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MARKOV CHAIN APPLICATION IN FATIGUE RELIABILITY ANALYSIS FOR DURABILITY ASSESSMENT OF A VEHICLE CRANKSHAFT

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ABSTRACT

This paper presents the durability assessment in terms of assessing the reliability analysis under random loading stress using the probabilistic approach of the stochastic process for structural health monitoring. The Markov process proposed in this study has the capability of generating synthetic loading stress data by embedding the actual maximum and minimum loading stresses. This is done by continuously updating the synthetic loading stress in order to generate the stress loading data history for each rotational speed. The purpose of this is to reduce the credible intervals between each data point for reliability analysis through the linear fatigue damage accumulation rule. The accuracy of the Markov process was validated through the finite element analysis and the accuracy and is statistically correlated between the actual and synthetic loading stress. The Markov process showed that the accuracy of the simulated fatigue life has an accuracy of 95% boundary condition when the actual and synthetic loading stress is statistically correlated using finite element analysis. Hence, it was able to provide a highly accurate assessment of durability for the improvement and control of risk factors for overcoming the extensive time and cost required in reliability testing.

Keywords: durability, markov, random, reliability, and stochastic.

INTRODUCTION

Durability assessment is becoming an important aspect in understanding fatigue reliability of mechanical structures and components especially in the automobile sector, Fatigue reliability is considered as a major aspect that analysis the fatigue life prediction data in order to understand structural health monitoring for structures and components. Hence it is an evaluation of fatigue failure in terms of forensic engineering. However, as fatigue tests require a lot of time and effort, much research has been focused on assessing the fatigue in terms of strain-life (ε-N) curve. Various experimental techniques including structural neural system have been proposed to monitor and detect the occurrence of cumulative damage in real time during the operation of a given structure [1]. This is due to the limitations of the strain gauge in terms of its sensitivity in capturing and handling real time data.

Various expert systems have been developed using artificial intelligence for the fatigue life prediction of metallic materials in structural health monitoring using their actual loading history data [2]. Hence, it is becoming increasingly important to develop systems that are capable of monitoring the durability mechanisms of components and structures under spectral loading by incorporating the probabilistic method of the stochastic process for appropriate data processing [3]. The random loading history data is considered stochastic in nature and is mainly influenced by the stress loading history acting on the component based on its geometric and material properties [4]. Hence, the non-deterministic method will provide a basis for the development of a safer and more reliable fatigue life prediction since disparity between the theory and practice of the deterministic method due to the large intervals and missing data between each data point during the experimental analysis [5].

Fatigue reliability assessment is a primary mode in assessing the fatigue life cycle of the automobile crankshaft under random stress loading. However, to model the fatigue reliability, an understanding of the physics and mechanism of the fatigue failure during its life cycle is important. The fatigue reliability using the Discrete Markov Chain can be used to assess the life cycle of the component in terms fatigue damage using cycle counting technique. The cycle counting technique is used to assess the fatigue reliability life by pairing the local minima and local maxima to obtain the equivalent stress load cycles. This equivalent stress load cycles are the rotating bending and axial loading are based on the multi-axial cyclic load along with the mean stress of the maximum stress that acts on the crankshaft during operating condition [6].

This purpose of this paper is to assess fatigue reliability of the automobile crankshaft by considering the effects of mean stress correction factor under random loading. The fatigue failure of the crankshaft is generally due to the service loading experienced by the component and it is categorized as non-deterministic failure. Hence, this vibration fatigue will lead towards a severe failure in the engine of the vehicle. Due to the limitations of the actual loading history, the fatigue reliability assessment has lower accuracy and efficiency in terms of fatigue life prediction. The stochastic process proposed in this study has the capability of generating the synthetic loading history data using the stochastic process by incorporating the actual sampling data for fatigue life prediction. As more data is generated within the given boundary condition with minimal intervals, the credible error in

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terms if accuracy would be reduced. More importantly, the stochastic process not only deals with fatigue life prediction using strain analysis but provides an accurate, efficient and fast fatigue reliability assessment in terms of structural health monitoring for the automobile industry.

