ARPN Journal of Engineering and Applied Sciences

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ASSESSMENT OF IMPACT STRENGTH OF FIBRE REINFORCED CONCRETE BY TWO PARAMETER WEIBULL DISTRIBUTION

Murali G.[1], Monika Vincy T.[1], Suraj K.[1], Ramkumar V. R.[2] and Karthikeyan K.[3]
[1]School of Civil Engineering, Sastra University, Thanjavur, Tamil Nadu, India
[2]Division of Structural Engineering, Anna University, Chennai, India
[3]The School of Mechanical and Building Sciences (SMBS), VIT University, Chennai, Tamil Nadu, India
E-Mail: murali_220984@yahoo.co.in

ABSTRACT

The investigation of impact strength (first crack strength N1 and failure strength N2) of two different type of steel fibre reinforced concretes (FRC) subjected to drop weight test was statistically commanded in this paper. For this purpose, a former researcher results were statistically investigated using two parameter Weibull distributions and presented the impact strengths in terms of reliability function. Furthermore, the Weibull parameters were determined by two estimation approaches such as least-squares (LS) regression of Y on X and least-squares (LS) regression of X on Y.Analysis suggested that the both the methods are more effective to estimate the Weibull parameters accurately due to that the deviation between the Weibull parameters obtained from the two methods was very less. In this respect, designer can choose the impact strength design value based on the required reliability.

Keywords: reliability, Weibull parameters, fibre, least square method, statistical analysis.

1. INTRODUCTION

Recently, there is a growing trend to enhance the impact resistance of civil and military infrastructure owing to the increasing number of terrorist attack [1].One of the positive solutions for improving the impact strength of such structures is incorporating different types and dosages of fibres into concrete which can significantly diminish the damage of concrete structures due to impact load [2-4]. The impact strength of concrete can be measured by the drop-weight test due to its simplicity and attractive method, as recommended by the ACI Committee 544 [5]. As a result of the nature of the drop weight impact test, and particularly because of the non-homogeneity of concrete, the results obtained from the drop weight impact test can be scattered prominently, as reported by [6-9]. Therefore, statistical analysis is the evolving technique to describe the variations in impact test results.

In recent years, Weibull distribution is widely recognized as an effective statistical tool in life testing, fatigue testing and in reliability studies, [10-11]. Several revisions have been carried out by[12], which significantly expand the applications in concrete structures [13-14]. The Weibull distribution is characterized by scale and shape parameter; the cumulative distribution function of the two parameter Weibull distribution is.

$$F(N;\propto;\gamma) = 1 - \exp[-(\tfrac{N}{\alpha})]^{\gamma} \quad (1)$$

Where N is the strength, α is the scale parameter and γ is the shape parameter.

Several methods have been proposed for estimating Weibull parameters namely graphical method, maximum likelihood method, empirical method, energy pattern factor method, least squares and equivalent energy method [15-16]. Out of these, Least-squares (LS) method estimation which is called LS Y on X and LS X on Y methods are of recent interests [16-17].

Table-1. Number of blows required to cause initial crack and failure [14].

S. No.	N_1/N_2						
	PC	CF2	CF3	CF4	HF5	HF6	HF7
1	8/12	11/20	17/23	15/26	12/20	16/25	17/28
2	9/17	12/23	20/26	19/29	14/22	19/27	19/31
3	11/20	15/24	21/29	22/35	15/27	22/30	22/38
4	13/25	16/26	24/31	27/38	17/31	26/34	28/42
5	15/28	18/28	26/34	29/41	19/33	28/36	33/45
6	17/31	20/32	27/37	30/44	21/36	31/39	35/46
Mean	12/22	15/26	23/30	24/36	16/28	24/32	26/38

To the authors' best knowledge; though there are few studies available for evaluating the variations in impact test results statistically, there is only one study reporting impact strength in terms of reliability by

VOL. 12, NO. 5, MARCH 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

graphical method of Weibull distribution [14]. Therefore, the two methods used in this study namely, LS Y on X and LS X on Y to determine the Weibull parameters and described the impact strength of steel FRC in terms of reliability has not been performed by any of the earlier researchers. The statistical analysis was performed from the experimental test results [14] as shown in Table 1.

