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A TECHNIQUE FOR ROTOR SYSTEMS RELIABILITY ESTIMATION BASED ON STATISTICAL MODELING OF VIBRATIONS

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ABSTRACT

Vibrations excited by rotating parts determine or limit reliability of machine. Mathematical models of rotor system vibrations are usually do not consider random parameters such as dimensional and weight discrepancy, misbalances magnitudes etc. Although the random factors in particular cases define vibrations characteristics variation and probability of failures related to them. The goal of the work is to develop a technique for reliability characteristics estimation based on mathematical modeling of rotor systems vibrations considering all random parameters mentioned above. The methodology includes three stages. On the first stage an analysis of rotor system failure probability found on fault tree diagram. On the second stage a determinate mathematical model is developed for rotor system vibration characteristics and structural loads evaluation. On the third stage components loads are considered as random values. Using Monte-Carlo simulation factors distribution curves are calculated and fault probability estimation for the rotor system itself is performed. Applicability of the methodology is demonstrated on simple rotor system analysis. System consists of shaft with a disk, mounted in bearing supports. Specifics of parameters dispersion of the system are presented. The developed technique could be used for vibration reliability analysis of mechanical systems like gas turbine engines including complex rotor systems.

Keywords: rotor system, vibration, failure probability, amplitude-frequency response, fault tree analysis, statistical modeling method.

1. INTRODUCTION

Rotor systems are the part of many different power, transporting and technological machines, particularly gas turbine engines and power plants. One of the main problems which are solved during the machines development and operation is safety and reliability insurance. Substantially the solution of the mentioned problem depends on elimination of unwanted vibrations usually excited by rotors with some level of misbalance under centrifugal forces. Tendency of engines weight decrease together with power and parts loads increase causes vibration reliability problem becomes more acute [1-5]. Vibration characteristics of machines with rotor systems are usually defined during design concept validation and their further changes are connected with comprehensive processes and time-costs increase. All mentioned confirm that effective analyses and prediction techniques development regarding vibration reliability is still an actual problem.

Determinate mathematical models of various levels of complexity are typically used during rotor systems investigations. They allow describing basic effects of design and operational factors on rotors vibration characteristics [3-8]. By the way from field experience it is known that random factors have significant effect on vibration parameters of rotor systems. For instance, data described in [5] show vibratory velocity amplitudes with five to ten times greater dispersion in aircraft engines on the same operational regimes. Wide amount of publications is dedicated to vibrations investigation as a random process [9]. Impacts of assembly factors as a combination of parts tolerances defining main rotor parameter – misbalance are described in [10-12].

Usually mathematical models of rotor systems vibrations don't allow considering combination of random factors as components dimensional and weight tolerances variation misbalances, bolts connection toques etc. But those factors are main contributors of vibration characteristics dispersion and fault probability connected to them.

The main goal of the investigation consists of vibration reliability estimation technique development based on mathematical modeling of vibrations considering random factors mentioned above.

2. RELIABILITY PARAMETERS COMPUTATION TECHNIQUE

Proposed technique of rotor system reliability parameters estimation is based on vibrations modeling and consists of following steps (Figure-1):

- Rotor system design and fault probability analysis via fault tree diagram;
- Mathematical model development for rotor system parameters (loads) estimation which are key factors for its components reliability;
- Establishing of statistic distribution of system's parameters based on experimental data;
- Fault probability estimation based on statistic distribution of loads acting on system's components via statistical modeling
- Overhaul system's fault probability calculation by components faults probabilities.

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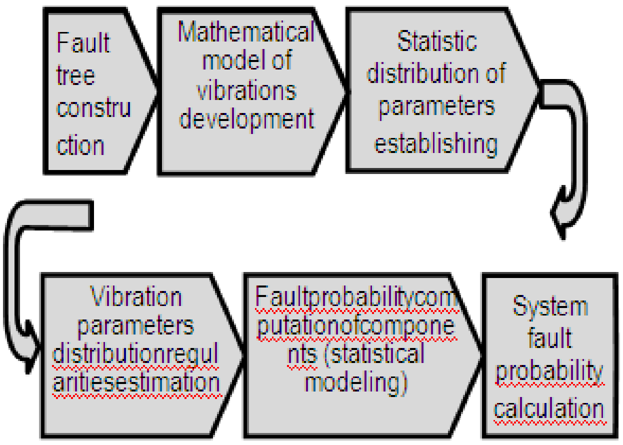

Figure-1. Rotor system fault probability estimation technique.