METHODOLOGY

The framework of the present paper proposes the fatigue reliability assessment for the crankshaft during its life cycle using the cycle counting technique with the effects of mean stress correction factor under random loading. Figure-1 illustrates the operational loading stress condition of the crankshaft through a combination of the bending and torsional stresses, where in general the combination of the stresses will lead towards the failure of the crankshaft.

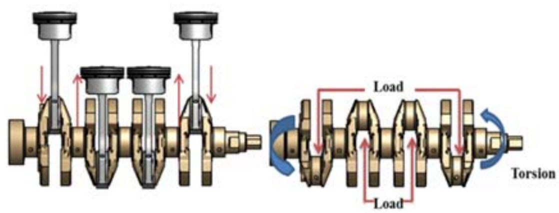

Figure-1. Operational condition of the crankshaft.

The Discrete Markov Chain is used to computationally model the failure probability criterion through the characterization of failure states [24] in the stress state condition of the component. This is because the Markov Chain has the capability of generating synthetic and near similar loading history data by incorporating the sampling maximum and minimum loading data [8]. Hence, as more data is generated, the credible intervals are reduced, making the durability assessment in fatigue damage more accurate through the use of the parametric distribution and correlation properties obtained from the linear cumulative damage rule by means of the local strain technique. The fatigue reliability life cycle assessment provides an important factor when durability is concerned. This is because the loading stresses, material fatigue properties and loading condition are directly associated to the life cycle of the component. Figure-2 illustrates the stages involved in the development of the mathematical model for the probabilistic computational mechanics schematic algorithm in assessing the fatigue reliability of the component. This is especially when there are constrains due to cost and lengthy duration of experimental setups.

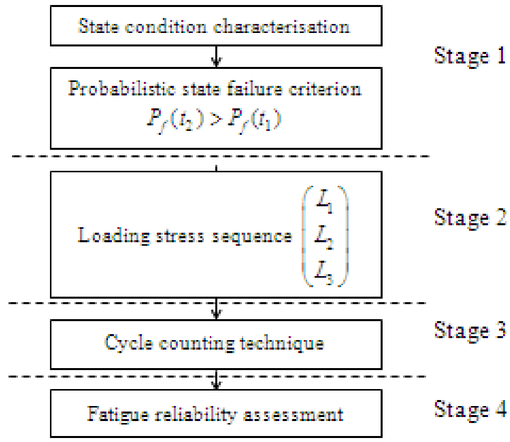

Figure-2. Development of the probabilistic computational mechanics schematic algorithm.

The following steps as shown in Figure 2 are used to in the development of the algorithm to assess the fatigue reliability of the crankshaft:

Stage 1: Model the failure modes that occur on the crankshaft during its life cycle subjected to the failure state condition of bending and torsion stresses through the Discrete Markov Chain state condition. This is done using the probabilities of failure state condition for each individual probability criterion based on the operating time transition. The mathematical model 1 of the Discrete Markov Chain predicts the condition of the future state, with the understanding that the current state condition is independent of the past state condition [7]. This is illustrated through Equation. (1):

$$P_r\{X_{t+1} \mid X_0 = B, X_1 = X_i = i_1, \ldots, X_n = i_n\} = P_r\{X_{t+1} = T, X_t = B\}$$
$$P_r\{X_1 = T \mid X_0 = B\} = P_r\{X_{t+1} = T \mid X_t = B\} \tag{1}$$

This is further expanded to represent the expressed in Equation. (2) over a given period of time when time $t \geq 0$ where i,j are non-negative integers.

$$P_r\{X_i = T|X_j\} = P_r\{X_1 = T|X_0 = B\} \times P_r\{X_2 = T|X_1 = B\} + \ldots$$
$$\ldots + P_r\{X_1 = B|X_0 = B\} \times P_r\{X_2 = T|X_1 = B\} + \ldots$$
$$= P_r(B,T)P_r(T,T) + P_r(B,T)P_r(T,T) \tag{2}$$

where probability P_r of X as the state condition of the Markov Chain for the transition phase of torsion, T and bending, B based on the operating condition of the component.

