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NUMERICAL SOLVING OPTIMAL CONTROL PROBLEMS BY THE METHOD OF VARIATIONS

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ABSTRACT

In the article based on the method of variations in the space of controls the algorithm is developed and program was implemented to determine the optimal control problems with free right end. As an illustration method, presents the results of numerical solution of the three examples with constraints on the control and phase variables. The advantage of this algorithm is the lack of requirements for the selection of the initial approximation control parameter and phase variables. The algorithm has good convergence and can be used to solve a large class of applications in various branches of the economy. By using the developed algorithm determined the optimum trajectory and the numerical values of the control parameter for the test problems. A comparative analysis of the results of the numerical solution of the examples for different values of initial approximation control and precision.

Keywords: method of variations, optimal control, phase variables.

1. INTRODUCTION

Methods of the optimum control theory are intensively used in various application areas. Control theory is application-oriented mathematics that deals with the basic principles underlying the analysis and design of (control) systems. Systems can be engineering systems, economic systems, biological systems and so on. To control means that one has to influence the behaviour of the system in a desirable way: for example, in the case of an air conditioner, the aim is to control the temperature of a room and maintain it at a desired level, while in the case of an aircraft, we wish to control its altitude at each point of time so that it follows a desired trajectory. As a result, more and more people will benefit greatly by learning to solve the optimal control problems numerically.

This work is devoted to an actual problem - development of efficient and universal algorithms of numerical problem solving of optimal control.

2. PROBLEM STATEMENT

Consider the following optimal control

$$\text{minnimize } I(u) = G(x(t)) \tag{1}$$

Subject to

$$\frac{dx_i}{dt} = f_i(t, x(t), u(t)), t \in [t_0, T], x(0) = x_0, \tag{2}$$

$$\phi(u) \le 0 \tag{3}$$

Where $u(t) \in R$ is the function characterizing the operating influence, $x(t) \in R^n$ is function describing a condition of process and t is time.

Let's consider various algorithms for problem solving of optimal control.

2.1 Performance criterion

A performance criterion (also called cost functional or simply cost) must be specified for evaluating the performance of a system quantitatively. By analogy to the problems of the calculus of variations, the cost functional $I : U[t_0, t_1] \to R$ may be defined in the so-called Lagrange form:

$$I(u) = \int_{t_0}^{t_1} f^0(t, x(t), u(t))dt \,. \tag{4}$$

3. THE ALGORITHM OF THE METHOD OF VARIATIONS

The algorithm consists of 9 steps:

1. Guess an initial approximation of control U_0.
2. Break interval $[t_0, t_k]$ to n parts, constituting a uniform system of units.
3. Select starting node t_0, which will be a variation of controls.
4. Compute $U(t_0) \pm \delta U$.
5. Compute $x(t)$, $u(t)$ by solving (3).
6. Calculate $I(u)$ according to (4).
7. Go to t_1 and go to step 4 for all remaining points t_i.
8. Determine the minimum value of the criterion calculated for all points t_i and define a new control U_1 corresponds to the lowest value criterion.
9. Set $\delta U = \frac{\delta U}{2}$. Then, with the control U_1, go to step 3 until will not find variation in which the performance criterion will not be improved.

4. DISCUSSIONS

The software for the numerical calculations presented below in this article was developed in Borland

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Delphi environment. For each of the following cases, we will compute the Euclidean norm of the solution error:

$$\varepsilon_{x_1} = \sqrt{\sum_i \left(x_{1i} - x_1^*(t_i)\right)^2},\ \varepsilon_{x_2} = \sqrt{\sum_i \left(x_{2i} - x_2^*(t_i)\right)^2},$$

$$\varepsilon_u = \sqrt{\sum_i \left(u_i - u^*(t_i)\right)^2}.$$

Example-1: Consider the following optimal control system:

$$\dot{x} = u(t),\ x(0) = 0,\ x(1) = 0.5,\ \ x \in R, u \in R, t \in [0,1]. \qquad (5)$$

The performance measure is:

$$I = \int_0^1 u^2(t) + x^2(t)dt \to \min. \qquad (6)$$

The optimal control problem is to find a control law $u^*(\cdot)$ which minimizes cost functional (6).