Fault tree diagram is suitable tool for components faults connection to critical system's fault. Fault tree methodology is described for instance in [2]. An analysis result on the first step is Boolean formula for system's fault probability estimation through its components fault probabilities. For rotor systems in our analysis let's consider faults caused by prohibited vibrations such as bearing supports faults, shafts and connection elements.

For the fault probability analysis of rotor elements it is proposed using of Monte-Carlo statistic modeling approach [13]. In [14] the same approach was used for reliability analysis of power plants.

As applied to the rotor dynamics the methodology allows vibration characteristics vector definition - $\mathbf{\hat{Y}}$ as combination of random factors by vector of rotor system design parameters $\mathbf{\check{Z}}$ which elements are random variables. The methodology considers consisting of deterministic mathematical model **Y(X)** which connects parameters of system **Z** with vibration parameters **Y**. Algorithm of the methodology consists of number of calculations - *N*(random process realization) according to deterministic model. In each of calculation random elements of vector **Z**are used as an input. For each element of vector **Y** set of *N* calculations is a selection of random variable of vector $\mathbf{\hat{Y}}$. Distribution of the variable defines vibration characteristic.

In each calculation random selection of design parameters $\mathbf{\check{Z}}$ based on deterministic model considers their known statistical distribution. Minimally it is necessary to know mathematical expectation (could be nominal value of a parameter) and estimated or expected dispersion. If no consistent data about parameter of vector $\mathbf{\check{Z}}$ are known as a first estimation it is allowed to apply uniform or normal distribution. In statistical modeling algorithms pseudorandom number generators are used.

For fault probability estimation of *i* element of rotor system it is necessary to analyze that the probability of component $\hat{Y}_i$(responsible for the element fault) of vector $\mathbf{\hat{Y}}$ defined by statistical modeling is greater than critical value Y_i^*: $q_i(\hat{Y}> Y_i^*)$. Rotor system fault probability *q* is calculated through faults probabilities of its components using Boolean formulas, developed by first step of fault tree analysis.

3. METHODOLOGY APPLICATION EXAMPLE

Reliability characteristics estimation technique application based on mathematical modeling of rotor system vibration considering random factors will be described on simple system shown on Figure-2. The system presents of rotor consisting of weightless shaft of length *l* and disk of weight *m*. The shaft is supported by two elasto-damping supports with stiffness C_{o1} and C_{o2}. The rotor has misbalance of *Q*.

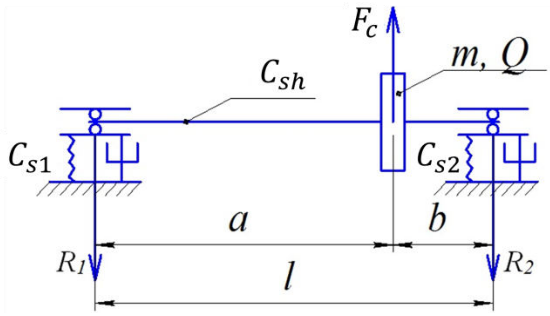

Figure-2. Rotor schema.

Considering system's weight, stiffness, misbalance and decrement of damping as a random variables with known experimental distributions it is necessary to deterministic statistical parameters of vibration responsible for system's reliability: reaction forcesR_1 and R_2 plus centrifugal force F_c. Moreover it is important to calculate fault probability of that system taking into account extreme values of supports loads R_1^* and R_2^* and shaft load F^*. Thus system's parameters vector is defined as $\mathbf{Z}^T$={m,C_{sh}, C_{s1} ,C_{s2} , δ, *Q*}and vibration vector as $\mathbf{Y}^T$={ R_1 , R_2 , F_c}.

Calculation of vibration characteristics using deterministic model is performed for nominal values shown in Table-1.

Table-1. Nominal values of the rotor system parameters.

Parameter	Value	Parameter	Value	Parameter	Value
m,kg	6,168	C_{s2}, MN/m	2,0	*L*,mm	750
C_{sh}, MN/m	1,0	δ	0,1	*a*,mm	250
C_{s1}, MN/m	3,0	*Q*,kg∗mm	3,1	*b*, mm	500

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System with parameters same as in Table-1 was used in experimental investigations [15, 16].