Stage 2: Simulate the actual maximum and minimum loading stress using the Discrete Markov Chain probabilistic model based on the probability failure criterion from the scalar matrix

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multiplication. The failure state condition for the crankshaft caused by bending and torsional stress loading is mathematically expressed through the probabilistic criterion from Equation. (3) in terms of the probability matrices:

$$P^n = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}^n \tag{3}$$

where P^n is transition of the probability matrices based on the operating time period of the crankshaft. The loading vector matrix L, is based on the actual maximum and minimum loading stress that that is incorporated into the chain as expressed in Equation. (4).

$$L = \begin{pmatrix} L_{max} \\ L_{min} \end{pmatrix} \tag{4}$$

where the loading matrix, L is based on the actual maximum, L_{max} and minimum, L_{min} loading data. This is used to synthetically generate the loading stresses every time the specific state condition is visited. The probability of failure of the crankshaft based on its fatigue reliability life cycle provides a finer discretization in time for the practical applications of the component by incorporating the actual loading data. Hence, the Discrete Markov Chain is expressed in a generalized term, by including the probability vector, μ in order to avoid confusion over the main failure state condition of the crankshaft.

$$E(X) = \begin{pmatrix} \mu_{11} & \mu_{12} \end{pmatrix} \times \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}^t \times \begin{pmatrix} L_{max} \\ L_{min} \end{pmatrix} \tag{5}$$

where the expected failure, E(X) is a scalar vector through the incorporation of the probability and loading matrices over a given period of time, t.

Stage 3: Use the scalar matrix multiplications by transforming the time series sequence to cycle counting method to calculate the life cycle of the crankshaft as illustrated in Figure-3.

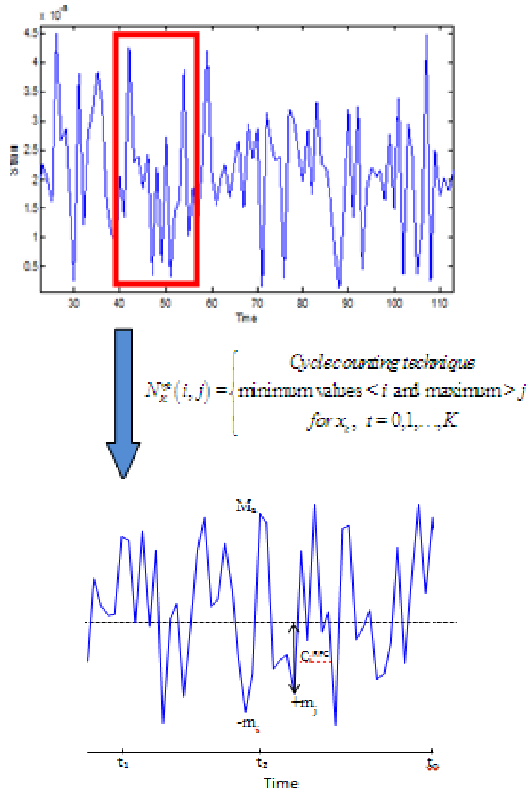

Figure-3. Cycle counting technique of pairing the local minima and maxima based on the upwards or downwards.

Stage 4: Compute the fatigue reliability by considering the effects of mean stress correction factor and statistically validate the appropriate and suitable model that will be used for assessing the life cycle of the crankshaft. The accuracy of the damage assessment for the ductile cast iron Grade ASTM 100-70-03 [8] is calculated using the mean stress correction factor for the monotonic and cyclic properties of the material for the crankshaft as shown in Equation. (6).

$$\varepsilon_{CM} = \frac{\sigma'_f}{E}\left(\frac{2}{D}\right)^b + \varepsilon'_f\left(\frac{2}{D}\right)^c \tag{6}$$

where b is the fatigue strength exponent; c is the fatigue ductility exponent; σ'_f is the fatigue strength coefficient; ε'_f is the fatigue ductility coefficient and N_f is the cycle life. Likewise, with the presence of of a nonzero normal stress influences the fatigue behaviour of materials, where the mean stress levels are relatively low compared to the cyclic yield stress. The mean stresses can significantly increase or decrease the life of a component or structure which includes the crack initiation as well as in crack propagation in fatigue loading. Therefore, the Morrow and Smith-Watson-Topper models as shwon in Equation. (7) and (8) should be consided for assessing the

