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TEXTILE MATERIAL HUMIDIFICATION PATTERNS

Aleksandr I. Eremkin[1], Svetlana V. Bakanova[1], Aleksandr N. Koshev[1], Yuri V. Rodionov[1] and Alexey P. Levtsev[2]
[1]Penza State University of Architecture and Construction, Penza, Russian Federation
[2]National Research Mordovia, State University of Saransk, Saransk, Russian Federation
E-Mail: svbakanova@mail.ru

ABSTRACT

Based on the presentation of a compactly formed textile material as a volume-porous medium with averaged physicochemical and technological parameters, a mathematical model has been proposed for conducting theoretical and numerical studies of textile material humidification patterns. The mathematical model represents a boundary value problem of mathematical physics, the solution of which enables us to calculate relative distribution of conditioned air flow humidity in the porous medium depth. Methods of solving the problem have been discussed. Some results of numerical studies have been provided.

Keywords: textile material, humid air, porous medium, thermal diffusion, conditioned air flow.

INTRODUCTION

In the course of the technological process in yarn production, it is required to ensure the required humidity of fibers at different stages of its manufacture. One of the form of the humidified material is represented by spools with tightly wound yarn semi-products and threads (Figure-1).

We suggest considering the processed amounts as a single porous and pseudo-homogenous medium, where the solid component is represented by humidified threads, whereas the pores are filled with humid air. Thus, each elementary volume of such medium includes both solid and gaseous phases. The properties of such pseudo-homogeneous medium are effective characteristics distributed by volume, such as porosity, thermal conductivity, mass conductivity, adsorption and diffusion characteristics, etc [1, 2].

The main factors of humid air penetration into the inner, hard-to-reach areas of the porous material are forced air flow and diffusion mechanism. For this purpose, the rate of the microscopic reaction of thread surface humidification shall be recorded in accordance with the theory and practice of the material's moisture content dependence on the relative air humidity and thermal conductivity of the medium. For our representation of the reaction area as a pseudo-homogenous medium, an important factor is the high rate of the humidification reaction, as compared with the rate of the humid air flow and that of the moisture diffusion inside the medium[3,4,5] .

In accordance with the above, we will consider the entire porous medium as homogenous, at each point of which a kinetic reaction of humidification occurs [6].

The temperature of the porous material internal area is determined by thermal diffusion and, obviously, it should be taken into account in case of significant temperature changes in the humid air being supplied to the processed area. If this does not occur, we can neglect the temperature changes outside and inside the medium.

The validity and applicability of such reaction medium model representation is limited and may only be used to describe the macro-kinetic patterns of physicochemical processes. In this case, important is theoretical and numerical estimation of the medium's elementary volume dimensions. On the one hand, the elementary volume should be small enough to ensure accuracy of the numerical calculations in mathematical modeling of the considered processes; on the other hand, the elementary volume, as taken at any point of such porous medium, must include both solid and gaseous components. For this reason, the theoretic and numerical estimates of elementary calculation steps during numerical implementation of mathematical models and methods should correlate with physical properties of the materials that form the pseudo-homogenous medium [7, 8].

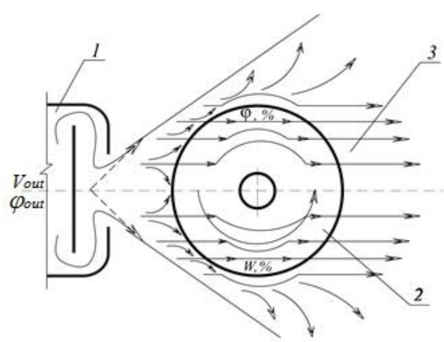

Figure-1. Scheme of forced air flow supply to the spool: 1- air diffuser; 2- rove; 3- the estimated scheme of air velocity diagram.