Deterministic model for vibration characteristic estimation for described rotor system is well known, see [1, 3]. If gyroscopic moments are considered as negligible the model is defined by following relations.
Critical speed of the rotor n_c:

$$n_c = \frac{p}{2 \cdot \pi} \tag{1}$$

$$p = \sqrt{C/m} \tag{2}$$

Where Stiffness of the system C is defined by shafts flexibilities and supports flexibilities:

$$C = (1/C_{o1} + 1/C_{o2} + 1/C_{в})^{-1} \tag{3}$$

Amplitude of vibration $A$is computed as:

$$A = \frac{e\omega^2}{\sqrt{\left(p^2 - \omega^2\right)^2 + \left(2\delta p \omega\right)^2}}, \tag{4}$$

where ω – rotor speed, $e = Q/m$ – eccentricity.
Centrifugal force of a disk F_c:

$$F_c = M \cdot \omega^2 \cdot (A + e) \tag{5}$$

Reaction forces in supports:

$$R_1 = F_c \frac{b}{l}; \qquad R_2 = F_c \frac{a}{l}. \tag{6}$$

In described rotor system shaft supports are subjected to vibrations. Extreme loads causing rotor fault for supports are R^* for a shaft is F^*. Determination of extreme loads values is not included into scope of the investigation and further they are considered as known.

Fault tree for the analyzed system is presented on Figure-3. System fault occurs when F_c is greater than extreme value F^* or supports loads R_1 or R_2 exceed extreme value of R^*.

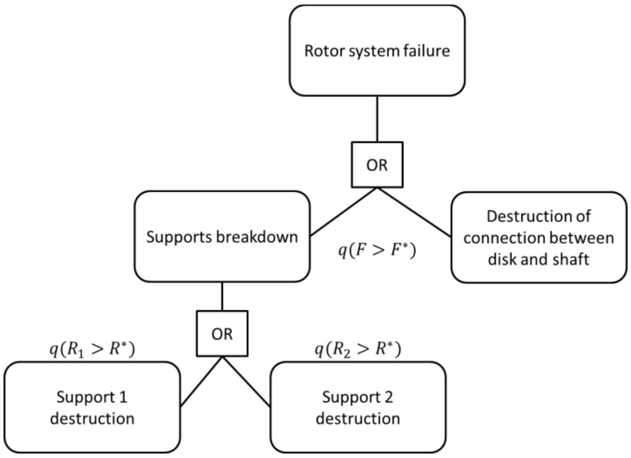

Figure-3. Fault tree of analyzed rotor system.

The system's fault probability P is determined as:

$$q = (F_{ц} \geq F^*) \vee (R_1 \geq R^*) \vee (R_2 \geq R^*) \tag{7}$$

Statistical modeling of vibrations was performed via mathematical model in accordance with equations (1) - (4). Statistical parameters for rotor system characteristics used in modeling are listed in Table-2. Statistical modeling included N=5000 realizations.

Table-2. Statistical parameters for rotor system characteristics.

Parameter	Distribution law	Mathematical expectation	Standard deviation	Variation coefficient
m, kg	normal	6,168	0,31	0,05
C_{sh}, MN/m	normal	1,0	0,05	0,05
C_{s1}, MN/m	normal	3,0	0,15	0,05
C_{s2}, MN/m	normal	2,0	0,10	0,05
δ	normal	0,1	0,02	0,2
Q,kg*mm	normal	3,1	0,31	0,1

Histogram of critical rotor speeds distribution is shown on the Figure-4. Values determined by statistical modeling are in agreement with calculated values by deterministic mathematical model (see Table-3). Variation coefficient K_v=0, 03 is less that variation coefficients of weight and stiffness K_v=0, 05.

