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A SOLUTION OF AN IMPROPER INTEGRAL EQUATION OF SHIP WAVE RESISTANCE

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ABSTRACT

Ship wave resistance had been a subject of continuous in depth study since the late 1800. The pioneer work on ship wave resistance was by Michell (1898) and followed by many studies thereafter. The ship wave equation derived and used by Michell is an improper integral form. Wigley (1926-1948) carried out further works extensively, improvised and solved the Michell ship wave resistance equation theoretically and experimentally using mathematical hull forms of thin ships. Wigley solved the equation with the assumption that the integral is convergent and errors in the remainders were appreciably negligible. Nevertheless the tendency of the ship wave resistance becoming divergent is obvious for hulls of larger angles of entrance. The objective of the paper is to present a solution for a divergent or an improper integral equation of ship wave resistance. The results matched closely with experimental results and of better accuracy as compared with theoretical results obtained from different methods performed by several other authors. This method of solution would be practical and useful engineering tool for the prediction of ship hull performance and optimization in terms of the wave resistance and hence the total resistance with the ultimate aim for application in ship powering estimates.

Keywords: ship wave resistance, michell integral, definite integral, improper integral, final root method.

INTRODUCTION

Linear thin ship theory by Michell (1898) and further worked by various successors had been reviewed. The Michell ship wave resistance equation or popularly known as the Michell Integral as further improved by Wigley (1926 - 1948) is the basis for solving the ship wave resistance problems within the ambit of linear thin ship theory in the present study. Michell derived the velocity potential by Fourier- integral theorem. He formulated the wave resistance of a thin ship in motion on the surface of an ideal fluid of infinite depth and found the relationship between wave resistance and the hull form. Michell ship wave resistance can be determined by integrating the normal pressure distribution over the hull surface along the length of the ship and substituting the derivatives of the velocity potential ϕ so derived which is of the following general form.

$$R_w = -2\rho U \iint (d\phi/dx)(d\eta/dx)\,dx\,dz \tag{1}$$

$$\begin{aligned}
\phi = {} & (2U/\pi^2)\iiiint f(\xi,\zeta)\{[\cos(nz-\varepsilon)\cos(n-\zeta)\\
& \cos m(\xi-x)]/[\sqrt{(m^2+n^2)}]\}\exp[y\sqrt{(m^2+n^2)}]\\
& d\xi d\zeta dm dn;\\
& 0 \le n \le \infty,\ 0 \le m \le \infty,\ 0 \le \zeta \le \infty,\ -\infty \le \xi \le \infty\\
& -(2U^2/\pi g)\iiint f(\xi,\zeta)[m\exp(-m^2U^2(z+\\
& \zeta)/g]/[\sqrt{(m^2U^4/g^2-1)}]\sin\{m(x-\xi)+\\
& ym\sqrt{(m^2U^4/g^2-1)}\}d\xi d\zeta dm;\ g/U^2 \le m \le \infty,\ 0 \le \zeta\\
& \le \infty,\ -\infty \le \xi \le \infty\\
& +(2U^2/\pi g)\iiint f(\xi,\zeta)[m\exp(-m^2U^2(z+\\
& \zeta)/g]/[\sqrt{(m^2U^4/g^2-1)}]\cos\{m(x-\xi)+\\
& ym\sqrt{(1-m^2U^4/g^2)}\}d\xi d\zeta dm;\\
& 0 \le m \le g/U^2,\ 0 \le \zeta \le \infty,\ -\infty \le \xi \le \infty
\end{aligned} \tag{2}$$

$$R_w = (4\rho g^2/\pi U^2)\int |(I^2 + J^2)|\sec^3\theta\, d\theta;\ 0 \le \theta \le \pi/2 \tag{3}$$

$$\begin{aligned}
J = bd \iint & (\delta\eta/\delta\xi)\exp(-dg\zeta\sec^2\theta/U^2)\sin(\ell g\xi\sec\theta/U^2)\,\delta\xi\delta\zeta;\\
& -1 \le \xi \le 1,\ 0 \le \zeta \le 1
\end{aligned} \tag{4}$$

$$\begin{aligned}
I = bd \iint & (\delta\eta/\delta\xi)\exp(-dg\zeta\sec^2\theta/U^2)\cos(\ell g\xi\sec\theta/U^2)\,\delta\xi\delta\zeta;\\
& -1 \le \xi \le 1,\ 0 \le \zeta \le 1
\end{aligned} \tag{5}$$