MATERIALS AND METHODS

Taking into account the above suggestions, based on the law of conservation of matter, at each point of the medium in question, humidity change φ over time t satisfies the equation:

$$\frac{\partial \varphi}{\partial t} = -\text{div}\left(j_{forced} + j_{diff} \right) + j_{source} \quad (1)$$

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where $div = (\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})$ is the vector divergence operator;

j_{forced} is the forced air flow due to the availability of the initial air supply rate into the porous medium; j_{diff} is the flow due to moisture diffusion; j_{source} is a negative source due to moisture loss from the supplied humid air by absorption of moisture at each point of the pseudo-homogenous medium volume.

Let us consider the equation for each of the flows separately.

For the forced air flow, we have

$$j_{\text{forced}} = \varphi \cdot \vec{w} \quad (2)$$

where $\vec{w} = (w_1, w_2, w_3)$ is the velocity vector of the conditioned air flow movement through the porous medium.

For the moisture diffusion flow inside the medium, there is the following known expression [9,10]:

$$j_{\text{diff}} = -D \cdot grad(\varphi) \quad (3)$$

where D is the averaged effective diffusion factor.

As we are considering a porous medium pseudo-homogenous model, the value D is, certainly, a certain operational parameters allowing for efficiently describing the moisture diffusion process in a porous medium, rather than a true diffusion coefficient.

The density of the source of moisture adsorption from the humidified air:

$$j_{sourse} = k \cdot F_{sp} \cdot f(\varphi)$$

where k is the moisture adsorption rate constant per unit area; F_{sp} is the specific surface area of the porous medium volume, m^2/m^3.

Obviously, the form of the function $f(\varphi)$ depends on the material humidification mechanism. There are numerous models describing this mechanism, for example [11 ,12, 13, 14], that, however, describe mainly particular physicochemical situations and are rather unsuitable to be used in a general, averaged case. In this paper, we suggest an original approach to solving the problem, based on which we obtained the general form of the function $f(\varphi)$, taking into account physical patterns of the adsorption process and allowing for finding an analytical equation for the source function in each technological case, in accordance with the experimentally determined dependence.

RESULTS AND DISCUSSIONS

Based on the conducted theoretical studies, we have developed a mathematical description of the dependence of the equilibrium moisture content of fibers $W_p(x)$ on the relative air humidity $\varphi(x)$ at a constant temperature of t_a, that corresponds to the experimentally obtained sorption isotherms. The mathematical modelling results $W_p = f(\varphi)_t$ are presented as an analytical functions that are characterized by a set of parameters, the numerical values of which are determined by the mechanism of a particular moisture absorption (Figure-2).

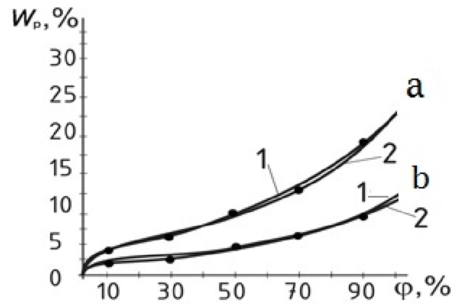

Figure-2. Graphs of experimental (1) and theoretical (2) dependences of moisture content $W_p(x)$ on the relative air humidity $\varphi(x)$: a - viscose; b - nylon.

For mathematical description of the experimental dependences, we have identified two sections of each curve: with $0 < \varphi(x) < \varphi_c$, a section with a convexity upwards from the axis W_p, and with $\varphi < \varphi(x) < 100\%$, a straight section passing to the section with a convexity downwards from the axis φ. Here, φ_c is a certain averaged parameter having a corresponding value for each type of textile material. On the both identified sections, we determined analytical dependences obtained on the grounds of the following considerations.