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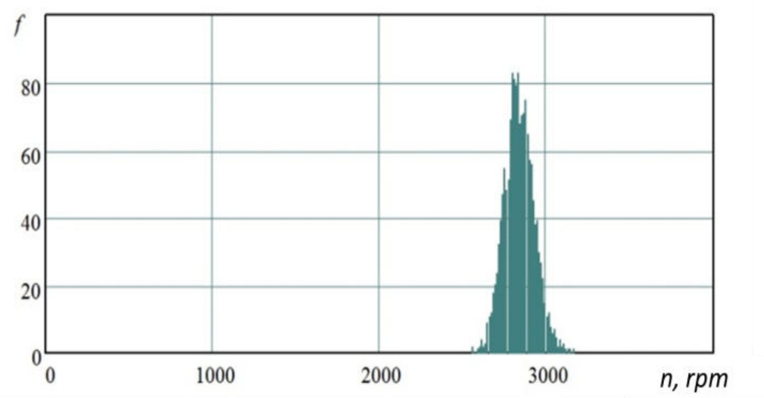

Figure-4. Critical rotor speeds distribution.

Table-3. Statistical characteristics of rotor system vibrations parameters*.

Parameter	X_{det}	X_{av}	X_{max}	X_{min}	σ	K_v	A_s
n_c, rpm	**2839**	**2839**	**3157**	**2571**	**84**	**0,03**	**0,21**
A_{3000}, mm	2,33	2,35	6,87	1,07	0,58	0,25	1,4
F_c , N	1722	1730	4948	951	333	0,19	1,26
R_1, N	574	576	1649	317	111	0,19	1,26
R_2, N	1148	1154	3298	634	221	0,19	1,26

***Note:** in table: X_{det} - calculated value of parameter according to deterministic model; X_{av}, X_{max},X_{min} - average, maximum and minimum values of parameter determined by statistical modeling; σ - standard deviation; K_v - variation coefficient; A_s - asymmetry ratio; A_{3000} - vibration amplitude at rotor speed of 3000 RPM.

Vibration amplitudes distributions for various rotor speeds are shown on the Figure-5. It is visible that the amplitude has considerable dispersion. At the rotor speed of 3000 RPM variation coefficient is K_v=0, 25 (see Table-3) and that is lighter higher than K_v=0, 2 in deterministic model but have there is nearly the same sample mean.

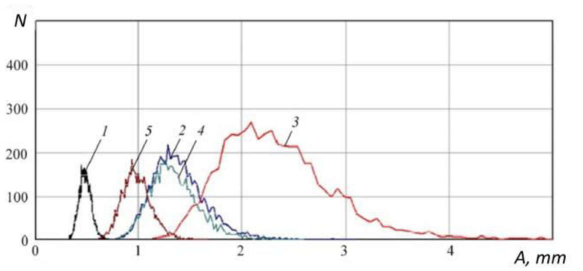

Figure-5. Rotor vibration amplitudes distributions for various rotary speeds: 1- 2000 RPM, 2 - 2500 RPM, 3 - 3000 RPM, 4 - 3500 RPM, 5 - 4000 RPM

It is typical for amplitudes distribution to have asymmetry with ratio A_s=1, 4. On Figure-6 this asymmetry is obvious particularly around critical rotor speed. Maximum random value of vibration amplitude was obtained at rotor speed of 3000 RPM and it is three times greater than average. Average value is very close to calculated by deterministic model.

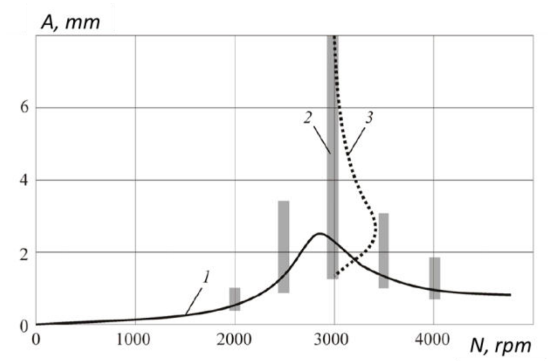

Figure-6. Amplitude-frequency characteristic (1) of rotor system with dispersion ranges at various rotor speeds (2) and distribution curve (3) of vibration amplitudes

Similar behavior was obtained for forces distributions: F_c, R_1 and R_2 (see Figure-7). Average values of forces established by statistical modeling are close to values calculated by deterministic model, but maximum value exceed then almost three times. Variation coefficient determined by forces vibrations is K_v=0, 19 and it is close to variation coefficient of logarithmic decrement of vibrations K_v=0, 2.