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fatigue reliability of he component. Both this model resides on the parameters for multiaxial loading that based on the total fatigue strain amplitude and the tensile stress during a loading cycle.

$$\varepsilon_{morrow} = \frac{\sigma'_f - \sigma_m}{E}\left(\frac{2}{D}\right)^b + \varepsilon'_f\left(\frac{2}{D}\right)^c \quad (7)$$

$$\sigma_{max}\varepsilon_{SWT} = \frac{\sigma'^2_f}{E}\left(\frac{2}{D}\right)^{2b} + \sigma'_f\varepsilon'_f\left(\frac{2}{D}\right)^{b+c} \quad (8)$$

where b is the fatigue strength exponent; c is the fatigue ductility exponent; σ'_f is the fatigue strength coefficient; ε'_f is the fatigue ductility coefficient and N_f is the cycle life. The actual loading stress data will be compared against the numerically generated variable amplitude loading data obtained from the Discrete Markov Chain model.

The fatigue reliability assessment for the crankshaft shaft is based on the durability assessment which was accounted for through the reliability function, hazard rate function and mean time to failure based on the parametric properties of the stress loading. The parametric properties were based gamma function, shape and scale parameter of the captured data {β, θ, γ and Γ}. The estimated shape parameter is used to evaluate the reliability and hazard rate based on the monotonic failure properties. Likewise, reliability and hazard rate functions were rewritten in the following forms, where hazard or failure rate (λ) was used to illustrate the failure of either being a decreasing or an increasing failure rate through the properties of the shape parameter. The mean time to failure (MTTF) for fatigue reliability assessment is through the mean cycle to failure. This is for the durability assessment of the component in terms of structural health monitoring which can be illustrated through Equation. (9), (10) and (11).

$$R(t:\beta,\theta,\gamma) = \exp\left[-\left(\frac{t-\gamma}{\theta}\right)^{\beta}\right] \quad (9)$$

$$\lambda(t:\beta,\theta,\gamma) = \left(\frac{\beta}{\theta}\right)\left(\frac{t-\gamma}{\theta}\right)^{\beta-1} \quad (10)$$

$$MTTF = \theta\Gamma(1-\frac{1}{\beta}) \quad (11)$$

where β, θ, γ, R, λ and Γ are the shape, scale, location parameters, reliability function, hazard rate function and gamma function for the given random stress loading data, respectively.

RESULTS AND DISCUSSIONS

ANALYSIS OF SYNTHETIC STRESS LOADING

The stochastic process of Markov Chain based on Equation. (4) was used to generate synthetic loading history for assessing the fatigue reliability life cycle based on the actual loading stress data. This is because the Markov process provides the capability to generate and update the loading stress history data by replicating the actual loading data with minimal intervals. The predicted maximum loading stress was compared against the actual data using the boundary condition of 90% and correlation line. The boundary condition indicated that all the generated loading stresses were within the given boundary condition with a scatter close to the diagonal line with an accuracy of more than 95%. Based on the results shown in Figure-4 and 5, it clearly indicates that the Markov process is capable in generating synthetic maximum loading stress data with high accurate that can be used in fatigue reliability life cycle assessment [9].

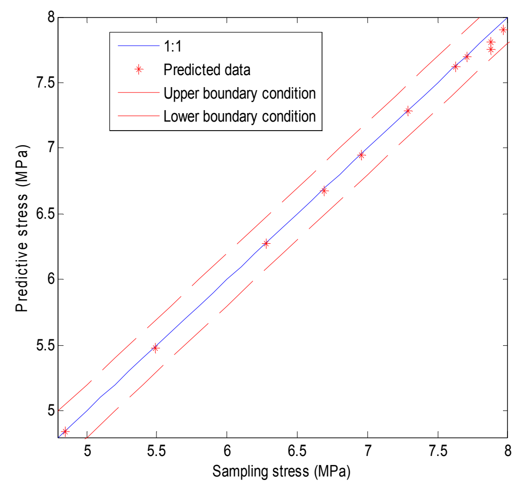

Figure-4. Statistical correlation with 90% confidence interval for synthetic maximum stress load.