At the first section of the adsorption curve, with $0 < \varphi(x) < \varphi_c$, it has been considered that the increase rate of the adsorption moisture volume is proportional to its value at the given air humidity φ, as well as to the difference $W_m - W_p$, where W_m is the constant that characterizes the limit moisture content in the porous material. As a result, we obtained the equation $dW_p / d\varphi = k \cdot W_p \cdot (W_m - W_p)$, where k is the proportionality factor. The solution of this equation depends on the constant W_H, that characterizes the initial equilibrium moisture content of fibers W_p:

$$W_p = W_n / [1 + (W_n / W_m - 1)e^{-k\varphi}] \quad (4)$$

In particular, it follows from equation (4) that the maximum rate of change W_p shall be observed at $\varphi = (1/W_m) \cdot \ln(W_m / W_n - 1)$.

At the second section, with $\varphi \geq \varphi_c$, a multi-molecular and capillary mechanism of moisture saturation of the capillary-porous medium is activated. In such case,

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the increase of W_p with the growth of φ is exponential in nature:

$$W_\partial = k_2 \cdot e^{k_1(\varphi - \varphi_c)} \quad (5)$$

where k_1, k_2 and φ_c are some constants, the values of which may be calculated using the methods of analytical processing of experimental data.

Summing up the cases (4) and (5), as considered above, we obtain a general equation of the dependence of W_p on φ:

$$W_\partial = k_2 \cdot e^{k_1(\varphi-\varphi_c)} + W_n / \left[(1 + W_n / W_m - 1)e^{-k\varphi}\right] \quad (6)$$

The data provided in Fig. 2 and other calculated data lead to the conclusion that the mathematical dependence (6) reflects quite accurately the equilibrium process of interaction between the textile fibers and the conditioned air with the maximum deviation of 5–9 % from the experimental curve.

Let us return to the equation (1). We have:

$$\frac{d\varphi}{dt} = -div\left(j_{forced}\right) - div\left(j_{diff}\right) + j_{source}$$

or

$$\frac{d\varphi}{dt} = -div\,(\varphi \cdot \vec{w}) + D \cdot \left(\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2} + \frac{d^2\varphi}{dz^2}\right) - k \cdot F_{sp} \cdot f(\varphi) \quad (7)$$

Thus, the equation (7) will be adopted as a basic model for calculating the humidifying air distribution within the volume of the porous medium in question.

The general scheme of the air supply process for textile yarn humidification is shown in Figure-1. Based on this scheme, as a first approximation, let us consider the problem of humid air penetration into the medium depth as a one-dimensional one, i.e. let us consider the mechanism of moisture penetration and the pseudo-homogenous medium humidification along the spool radius in the direction of the medium's center. Such simplification does now allow for accurately describing the material humidification process at each point of the spool, but it makes it possible to analyze theoretically and numerically the process patterns and to assess its effectiveness.

Within the framework of such assumptions, the multidimensional equation (7) shall be simplified to a one-dimensional one by the spatial coordinates:

$$\frac{d\varphi}{dt} = D\frac{d^2\varphi}{dx^2} - \frac{d(\varphi, w)}{dx} - F_{sp} \cdot f(\varphi). \quad (8)$$

The equation (8) may be solved using finite difference methods, if we know the form and the tabulated values of the function $f(\varphi)$, as well as the values of the averaged diffusion factor D, the flow rate w, and the specific surface of the medium $F_{sp.}$

For numerical solution of the equation (8), it is necessary to additionally define the initial and boundary conditions. At the initial time, the humidity in any point of the pseudo-homogenous medium in question may be considered as identical and equal to the air humidity in the production room:

$$\varphi(0, x) = \varphi_0$$

At the point, x=0 i.e. at the boundary of the porous medium, the humidity is also equal to that of the air supplied to the technological area – φ_0, i.e.

$$\varphi(t, 0) = \varphi_0 \quad (9)$$

Let us derive the condition for the derivative $\frac{d\varphi}{dx}$ at the point $x = 0$.

Let us identify the elementary volume in the porous medium (PM) with a unit area and a small thickness V_m.

Let us consider the changes in the moisture content of the air during its passage through the porous medium with a given volume.