Where;

b – maximum half breadth (B/2), ℓ- half length (L/2), d – draft, $\xi = x/\ell$, $\eta = y/b$, $\zeta = z/d$, $\eta_x = \delta\eta/\delta\xi$, x – axial position from amidships along the ship length, y – hull offset measured from ship centerline, $\eta = f(\xi, \zeta)$ is the equation that represents one-half of the surface hull form

The above improvised Michell Integral by Wigley is herein called the "Michell-Wigley" ship wave integral or equation to give due recognition and acknowledgement to both of them in particular to Michell for pioneering the study of theoretical solution of ship wave resistance problems following linear thin ship theory as well as further works carried out by Wigley. In the current method of solution of the wave resistance of a moving ship in water is also dependent on a precise propagation angle of the combined divergent and transverse waves. This angle which sets the maximum limit of the mathematical integration of the wave resistance and the wave amplitude integral function replaces the maximum limit of integration $\pi/2$ of the original expressions by Michell-Wigley in which for a particular ship's speed the angle can be determined by solving the final or last root of the solution of the ship wave resistance integral. This unique angle is termed as the precise maximum resultant wave propagation angle abbreviated by θ_{pmax}.

Mathematically, the integral of the ship wave resistance R_w contains infinite roots of solution. These highly oscillatory complex integrals are not easily differentiable directly to determine the roots. Nonetheless, practically one may solve for the roots iteratively and observe the roots graphically by plotting the calculated values of the ship wave resistance integral against the wave propagation angle θ i.e. R_w versus θ. The positions of the infinite roots would be at the points of inflexion whereby the values of θ for the zeroes of R_w.

The study zoomed on the final root of minima which generally exists in the range of θ approaching the value of π/2. The precise maximum resultant wave propagation angle θ_{pmax} would be the final root of the ship wave resistance integral function and that of the point of minima of the R_w curve, beyond this point there should uniquely be no other root of minima exists. θ_{pmax} dictates the absolute solution of the ship wave resistance integral and hence the original governing equation of the ship wave resistance is then integrated up to the maximum limit of θ_{pmax}. This concept or methodology is a new and novel idea applied for solving such improper integral or equation of ship wave resistance for mono-hull ships. This method of solution is hereby called "Final Root Method".

The modified governing equation

The modified or revitalised Michell-Wigley ship wave resistance integral or equation may then be written mathematically in the form as follows with the upper limit of the mathematical integration is reassigned with θ_{pmax} and as for the the integrals J and I remain unchanged as originally given;

$$R_w = (4\rho g^2/\pi U^2)\int (I^2 + J^2)\sec^3\theta d\theta;\ 0 \le \theta \le \theta_{pmax} \tag{6}$$

Computational technique

In implementing the calculation technique the model principal dimensions as given below are divided into frames or stations of equal intervals or spacings longitudinally along the length of the ship and divided into waterlines of equal intervals vertically down from the free surface to the keel bottom. The mathematical calculation technique makes use of Simpson's First and Second Rules of Integration as appropriate. The coordinate system used in the analysis is shown in Figure-1. The integration performed in strip-wise sequence, first in the η-ζ direction at each cross-sectional plane of the hull followed by the integration in the ξ direction along the length of the ship hull to compute the integrals I and J with the origin at amidships. The integration is then continued in the angular θ direction from zero up to the limiting resultant propagation angle i.e. θ_{pmax} to finally determine R_w. The integration in the angular θ-direction is done at every interval of one degree and finer intervals near the resultant wave propagation angle θ_{pmax}. The values of integral I and J are read from calculation of separate subroutines for the same hull form of the mono-hull ship.