1.1 Weibull distribution

Weibull distribution is being used to model extreme values such as failure times and fracture strength. Two popular forms of this distribution are two and three parameter Weibull distributions. The cumulative density function of two-parameter distribution has been shown in Equation (1). Therefore, the two-parameter Weibull distribution is used to clarify the distribution characteristics of the impact strength in seven groups of samples.The impact strengthin terms of probability of survival (R)i.e., reliability [12].

$$N_R = \alpha(-\ln(R)^{\frac{1}{\gamma}}) \qquad (2)$$

1.2 Estimators of LS Y on X

Least square method is also used to calculate the Weibull parameter when modeling an experiment of a phenomenon and it can give an estimation of the parameters. When using least square method, the sum of squares of the deviations S which is defined as below [21], should be minimized.

$$S = \sum_{i=1}^{n}[Y_i - (\gamma_0 - \gamma_1 X_i)]^2 \qquad (3)$$

Where γ_0 and $\gamma_1$are the least squares estimators' that minimize the error sum of squares. By adding and subtracting the $\overline{X}$&$\overline{Y}$ gives

$$= \sum_{i=1}^{n}[(Y_i + \overline{Y} - \overline{Y}) - \gamma_0 - \gamma_1(X_i + \overline{X} - \overline{X})]^2$$
$$= \sum_{i=1}^{n}[(\overline{Y} - \gamma_0 - \gamma_1\overline{X}) + Y_i - \overline{Y} - \gamma_1 X_i + \gamma_1\overline{X}]^2$$
$$= \sum_{i=1}^{n}[(\overline{Y} - \gamma_0 - \gamma_1\overline{X}) - (\gamma_1 X_i - \gamma_1\overline{X} - Y_i + \overline{Y})]^2$$
$$= \sum_{i=1}^{n}[(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2] + \sum_{i=1}^{n}[(\gamma_1 X_i - \gamma_1\overline{X} - Y_i + \overline{Y})]^2$$
$$= n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2 + \sum_{i=1}^{n}[(\gamma_1 X_i - \gamma_1\overline{X} - Y_i + \overline{Y})]^2$$
$$= n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2 + \sum_{i=1}^{n}[\gamma_1(X_i - \overline{X}) - (Y_i - \overline{Y})]^2$$
$$= n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2 + \sum_{i=1}^{n}[\gamma_1{}^2(X_i - \overline{X})^2 - 2\gamma_1(X_i - \overline{X})(Y_i - \overline{Y}) +)(Y_i - \overline{Y})^2]$$
$$= n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2 + \gamma_1{}^2\sum_{i=1}^{n}(X_i - \overline{X})^2 - 2\gamma_1\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y}) + \sum_{i=1}^{n}(Y_i - \overline{Y})^2$$
$$= n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2 + \left(\gamma_1{}^2 - \frac{2\gamma_1\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - X)^2} + \frac{\sum_{i=1}^{n}(Y - \overline{Y})^2}{\sum_{i=1}^{n}(X_i - X)^2}\right)\sum_{i=1}^{n}(X_i - X)^2$$

By adding and subtracting the term $[\frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{\sum_{i=1}^{n}(X_i-X)^2}]^2$ we get

$$= n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2 + \left(\gamma_1{}^2 - \frac{2\gamma_1\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - X)^2} + [\frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - X)^2}]^2 + \frac{\sum_{i=1}^{n}(Y - \overline{Y})^2}{\sum_{i=1}^{n}(X_i - X)^2} - [\frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - X)^2}]^2\right)\sum_{i=1}^{n}(X_i - X)^2$$

This can be written as

$$S = n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2 + \left((\gamma_1 - \frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{\sum_{i=1}^{n}(X_i-X)^2})^2 + \sum_{i=1}^{n}(Y_i - Y)^2 - [\frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{(X_i-X)}]^2\right)$$