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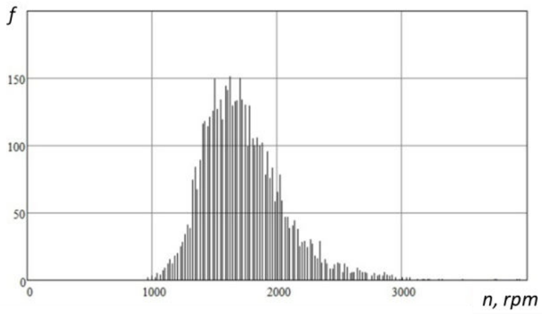
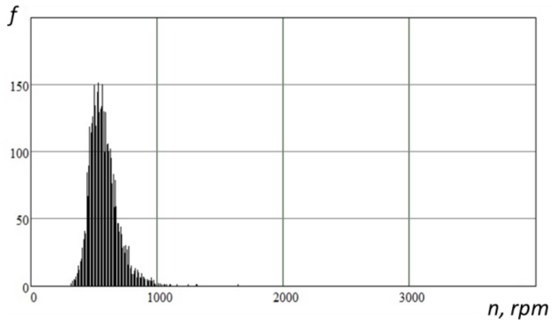
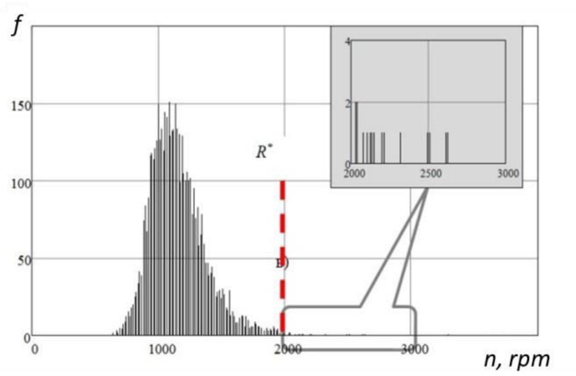


(a)



(b)



(c)

Figure-7. Forces distribution histograms F_c - (a), R_1 - (b), R_2 - (c) at rotor speed of 3000 RPM.

On the Figure-7 zoomed part of distribution with high values of random force R_1 is highlighted. It is seen that there is a probability of extreme magnitudes of force R_1 significantly exceeding average values.

Fault probability analysis for the component is performed based on estimation of case probability when its random parameter exceeds critical value. For the described system critical values and faults probabilities are presented in Table-4. Fault probability of the whole system is calculated by (5).

Analysis of the results presented in Table-4 shows that connection between shaft and disk is appeared to be less reliable.

Table-4. Fault probabilities for the rotor system components.

Case	Condition	Critical value, N	Probability
Support#1 fault	$(R_1>R^*)$	2000	0
Support#2 fault	$(R_2>R^*)$	2000	0,04
Shaft fault	$(F_c>F^*)$	2500	0,028
Rotor system fault	$(F_c \ge F^*)\vee$ $(R_1 \ge R^*)\vee$ $(R_2 \ge R^*)$		0,031

It is worth to notice that obtained fault probability q is a random value itself, because statistical modeling is performed on limited numbers of calculations N. Estimation equivalent to q=0, 031 described above is one of the calculation alternatives using N=5000. Ten additionally repeated calculations in accordance with the methodology gave average value of q=0,029 with variation coefficient of 0, 11. With increasing number of realizations N during statistical modeling probability estimations become more reliable.

CONCLUSIONS

Rotor system reliability characteristics estimation technique is developed. The technique considers impact of random factors determining rotors systems vibrations. The technique is based on fault tree analysis in combination with statistical modeling of components faults in vibration operational conditions.

Application of the methodology is presented on simple rotor system consisting of rotor with one disk supported by elastic supports. Histograms of distributions for the vibration amplitudes and dynamic loads responsible for system's components faults were obtained by statistical modeling. Average values of the distributions are close to values calculated by deterministic model and variation coefficients are close to variation coefficients of logarithmic decrement of vibrations. It is typical for rotor amplitudes and dynamic forces distributions to have considerable asymmetry. In described rotor system selected maximum force acting on a support exceeds average value almost three times.

Developed technique could be used as an engineering tool for vibration reliability analysis of machines consisted of more complex rotor systems like gas turbine engines.

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