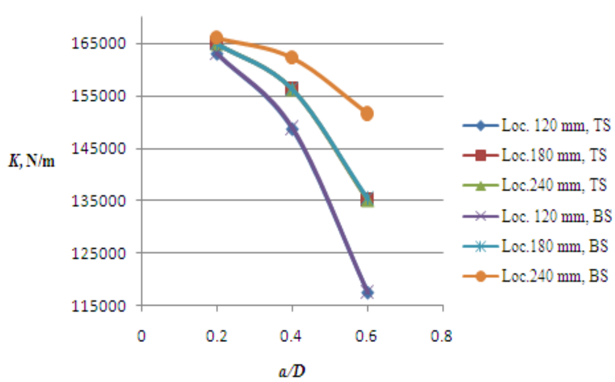

Figure-13. Variation of stiffness versus crack depth ratio for 10^0 cracked cases.

VOL. 12, NO. 4, FEBRUARY 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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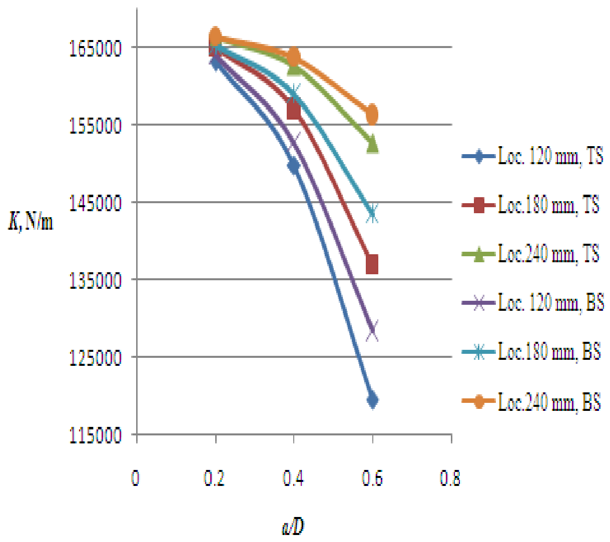

Figure-14. Variation of stiffness versus crack depth ratio for 20^0 cracked cases.

From Figure-13, it is found that as depth of 10^0 oblique cracks increases at any unique location then value of stiffness of the beam decreases. The reduction in stiffness is found same for top side and bottom side cracked cases. It means that algebraic sum of stiffness of top side cracked cases is equal to the algebraic sum of stiffness of bottom side cracked cases. This is true when crack angle equals to 10^0. It is also found that the presence of top side crack and bottom side crack of the same configuration is not a function of stiffness for 10^0 oblique cracks.

From Figure-14, it is found that as depth of the 20^0 oblique crack increases at any location then value of stiffness of the beam decreases. The reduction in stiffness for this case is found less abrupt than 10^0 oblique cracks; it means that as crack angle increases by keeping the constant crack depth then stiffness of the beam increases. Also the reduction in stiffness is found different for top side and bottom side cracked cases of the same configuration. It is observed that algebraic sum of stiffness of top side cracked cases is not equal to the algebraic sum of stiffness of bottom side cracked cases. It means that the presence of top side crack and bottom side crack of the same configuration is a function of stiffness for 20^0 oblique cracks, even though cracked beam is of square cross section.

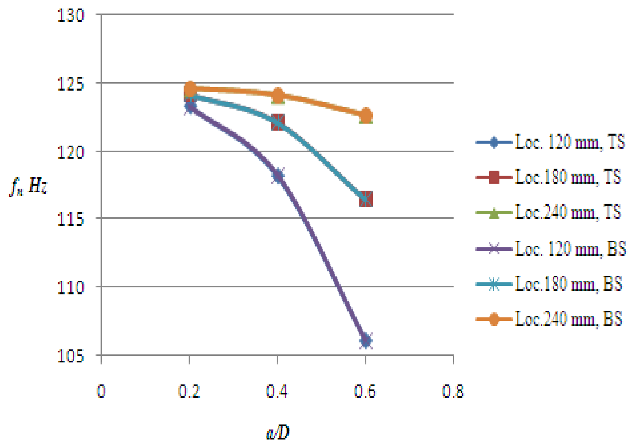

Figure-15. Variation of natural frequency versus crack depth ratio for 10^0 cracked cases by FEA.

From Figure-15, it is found that as depth of the oblique crack increases at any location then value of natural frequency decreases, it means that crack depth is the function of natural frequency. The decrease in natural frequency is comparatively least abrupt for 240 mm crack location. It means that 120 mm and 180 mm crack location contributes comparatively more damping effect in the beam than 240 mm location, due to which reduction in natural frequency is significant at such locations. Also natural frequency found for top side cracked case is identical with bottom side cracked case of the same configurations.