The analytical solution of this problem is presented in [2].

Figure-1 show the comparison between numerical solution and approximate solution for $u_0 = 0.2$. Table-1 presents simulation results for different initial guess and accuracy of this problem.

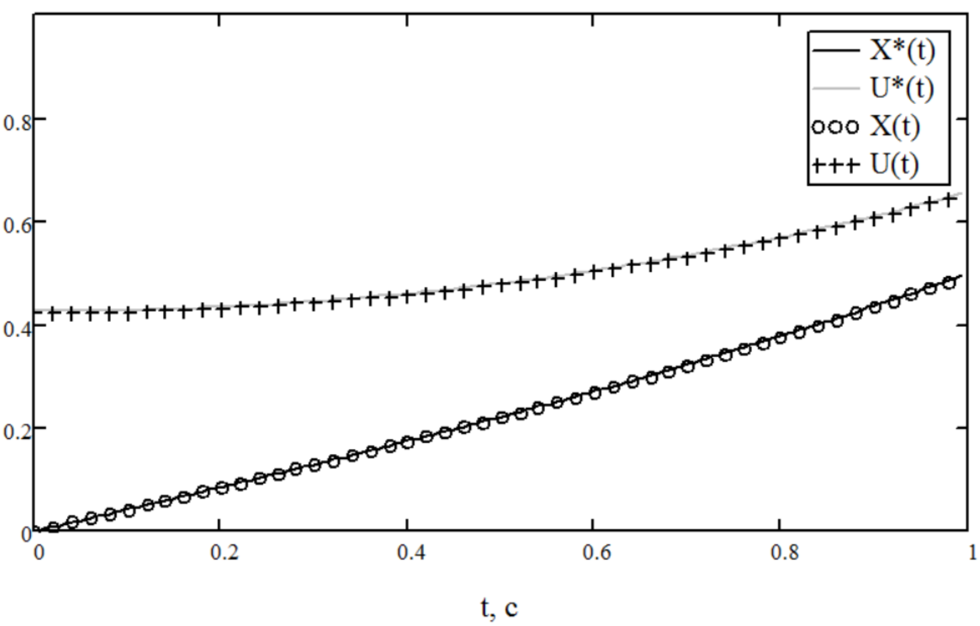

Figure-1. Comparison between numerical solution and approximate solution, Example 1.

Table-1. Simulation results for different initial guess and accuracy, Example 1.

№.	***u₀***	**Accuracy**	**Elapsed time, s.**	ε_u	ε_x
1	0	0,1	0,36	1,42	0,93
2	0	0,01	0,74	0,99	0,04
3	0	0,001	1,22	0,083	0,006
4	-0,6	0,001	1,42	0,004	0,002
5	-0,9	0,0001	4,75	0,0004	0,0005
6	0,1	0,00001	6,43	0,00008	0,00013

Example-2: Consider the following optimal control system:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = u^2(t) - 0.5 \cdot u(t); \end{cases} \qquad (7)$$
$$x_1(0) = 0,\ x_2(0) = 0,\ 0 \le t \le 1,\ 0 \le u \le 1.$$

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The performance measure is:

$$I(x_1, x_2) = x_1(1) \to \max. \quad (8)$$

The optimal control problem is to find a control law $u^*(\cdot)$ which minimizes cost functional (8).

The analytical solution of this problem is presented in [3].

Figure-2 shows the comparison between numerical solution and approximate solution for $u_0 = 0.6$. Table-2 presents simulation results for different initial guess and accuracy of this problem.

At the same time the estimated value of the control parameter in the range $0 \le t \le 1$ has a constant value equal to 1.