This is the minimum possible value for the S.[21]To minimize the S, it is split into sum of three terms. It is to be noted here that only the first two terms involve the parametersγ_0 and γ_1. The third term is a function of the data and not the parameter. In order to achieve the minimum possible value the first two terms of the above equation are set to zero. Thus we can solve for γ_0 and $\gamma_1$as,

$$0 = n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2$$
$$0 = (\gamma_1 - \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - X)^2})^2$$

From first the equation we get

$$0 = n(\overline{Y} - \gamma_0 - \gamma_1\overline{X})^2$$
$$0 = \overline{Y} - \gamma_0 - \gamma_1\overline{X}$$
$$a_{yx} \text{ or } \gamma_0 = \overline{Y} - \gamma_1\overline{X}$$

From second the equation we get

$$0 = (\gamma_1 - \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - X)^2})^2$$
$$0 = \gamma_1 - \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - X)^2}$$
$$b_{yx} \text{ or } \gamma_1 = \frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{\sum_{i=1}^{n}(X_i-X)^2} \qquad (4)$$

Let a_{yx} and b_{yx} denote the estimators of α and β.

VOL. 12, NO. 5, MARCH 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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$$b_{yx} = \frac{\sum_{i=1}^{n}[(X_i-\overline{X})(Y_i-\overline{Y})]}{\sum_{i=1}^{n}(X_i-X)^2} \quad (5)$$

$$a_{yx} = \overline{Y} - b_{yx}\overline{X} \quad (6)$$

$$\bar{X} = \sum_{I=1}^{n} X_i/n \quad (7)$$

$$\bar{Y} = \sum_{I=1}^{n} Y_i/n \quad (8)$$

Table-2. The sample spreadsheet for calculating the LS estimates for PC (N1).

i	U	$\hat{F}_i$	Xi=LN(N)	Y_i=LN(-ln(1-$\hat{F}_i$))	Xi-$\overline{X}$	Yi-$\overline{Y}$	(Xi-$\overline{X}$)(Yi-$\overline{Y}$)	(Xi-$\overline{X}$)^2	(Yi-$\overline{Y}$)^2
1	8	0.109	2.079	-2.156	-0.384	-1.655	0.636	0.147	2.739
2	9	0.266	2.197	-1.175	-0.266	-0.675	0.180	0.071	0.455
3	11	0.422	2.398	-0.602	-0.066	-0.101	0.007	0.004	0.010
4	13	0.578	2.565	-0.147	0.101	0.353	0.036	0.010	0.125
5	15	0.734	2.708	0.282	0.245	0.782	0.191	0.060	0.612
6	17	0.891	2.833	0.794	0.370	1.295	0.479	0.137	1.677

Table-3. The calculated Weibull parameters γ and α.

Method	P	LS-YX		LS-XY	
		N1	N2	N1	N2
PC	γ	3.56	2.98	3.68	3.01
	α	13.52	24.96	13.46	24.93
CF2	γ	4.52	6.39	4.67	6.60
	α	16.76	27.27	16.70	27.21
CF3	γ	5.97	6.06	6.09	6.10
	α	24.16	32.18	24.12	32.16
CF4	γ	3.83	5.14	3.95	5.26
	α	26.19	38.48	26.09	38.39
HF5	γ	5.10	4.47	5.20	4.63
	α	17.70	30.82	17.67	30.70
HF6	γ	4.21	6.04	4.26	6.25
	α	25.99	34.16	25.95	34.07
HF7	γ	3.45	5.05	3.67	5.31
	α	28.61	41.62	28.36	41.42

P: Parameters

1.3 Estimator of LS X on Y

The estimating equation of LS X on Y can be obtained in a similar approach, the formula as given as.

$$S = \sum_{i=1}^{n}[X_i - (\frac{1}{\gamma Y_i} + \ln \propto)]^2 \quad (9)$$

$$b_{xy} = \frac{\sum_{i=1}^{n}(Y_i-\overline{Y})^2}{\sum_{i=1}^{n}[(X_i-X)(Y_i-\overline{Y})]} \quad (10)$$

$$a_{yx} = \overline{Y} - b_{xy}\overline{X} \quad (11)$$

$\bar{X}$ and $\bar{Y}$ LS X on Y is calculated using the Equations (7) and (8). Table-2 tabulates the spread sheet for calculating a_{yx} (Intercept) and b_{yx} (γ) using the two LS regression methods for the mix PC (N1).