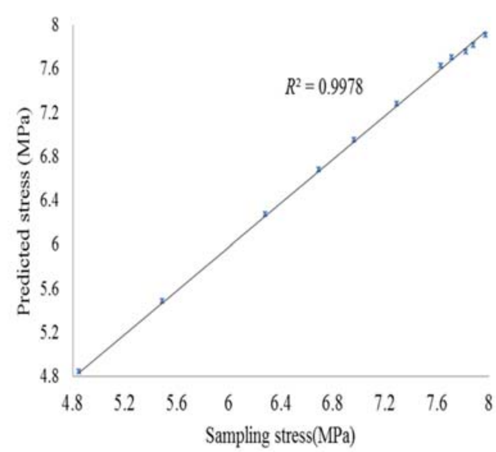

Figure-5. Statistical accuracy of the synthetic maximum stress load.

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FATIGUE LIFE ASSESSMENT

The test data for the fatigue life prediction using the local strain analysis from the synthetic loading stress history was calculated using the mean stress correction factors for every rotation per minute. This includes considering the presence of the of zero and non-zero normal stresses. The life cycle was obtained from $N = 1 / D$, where D is the damage per cycle to the crankshaft by the rotation, as shown in Table-1. The life cycle of the crankshaft was in the range of 1.23×10^2 to 1.42×10^4 as shown in Table-1. The presence of non-zero normal stress (whether positive or negative) will influence the fatigue behaviour of materials. The presence of mean stress can affect the fatigue life prediction as shown in Figure-6. Consequently, the highest cycle will is directly proportional towards critical location for fatigue life assessment of the crankshaft which illustrated in Figure-7.

Table-1. Fatigue life cycle based on mean stress correction factor.

Rotational speed (rpm)	Coffin-Manson (/cycle)	Morrow (/cycle)	SWT (/cycle)
1000	4.36×10^3	1.42×10^4	1.16×10^4
2000	6.10×10^2	1.87×10^3	1.46×10^3
3000	3.11×10^2	1.09×10^3	8.58×10^2
4000	1.76×10^2	6.42×10^2	4.02×10^2

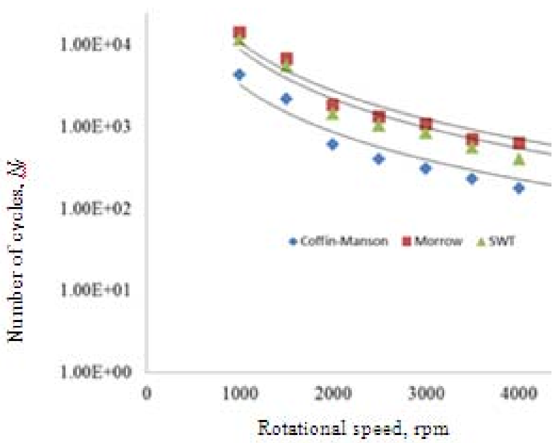

Figure-6. Fatigue life cycle for the 3 strain life models.

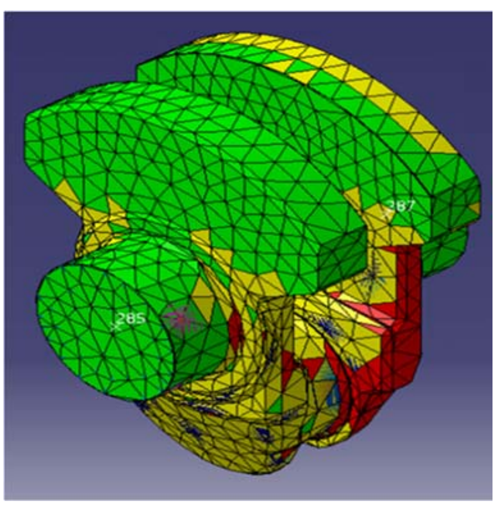

Figure-7. Critical location of stress based on the simulated stress loading history at 3000 rpm.