Model principal particulars (Wigley hull):

Length Waterline, L_{WL}	:	3.048 m
Draft, T	:	0.191 m
Beam Waterline, B_{WL}	:	0.305 m
Wetted Surface Area, S at draft T	:	1.381 m^2

Experimental analysis

The experimental resistance analysis could be carried out based on the procedures and formulation of ITTC 1957 as follows;

$$R_T = R_F + R_R \tag{7}$$

$$C_T = C_F + C_R = R_T/(½\rho U^2 S) \tag{8}$$

$$C_R = R_R/(½\rho U^2 S) \tag{9}$$

$$C_F = 0.075/(\log R_n - 2)^2 \tag{10}$$

Where;
ITTC – International Towing Tank Conference,
R_T - Total Resistance, R_F - Frictional Resistance,
R_R - Residuary Resistance, C_T - Total Resistance Coefficient, C_F - Frictional Resistance Coefficient,
C_R - Residuary Resistance Coefficient, R_n – Reynolds Number, U – Ship Speed, S – Hull Wetted Surface Area

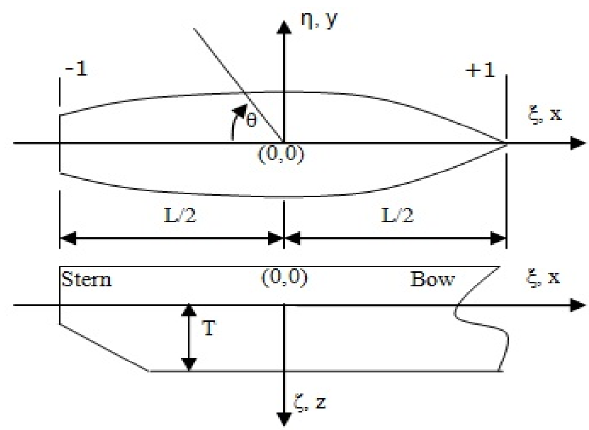

Figure-1. Coordinate system.

The experimental wave resistance R_w could be extracted from the analyzed total resistance R_T obtained from the model experiments by (1+k) factor method i.e. Prohaska's (1966) method [12]. The analysis could be performed in accordance with the following standard formulation of form factor method:

$$C_T = C_V + C_W \tag{11}$$

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$$C_V = (1+k)C_{Fo} \tag{12}$$

$$C_{Fo} = C_F \tag{13}$$

$$C_W = C_T - C_V = C_T - (1+k)C_{Fo} \tag{14}$$

$$R_w = ½\rho C_W S U^2 \tag{15}$$

Where;
R_w - Ship Wave Resistance, C_{Fo} - Model Correlation Coefficient of Frictional Resistance, C_W –Ship Wave Resistance Coefficient, C_V – Viscous Resistance Coefficient, (1+k) – Form Factor

Theoretical analysis

The theoretical analyses by final root method were performed according to the modified Michell-Wigley ship wave resistance integral for Froude number F_n ranging from 0.2 to 0.4 following the computational technique as described in the previous paragraphs.

RESULTS OF THE ANALYSIS

The results calculated by final root method are validated and compared against experimental and theoretical results of the various studies performed by other authors are presented in Tables-1 to 4 and the graphs are cross-plotted in Figure-2.

Table-1. SangseonJu (1983) experimental wave pattern.

F_n	C_w x 10^3 SangseonJu Wave Pattern (Experimental)
0.21	0.40
0.23	0.45
0.25	0.70
0.27	0.65
0.29	1.00
0.30	1.20
0.32	1.50
0.33	1.35
0.35	1.20
0.38	1.40
0.40	1.65

Table-2. SangsonJu (1983) and final root.

F_n	C_w x 10^3 SangsonJu (Experimental)	C_w x 10^3, Final Root (Modified Michell-Wigley)
0.20	0.360	0.335
0.22	0.460	0.446
0.25	0.790	0.762
0.27	0.780	0.748
0.28	0.940	0.896
0.32	1.470	1.428
0.34	1.410	1.383
0.37	1.520	1.490
0.40	2.120	2.073

Table-3. SW song and RE Baddour (1989) CFD Dawson and Newman Kelvin.

F_n	SW Song & RE Baddour	
	C_w x 10^3 (CFD Dawson)	C_w x 10^3 (CFD NewmanKelvin)
0.23	0.611	0.528
0.25	0.722	0.778
0.27	0.722	0.778
0.28	0.889	0.833
0.31	1.361	1.333
0.32	1.333	1.306
0.34	1.167	1.167
0.35	1.111	1.111
0.37	1.167	1.167
0.40	1.667	1.611

Table-4. Lalli, Campana and Bulgarelli (1989 - 1990).