The relative air humidity in this volume – $\varphi(x)$ will vary as it passes through the medium, namely it will decrease due to moisture deposition in the pores of the medium.

Let us adopt φ_0 as the initial relative air humidity at the entrance to the PM.

After passing into the medium depth by a distance Δx, the air humidity will change to φ_Δ due to moisture sorption in the medium's volume. Then,

$$\left(\varphi_0 - \varphi_\Delta\right) = \frac{Q_p^0 - Q_p^\Delta}{Q_{max}} \quad (10)$$

where Q_p^0 is the amount of water vapor in a unit volume of humid air before humidification of the volume V_m of the porous medium, Q_p^Δ is the amount of water vapor in a unit volume of humid air after humidification of the volume V_m of the porous medium, Q_{max} is the maximum amount of water vapor in a unit volume of humid air.

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Then, the value $Q_{max}\left(\varphi_0-\varphi_\Delta\right)\cdot V_m/V_{unit}$ is equal to the amount A of moisture adsorbed in the PM pores with the volume V_m (V_{unit} is a unit volume).

At each point of the porous medium, the moisture deposits according to the adsorption law, i.e. the adsorption curve that depends on the physicochemical properties of the medium:

$$W(x)=f(\varphi(x)) \tag{11}$$

where $W(x)$ is the specific moisture content in fibers at the point x.

Then, the adsorbed moisture amount in the volume V_m of the PM is equal to

$$A=\rho\, S_m\int_0^{\Delta x}\left(f\left(\varphi(x)\right)-W_0\right)\cdot dx \tag{12}$$

where S_m is the area occupied by the PM solid phase, ρ is the porous material density, W_0 is the specific moisture content in the material before humidification.
Therefore, we obtain:

$$Q_{max}\left(\varphi_0-\varphi_\Delta\right)\cdot V_m/V_{unit}=\rho$$
$$S_m\int_0^{\Delta x}\left(f\left(\varphi(x)\right)-W_0\right)\cdot dx \tag{13}$$

Let us divide the both parts of this equation by Δx and pass to the limit with $\Delta x\to 0$. Thus, it is logical to assume that, after the passage to the limit, the area V_m shall represent a portion of the unit area S_m, that corresponds to the surface area of pores, i.e. $\varepsilon\cdot S_{unit}$, where ε is the porosity ratio of the PM. Correspondingly, S_m is a portion of the unit area occupied by the medium's solid phase, i.e. $S_m=(1-\varepsilon)\cdot S_{unit}$.

Consequently, the condition for the derivative function $\varphi(x)$ at the point x=0:

$$\frac{d\varphi}{dx}(t,0)=-\frac{(1-\varepsilon)}{\varepsilon}\cdot\frac{\rho}{Q_{max}}\left(f\left(\varphi(0)\right)\ -W_0\right) \tag{14}$$

Besides, we can confidently assert that at a point that is remote from the border, in the volume of the porous medium, the air humidity in pores is also a certain constant value. Therefore, at some distance δ from the border, we have:

$$\left.\frac{d\varphi(t,x)}{dx}\right|_{x\geq\delta}=0\ .$$

This condition satisfies the assumption that, at a sufficient distance from the porous medium's border, the humidity of the air and that of the medium material may be adopted as equal and shall be used by us to control the numeric solution.

The problem (8), (9), (14) represents a two-point boundary problem for an ordinary differential equation.

It should be noted that the mathematical model of the material humidification process (8)-(14) is an approximation to the full mathematical description of the humidification process that occurs in the volume of a porous medium that, however, subject to the right choice of process parameters, allows us to calculate with sufficient precision the moisture distribution in the pores of the system under consideration.

The boundary "two-point" problem (8)-(14) represents an ordinary second-order differential equation with a "strongly non-linear" right part and boundary conditions defined in the form of the initial and final values of the desired function and its derivative.