The Weibull parameters were obtained from the LS-Y on X and LS-X on Y methods is shown in Table-3. For example, the γ value for the mix PC by LS Y on X and LS X on Y was 3.56 and 3.68 respectively in case of N1 and 2.98 and 3.01 respectively in case of N2. The deviation between the γ values obtained from two methods was very less and the same trend was obtained for remaining mixes (CF2, CF3, CF4, HF5, HF6, and HF7). The Weibull parameters deviations between LS Y on X and LS X on Y were observed very least and more or less same for all the mixes, hence these two methods are sufficient to estimate the Weibull parameters accurately.

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Table-4. Results of the Impact strength in terms of reliability (LS-Y on X).

Reliability	PC		CF2		CF3		CF4		HF5		HF6		HF7	
	N1	N2	N1	N2	N1	N2	N1	N2	N1	N2	N1	N2	N1	N2
0.01	21	42	24	35	31	41	39	52	24	43	37	44	45	56
0.05	18	36	21	32	29	39	35	48	22	39	34	41	39	52
0.10	17	33	20	31	28	37	33	45	21	37	32	39	36	49
0.15	16	31	19	30	27	36	31	44	20	36	30	38	34	47
0.20	15	29	19	29	26	35	30	42	19	34	29	37	33	46
0.25	15	28	18	29	26	34	29	41	19	33	28	36	31	44
0.30	14	27	17	28	25	33	27	40	18	32	27	35	30	43
0.35	14	25	17	27	24	32	27	39	18	31	26	34	29	42
0.40	13	24	16	27	24	32	26	38	17	30	25	34	28	41
0.45	13	23	16	26	23	31	25	37	17	29	25	33	27	40
0.50	12	22	15	26	23	30	24	36	16	28	24	32	26	39
0.55	12	21	15	25	22	30	23	35	16	27	23	31	25	38
0.60	11	20	14	25	22	29	22	34	16	27	22	31	24	36
0.65	11	19	14	24	21	28	21	33	15	26	21	30	22	35
0.70	10	18	13	23	20	27	20	31	14	24	20	29	21	34
0.75	10	16	13	22	20	26	19	30	14	23	19	28	20	33
0.80	9	15	12	22	19	25	18	29	13	22	18	27	19	31
0.85	8	14	11	21	18	24	16	27	12	21	17	25	17	29
0.90	7	12	10	19	17	22	15	25	11	19	15	24	15	27
0.95	6	9	9	17	15	20	12	22	10	16	13	21	12	23
0.99	4	5	6	13	11	15	8	16	7	11	9	16	8	17

VOL. 12, NO. 5, MARCH 2017 ISSN 1819-6608
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Table-5. Results of the Impact strength in terms of reliability (LS-X on Y).