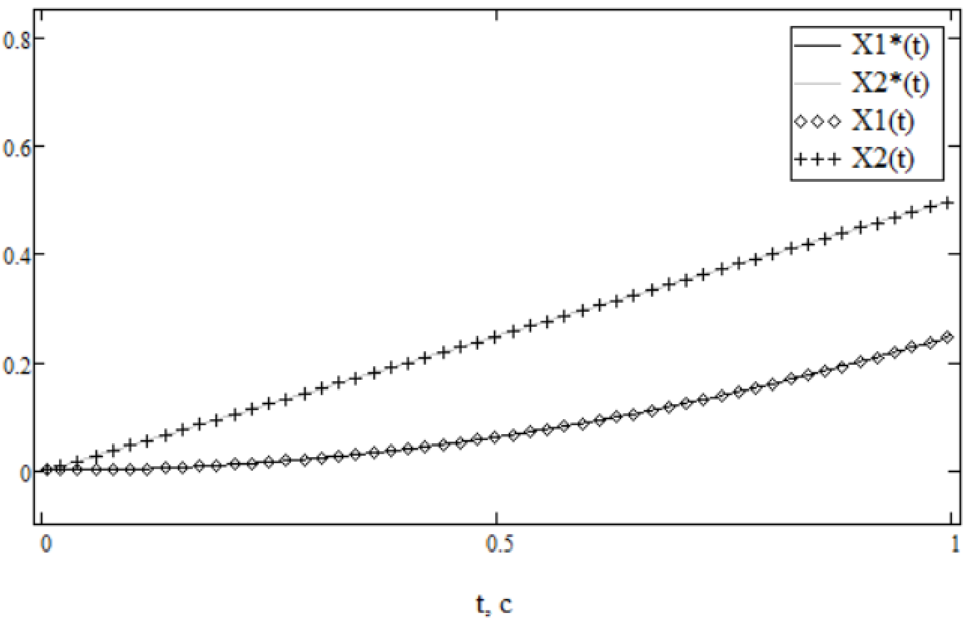

Figure-2. The suboptimal states.

Table-2. Simulation results for different initial guess and accuracy, Example 2.

№.	**u0**	**Accuracy**	**Elapsed time, s.**	ε_u	ε_{x_1}	ε_{x_2}
1	0,6	0,1	2,32	1,06	1,105	0,285
2	0,6	0,01	6,43	1,009	0,04	0,108
3	0,6	0,001	9,45	1	0,001	0,018
4	0,8	0,001	11,58	1	0,003	0,009
5	0,8	0,0001	18,23	1	0,0033	0,0091
6	0,6	0,00001	23,85	1	0,00002	0,00007

Example-3: Consider the following optimal control system:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -x_1(t) + u(t); \end{cases} \quad (9)$$

$x_1(0) = 0$, $x_2(0) = 0$, $0 \le t \le 2\pi$, $|u| \le 1$.

The performance measure is:

$$I(x_1, x_2) = x_2(2\pi) \to \min. \quad (10)$$

The optimal control problem is to find a control law $u^*(\cdot)$ which minimizes cost functional (10).

The exact solution are: $u^*(t) = \begin{cases} 1, t \le 0.5, \\ 0, 0.5 < t < 2, \\ -1, 2 \le t \le 2.5. \end{cases}$

Figure-3, Figure-4 shows the comparison between numerical solution and approximate solution for $u_0 = 0.1$. Table 3 presents simulation results for different initial guess and accuracy of this problem.