Before describing the method of solving the problem (8)-(14), let us present some arguments about the correctness of the formulation, in particular, the stability of solution. It is obvious that if the problem (8)-(14) does not belong to the category of stable differential equations, then, to perform calculations, it is necessary to use the co-called "hard" methods of differential equation numerical solution, whereas the use of common methods should be accompanied by a constant control over the solution. This is necessary due to the fact that, for classically unstable systems of differential equations, minor changes in the initial conditions for the desired function may result in significant errors in the system solution at the subsequent, and especially, final sections of the independent variable changes, until the complete corruption of the solution.

Let us show the possible instability of the problem (5)-(7). For this purpose, let us introduce a new unknown function:

$$\psi(x)=\frac{d\varphi}{dx} \tag{15}$$

and transform the second-order differential equation (8) into a system of two first-order differential equations relative to two unknown functions $\varphi(x)$ and its derivative $\psi(x)$:

$$\begin{cases} D\dfrac{d\psi}{dx}+w\dfrac{d\varphi}{dx}=F_{sp}\cdot f(\varphi) \\ \dfrac{d\varphi}{dx}=\psi \end{cases} \tag{16}$$

To study the stability of the system (16), let us simplify it, assuming that the conditioned air penetration into the porous medium due to forced air flow is

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insignificant, i.e. the value w is close to 0. For more convenience, let us define $\psi = Z_1, \varphi = Z_2$, then, the system (9) may be written as follows:

$$\begin{cases} D\dfrac{dZ_1}{dx} = F_{sp} \cdot f(Z_2) = f_1(Z_1, Z_2) \\ \dfrac{dZ_2}{dx} = Z_1 = f_2(Z_1, Z_2) \end{cases} \tag{17}$$

The system of differential equations (14) is referred to as an autonomous one and may be easily examined to assess its stability. To do this, it is enough to compose a characteristic equation relative to a certain formal unknown value λ [15]:

$$\det\left[\frac{df_i}{dx_k}(0) - \lambda \cdot \delta_k^i\right], \text{ where } \delta_k^i = \begin{cases} 1\, if\ i = k \\ 0\, if\ i \neq k \end{cases} \quad - \tag{18}$$

the Kronecker delta

and to check it for the availability of roots with a positive real part. If any, the system (17) is instable relative to the equilibrium point $Z_i(0)$. In our case, the characteristic equation has the following form:

$$\lambda^2 - F_{sp}\frac{df}{d\varphi} = 0 \tag{19}$$

Since the function $f(\varphi)$, represents the adsorption curve, then, due to the monotonic increase with the growth of φ, its derivative is positive within the entire change interval of φ and, consequently, one of the roots of the equation (19) is positive, which indicates a possible failure of stability when solving the system of equations (17), and hence a more complex system (16).

Despite the possible stability failure of the system (17), numerical calculations showed that to solve the system of equations (16), the Runge-Kutta finite difference method for solving systems of differential equations with automatic selection of the system integration step proved to be quite suitable.

Figure-3 shows the curves of changes in the relative air humidity in the porous medium depth for various values of the process parameters: D, m^2/s; w, m/s; l, m; F_{pore}, m^2/m^3 (specific surface of the pores).