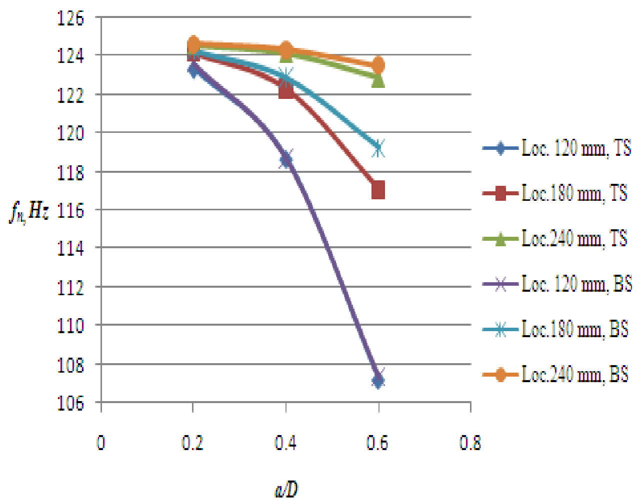

Figure-16. Variation of natural frequency versus crack depth ratio for 20^0 cracked cases by FEA.

From Figure-16, it is found that as depth of the oblique crack increases at any location then value of natural frequency decreases. The reduction in natural frequency for top side cracked cases is on higher side than bottom side cracked cases. It means that the presence of top side oblique cracks are more critical than bottom side oblique cracks; hence more attention should be given to

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the top side cracks, otherwise it may leads to early catastrophic failure of the beam.

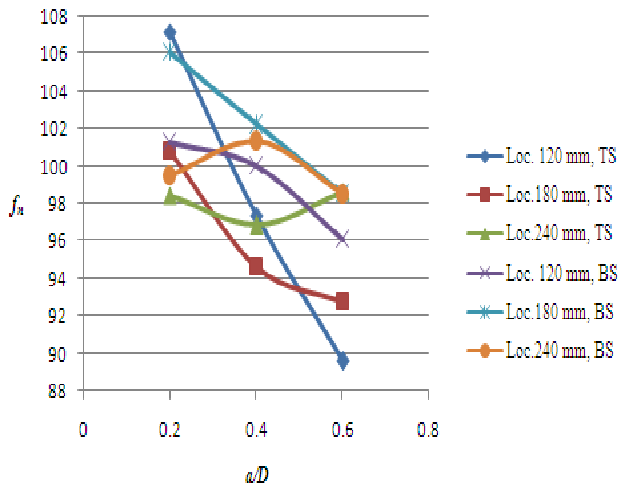

Figure-17. Variation of natural frequency versus crack depth ratio for 10^0 cracked cases by experimental method.

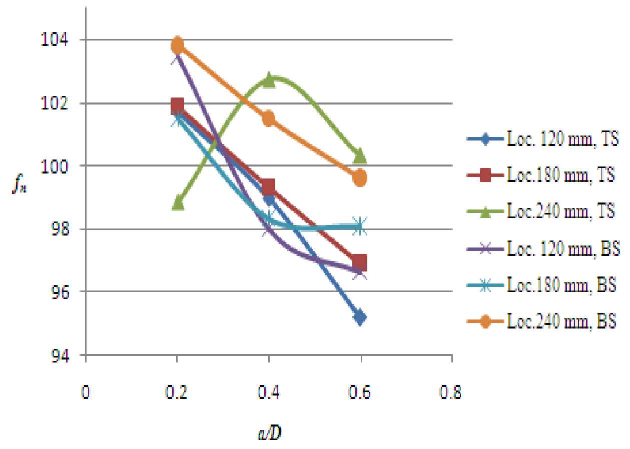

Figure-18. Variation of natural frequency versus crack depth ratio for 20^0 cracked cases by experimental method.

From Figures 17-18, it is found that as crack depth at any unique location, then natural frequency decreases. The natural frequencies found by the experimental methods are comparatively on the lower side than FEA results. This may be due to presence of some elastic properties of the fixture (specimen holder). Also presence of residual stresses increases damping effect in the beam.

4. CONCLUSIONS

Analysis focuses on study of free vibration only. In the FEA part of this study, the effect of the crack depth and location on modal properties of the beam was investigated. The following conclusions can be drawn from the analyses:

a) For 10^0 oblique cracks, the presence of top side and bottom side crack of the same configuration is not a function of stiffness.

b) For 20^0 oblique cracks, the presence of top side and bottom side crack of the same configuration is a function of stiffness even though beam is of square cross section.

c) Above mentioned conclusions are also valid for natural frequencies.

d) Presence of top side crack on the beam is comparatively more critical than presence of bottom side crack of the same configuration on the beam.

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