Reliability	PC		CF2		CF3		CF4		HF5		HF6		HF7	
	N1	N2	N1	N2	N1	N2	N1	N2	N1	N2	N1	N2	N1	N2
0.01	20	41	23	34	31	41	38	51	24	43	37	44	43	55
0.05	18	36	21	32	29	38	34	47	22	39	34	41	38	51
0.10	17	33	20	31	28	37	32	45	21	37	32	39	36	48
0.15	16	31	19	30	27	36	31	43	20	35	30	38	34	47
0.20	15	29	18	29	26	35	29	42	19	34	29	37	32	45
0.25	15	28	18	29	25	34	28	41	19	33	28	36	31	44
0.30	14	27	17	28	25	33	27	40	18	32	27	35	30	43
0.35	14	25	17	27	24	32	26	39	18	31	26	34	29	42
0.40	13	24	16	27	24	32	26	38	17	30	25	34	28	41
0.45	13	23	16	26	23	31	25	37	17	29	25	33	27	40
0.50	12	22	15	26	23	30	24	36	16	28	24	32	26	39
0.55	12	21	15	25	22	30	23	35	16	27	23	31	25	38
0.60	11	20	14	25	22	29	22	34	16	27	22	31	24	36
0.65	11	19	14	24	21	28	21	33	15	26	21	30	23	35
0.70	10	18	13	23	20	27	20	32	14	25	20	29	21	34
0.75	10	16	13	23	20	26	19	30	14	23	19	28	20	33
0.80	9	15	12	22	19	25	18	29	13	22	18	27	19	31
0.85	8	14	11	21	18	24	16	27	12	21	17	25	17	29
0.90	7	12	10	19	17	22	15	25	11	19	15	24	15	27
0.95	6	9	9	17	15	20	12	22	10	16	13	21	13	24
0.99	4	5	6	14	11	15	8	16	7	11	9	16	8	17

The impact strength was presented in terms of reliability as shown in Tables 4 and 5. The reliability that the true value lies is within the interval 0 or 0.99. For more certain assessment consider 0.90 reliability level and this value is substituted in Equation (2) and solved, the N1 values for the mix PC, CF2, CF3, CF4, HF5, HF6 and HF7 were 7, 10, 17, 15, 11, 15 and 15 respectively for both the methods LS-Y on X and LS-X on Y. Similarly the obtained N2 values were same for both the methods for the mix CF2, CF3, CF4, HF5, HF6 and HF7 were12, 19, 22, 25, 19, 24, 27 respectively.

On the other side, considering the 0.2 reliability level the N1 value for the mix PC, CF2, CF3, CF4, HF5, HF6 and HF7 were 15, 19, 26, 30, 19, 29, 33 respectively in case of LS- Y on X and were 15, 18, 26, 29, 19, 29, 32 respectively in case of LS-X on Y. Since this method discards of taking the average of experimental test results, in this respect two parameter Weibull distribution enable the designers to describe the impact strength in terms of a reliability level. The results obtained using Weibull reliability analysis is reliable for application of civilian infrastructures such as offshore structure exposed to ice and barge impact, airport run way exposed to dynamic loads due to aircraft take-off and landing, vehicle crash impact of concrete structures etc.

2. CONCLUSIONS

From the drop weight, experimental tests results, it is critical to choose the design value owing to its lack of reliability which led to increase the probability of failure. In this paper, a proficient and deepened method was employed to analyse the variations in drop weight test results with the help of two parameter Weibull distribution; also, the impact strength was described in terms of reliability level. The following conclusions were made based on statistical analysis.

- Weibull parameters were determined using LS- Y on X and LS- X on Y method. The γ value for the mix PC by LS Y on X and LS X on Y was 3.56 and 3.68 respectively in case of N1 and 2.98 and 3.01 respectively in case of N2. The deviation between the γ and α values obtained from two method was very less for all the mixes, analysis suggested that the both the methods are more effective to estimate the Weibull parameters accurately.

- Considering 0.90 reliability level, the N1 values for the mix PC, CF2, CF3, CF4, HF5, HF6 and HF7 were 7, 10, 17, 15, 11, 15 and 15 respectively for both the methods LS-Y on X and LS-X on Y. Similarly, the obtained N2 values were for the mix CF2, CF3, CF4, HF5, HF6 and HF7 were12, 19, 22, 25, 19, 24, and 27 respectively.
- By introducing two parameter Weibull distributions for analyzing the variations in experimental test results, it led to elimination of taking the average. In this respect Weibull distribution facilitate the designers to describe the impact strength in terms of a reliability level. Lastly, the Weibull distribution was employed here to model an impact strength property and reliability analysis, but it can also be used in areas with similar uncertainties.

ACKNOWLEDGMENTS

An author gratefully acknowledges support by Sastra University.