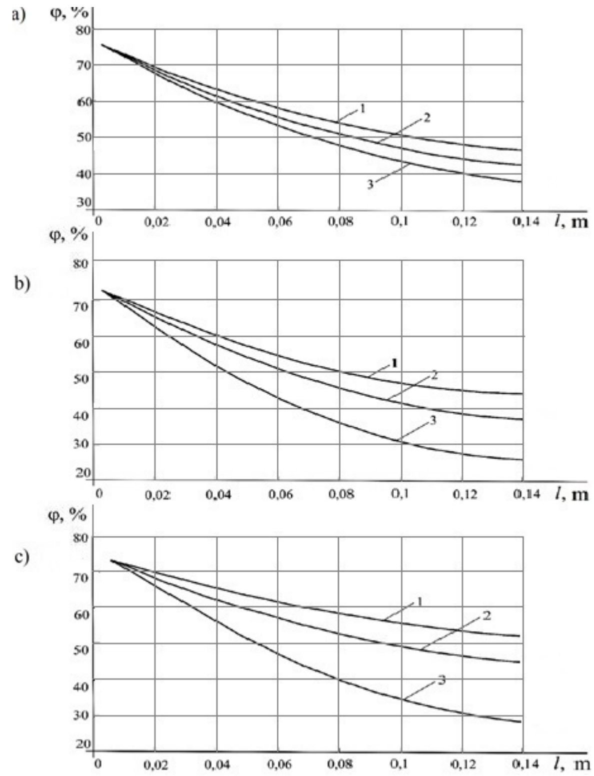

Figure-3. Distribution of relative humidity of the conditioned air (φ, %) within the depth (l, m) of a spool with a rove:
a - with values F_{pore}: 3,000 - the curve (1); 3,500 - (2); 4,000 – (3); $w = 0{,}24$ $D = 0{,}15$; b - with values w: 0,3 - the curve (1); 0,2 - (2); 0,1 - (3); $D = 0{,}15$; $F_{pore} = 3{,}500$; c - with values D: 0,05 - the curve (1); 0,15 - (2); 0, 45 – (3); $w = 0{,}2$; $F_{pore} = 3{,}500$

The analysis of the calculations leads to the conclusion that the value φ decreases monotonically as the distance from the surface increases, up to a certain limit value. In this case, as follows from Fig. 3, (a), big values of the specific surface area of a medium consisting of capillary porous textile material correspond to a more intensive decrease in the relative humidity of the conditioned air φ in a moving air flow, as the distance from the the porous medium's outer boundary increases.

A more intensive decrease of φ occurs also with decreasing flow rate, as well as with a lower diffusion factor.

We have found the influence of the effective moisture diffusion rate D on the porous medium humidification process. When calculating D, we used a correction factor that depends on the air flow rate. A good agreement between the results of calculations and experiments was ensured by the formula $D = D_{source} \cdot q \cdot w^p$. In our case, the obtained values are $p = 0{,}3$; $q = 71$.

We conducted a numerical study of the air flow rate influence on the relative air humidity distribution inside a compactly formed textile semi-product, consideration being given to the influence of the air flow rate on the source function, i.e. $W = f(\varphi, w)$:

$$W_p = K_m\left(k_2 \cdot e^{k_1(\varphi-\varphi_c)} + W_n / \left(1 + \left(W_n / W_m - 1\right)e^{-k\varphi}\right)\right) \quad (20)$$

where $K_m = 0{,}0001 \cdot w^{-0{,}7}$.

For the numerical solution of the problem, we need to know the dependence of the rate value of the air flow penetrating to the medium's depth from the coordinate along the spool thickness. The best approximation, consistent with the experimental data, proved to be the exponential dependence of the humid air flow rate on the coordinate along the porous medium's depth:

$w(x) = \gamma \cdot \exp(-\mu \cdot x)$, where $\mu = w(0) = w_0$, the value γ regulates the decrease rate of $w(x)$. From the results of the numeric calculations, as shown in Fig. 3, it follows that, with increasing conditioned air flow rate, the moisture penetration depth into the porous medium's volume increases.

CONCLUSIONS

Therefore, in this publication, based on the representation of a compactly formed textile material as a pseudo-homogenous medium, that is characterized by averaged physical and technological parameters, as well as based on physical and mathematical descriptions of processes in the volume of capillary porous colloid medium when humidified using a forced conditioned air flow, mathematical models have been built, and boundary value problems have been formulated. They allow for calculating the conditioned air relative humidity distribution within the volume of the porous medium under consideration. The methods for solving the problems have been discussed. Calculations have been provided for different values of technological and physical parameters, and their elementary analysis has been performed.

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