FATIGUE LIFE ASSESSMENT

The predicted reliability function is statistically correlated using the 90% boundary condition to observe the trend and accuracy towards the actual sampling data, as shown in Figure-8. It was observed that the prediction reliability agrees well towards the sampling data with an average error of 6% with all the points were within the boundary condition of 90% for reliability of the fatigue life prediction. The statistical confidence level assessment is constructed to verify the simulation model towards the reliability prediction of the crankshaft in order to observe the presence of outliers.

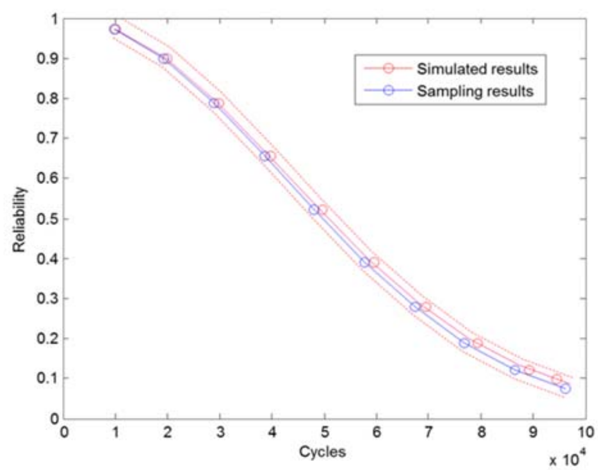

Figure-8. The predicted reliability against the sampling data at 3000 rpm.

The hazard rate is used to evaluate the fatigue failure of the component prior to the occurrence of any hazardous failure. From Figure-9, it is observed that the hazard rate had an incremental failure with a variance of 6.2% for both the synthetic and actual loading stress for fatigue life data. The statistical confidence level

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assessment of 90% was constructed to verify the simulation model with regard to the reliability and hazard rate assessment of the crankshaft. Hence, the capability to assess the hazard rate of the fatigue life data with high accuracy is used in assessing the crankshaft in terms of structural health monitoring.

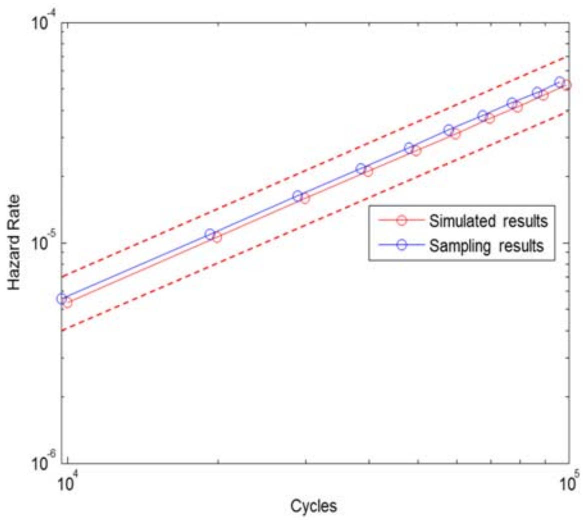

Figure-9. The predicted hazard rate against the sampling data at 3000 rpm.

With both the reliability and hazard rate, the improvement of the design life of the component can be achieved by controlling the risk factors in terms of preventive maintenance, operational maintenance and determining better safety limits.

CONCLUSIONS

This paper presented the fatigue reliability life cycle assessment using the probabilistic Markov Chain method to generate the synthetic loading stress history data by incorporating the actual sampling data for the automobile crankshaft. The fatigue reliability was predicted by considering the geometric structure, material properties and random loading on the component using this embedded Markov process. The stochastic approach manages to reduce the intervals between the loading data points, where the generated loading stresses were within the given boundary condition with a scatter close to the diagonal line with an accuracy of more than 90%. Therefore the overall methodology features in the fatigue reliability life cycle assessment were as follows: (1) The fatigue life prediction of the mean stress correlation factor was based on the existing fatigue data of the crankshaft material using the linear fatigue damage rule; and, (2) The fatigue reliability life cycle assessment was based on finite element modelling by considering the geometrical structure, material properties and random loading stresses.

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