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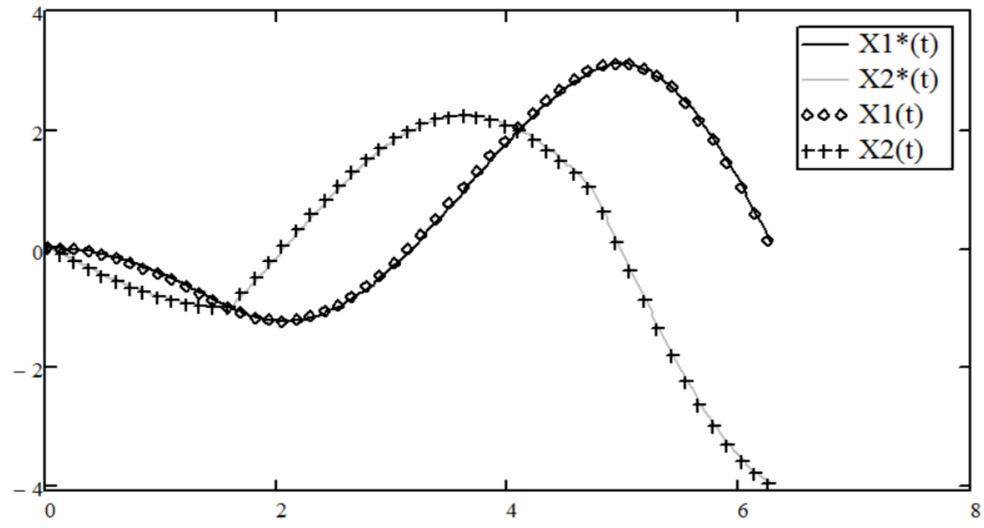

Figure-3. The suboptimal states.

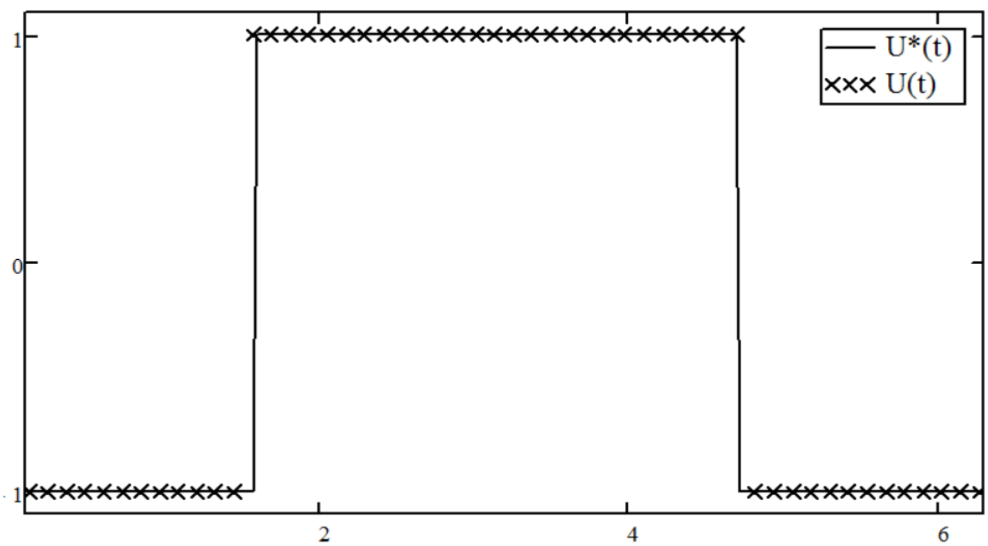

Figure-4. The suboptimal control.

Table-3. Simulation results for different initial guess and accuracy, Example 3.

№.	u_0	Accuracy	Elapsed time, s.	ε_u	ε_{x_1}	ε_{x_2}
1	0	0,1	2,06	3,06	1,11	1,21
2	0	0,01	2,85	2,99	0,14	0,15
3	0	0,001	4,12	2,987	0,018	0,019
4	-0,6	0,001	3,94	2,854	0,019	0,016
5	-0,9	0,0001	12,06	2,0024	0,1086	0,1089
6	0,1	0,00001	23,35	2,0023	0,1093	0,1088

5. CONCLUSIONS

For many optimal control problems, the method of variations is the best option we have. The advantage of this algorithm is that it does not have requirements with initial guess. The algorithm has good convergence and can be used to solve a large class of applications in various fields of national economy.

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