F_n	CFD - 2400 Panels on FS and 800 Panels on Body		
	C_w x 10^3 (Dawson AN)	C_w x 10^3 (Dawson FD)	C_w x 10^3 (NewmanKelvin)
0.20	0.500	0.400	0.525
0.22	0.550	0.600	0.600
0.24	0.950	0.800	0.950
0.25	0.925	0.825	0.900
0.27	0.850	0.950	0.850
0.28	1.050	1.100	1.050
0.31	1.550	1.600	1.400
0.32	1.500	1.650	1.350
0.34	1.300	1.450	1.200
0.35	1.250	1.400	1.200
0.37	1.400	1.500	1.350

DISCUSSION AND CONCLUSIONS

The results of the theoretical analysis by final root method versus the experimental data and results of calculation by different methods by others authors shown in Figure-2 are evident of the effectiveness of the final root method of solution together with the computational technique applied in the studies. The existence of the final roots θ_{pmax} had been mathematically identified. The "Final Root Method" employed in the study had been verified and validated successfully by comparing the results with those of the experimental and theoretical works of others i.e. wave pattern experiments and computational fluid dynamics (CFD) methods of solution as presented in Figure-2. Both the solving method and the computational technique are therefore potentially useful for solving such an improper integral of ship wave resistance for mono-hull ship definitively. Obviously, this new method of solution would be very practical and useful engineering tool for the prediction of ship hull performance and optimisation in terms of the wave resistance and hence the total resistance with the ultimate aim for application in ship powering estimates.

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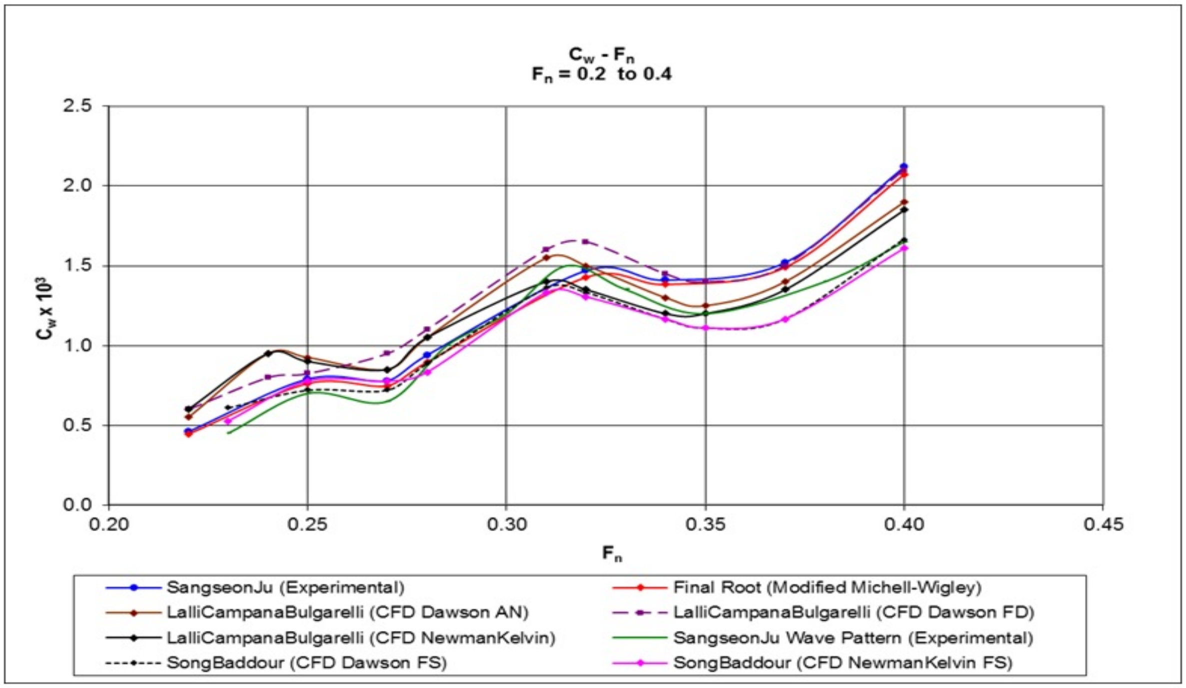

Figure-2. Graphs of experimental and theoretical C_w Versus F_n .

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