REFERENCES

[1] Y. Sun, Z. Yu, Z. Wang, X. Liu. 2015. Novel protective covering to enhance concrete resistance against projectile impact. Construction and Building Materials. 96: 484-490.

[2] G. Murali, A. S. Santhi, and G. Mohanganesh. 2016. Loss of mechanical properties of fiber-reinforced concrete exposed to impact load. Romanian Journal of Materials. 46(4): 491-496.

[3] U. A. S Faiz. 2013. Review of mechanical properties of short fibre reinforced geopolymer composites. Construction and Building Materials. 43: 37-49.

[4] 1989. ACI Committee 544.2R-89, Measurement of properties of fibre reinforced concrete, Detroit: American Concrete Institute.

[5] M. C. Nataraja, N. Dhang, A. P. Gupta. 1989. Statistical variations in impact resistance of steel fiber-reinforced concrete subjected to drop weight test, Cement and Concrete Research. 29: 989-995.

[6] P. S. Song, J. C. Wu, S. H. Wang, B. C. Sheu. 2005. Statistical analysis of impact strength and strength reliability of steel-polypropylene hybrid fiber-reinforced concrete. Construction and Building Materials. 19: 1-9.

[7] T. Rahmani, B. Kiani, M. Shekarchi, A. Safari. 2012. Statistical and experimental analysis on the behavior of fiber reinforced concretes subjected to drop weight test, Construction and Building Materials. 37: 360-369.

[8] M. Mastali. A. Dalvand. 2016. The impact resistance and mechanical properties of self-compacting concrete reinforced with recycled CFRP pieces, Composites Part B. 92: 360-376.

[9] A. S. Haidyrah, J. W. Newkirk, C. H. Castano. 2016. Weibull statistical analysis of Krouse type bending fatigue of nuclear materials, Journal of Nuclear Materials. 470: 244-250.

[10] K. S. P. Kumar, S. Gaddada. 2015. Statistical scrutiny of Weibull parameters for wind energy potential appraisal in the area of northern Ethiopia, Renewables. 2(14): 1-15.

[11] C. Fredy, J. L.Artur. 2015. A new generalized Weibull distribution generated by gamma random variables. Journal of the Egyptian Mathematical Society, 23: 382-390.

[12] X. Chen, Y. Ding C. Azevedo. 2011. Combined effect of steel fibres and steel rebars on impact resistance of high performance concrete. Journal of Central South University Technology. 18: 1677-1684.

[13] G. Murali, A. S. Santhi G. Mohan Ganesh. 2014. Impact resistance and strength reliability of Fiber-reinforced concrete in bending under drop weight impact load, International Journal of Technology, 5(2): 111-120.DOI:

[14] A. C. R. Paulo, C. D. S. Ricardo, F. D. A. Carla E. V. D. S. Maria. 2012. Comparison of seven numerical methods for determining Weibull parameters for wind energy generation in the northeast region of Brazil, Applied Energy. 89: 395-400.

[15] L. F. Zhang, M. Xie, L. C. Tang. 2007. A study of two estimation approaches for parameters of Weibull distribution based on WPP, Reliability Engineering and System Safety. 92: 360-368.

[16] A. K. Azad, M. G. Rasul, R. Islam, I. R. Shishir. 2015. Analysis of wind energy prospect for power generation by three Weibull distribution methods. Energy Procedia. 75: 722-727.

[17] J. C. Fothergill. 1990. Estimating the Cumulative Probability of Failure Data Points to be plotted on Weibull and Other Probability Paper. IEEE Transactions on Electrical Insulation. 25(3): 489-92.

[18] K. R. Skinner, J. B. Keats, W. J. Zimmer. 2001. A Comparison of Three Estimators of the Weibull Parameters, Quality Reliability Engineering International. 17(4): 249-56.

[19] A. Bernard, L. E. C. Bosi. 1943. The Plotting of Observations on Probability Paper, Stat Neerlandica. 53(7): 163-73.

[20] I. Eric. 1996. A Note on Derivation of the Least Squares Estimator, Working Paper Series No. 96-11. University of Hawai'i at Manoa Department of Economics.