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EFFECT OF SINUSOIDALLY HEATING ON MIXED CONVECTION IN SQUARE CAVITY FILLED WITH A POROUS MEDIUM

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ABSTRACT

Unsteady mixed convection flows in a lid-driven square cavity filled with a fluid-saturated porous medium is investigated numerically. The two vertical walls of the enclosure are insulated while the bottom wall is cooled. The top wall is heated sinusoidally and moving at a constant speed. The governing equations are solved numerically by using finite volume method with SIMPLE algorithm. The results with different parameters of Darcy and Richardson numbers are obtained and shown graphically. The presence of porous medium has influence to the movement of heat transfer and fluid flow inside the cavity. While, the effect of sinusoidal heating causes the flow patterns of streamline and isotherm increase and look strength.

Keywords: mixed convection, porous medium, sinusoidal temperature, lid-driven.

INTRODUCTION

Mixed convection in a square lid-driven has taken a great interest and important in a varieties applications. Meanwhile, heat and fluid flow studies in porous medium has received considerable attention in recent years. Those applications include nuclear reactor, lubrication technology, geothermal energy systems and many others. Much research has been studied about mixed convection in cavity enclosure but less study considered with porous medium.

Stochastic Mixed convection fluid flow and heat transfer in cavities have been studied extensively in the literature. [1]considered the effects of the Prandtl number on the flow and heat transfer in a square lid-driven cavity and solved numerically by using the control volume approach with the power-law and SIMPLER scheme. Later on, [2] investigated numerically mixed convection heat transfer in a driven cavity with a stable vertical temperature gradient. The results on an effect of the Richardson number have been presented. The problem of unsteady mixed convection in a square cavity in the presence of internal heat generation or absorption and a magnetic field is formulated by [3]. The governing equations are solved by the finite volume method approach along with the alternating direct implicit (ADI) procedure. While, the finite volume method by the SIMPLE algorithm on mixed convection in lid-driven cavity has been applied by [4] *et al*. which considered the inclined lid-driven cavity, [5] and [6] discusses the lid-driven square cavity filled with nanofluids. [7] investigated numerically by a second-order accurate finite-volume method on mixed convection flow in a lid-driven inclined square enclosure filled with a nanofluid. Recently [8] studied on the effects of magnetic field on mixed convection in a lid-driven square cavity filled with nanofluids.

Mixed convection in enclosures cavity filled with porous medium has received attention in recent years. [9] was the first investigated the problem of porous medium on mixed convection flow in a lid-driven enclosure with taking into the presence of internal heat generation. They solved numerically using the finite-volume approach with ADI procedure and found that the heat transfer mechanisms and the flow characteristics inside the cavity are strongly dependent on the Richardson number. The study has been extended by [10] on double-diffusive mixed convection heat and mass transport in a lid-driven square enclosure filled with a non-Darcian fluid-saturated porous medium.

The work by [9] has been extended by [10] studied on the laminar transport processes in a lid-driven square cavity filled with a water-saturated porous medium. Later on, [11] investigated on mixed convection flow in a vented enclosure with an isothermal vertical wall and filled with a fluid-saturated porous medium. [12] studied on laminar mixed convection in a parallel two-sided lid-driven differentially heated square cavity filled with a fluid-saturated porous medium by employing the finite volume approach with third order accurate upwind scheme. While, [13] considered mixed convection flows in a lid-driven square cavity filled with porous medium and solved numerically using penalty finite element analysis. The flow simulation and mixed convection in a lid-driven square cavity with saturated porous media has been investigated by [14]. Recently [15] investigated the mixed convection flow in a lid-driven square cavity filled with a porous medium under the effect of a magnetic field. From the previous studies, it is found that the presences of a porous medium within the cavity cause a force opposite to the flow direction which tends to resist the flow. Thus, the increase in the permeability of porous medium causes the fluid flow move fast.

There are a large number of studies on mixed convection in cavities enclosure with non-uniformly temperature distribution on their walls. [13] presented an analysis of mixed convection flows within a square cavity with uniform and non-uniform heating of bottom wall. In both uniform and non-uniform cases results show that the effect of heating is more pronounced at the bottom and left walls as the formation of thermal boundary layers is

restricted near the bottom and left wall. [17] performed a numerical study on mixed convection in a lid-driven cavity enclosure having vertical sidewalls with sinusoidal temperature. It is observed that the heat transfer increases as the non-uniform heating increases thus, the heat transfer rate for non-uniform heating of both walls is higher than non-uniform heating of one wall. Numerical study of mixed convection flow of nanofluid in a lid-driven square cavity with sinusoidal heating on sidewalls has been reported by [18]. While, [19] investigated on steady state two-dimensional mixed convection in a lid-driven square cavity filled with nanofluid with the presence of heat generation. It is found that a sinusoidal type of the local heat transfer rate produced for the non-uniform heating. The non-uniform heating condition attains maximum heat transfer rates at the center of the bottom wall than with uniform heating condition for all Richardson number. Later, [20] analyzed on laminar mixed convection of non-Newtonian nanofluids in a square lid-driven cavity with a sinusoidal heating at the right sidewall.

Heat and mass transfer in lid-driven cavity filled with porous medium have received less attention in the literature. So, the present investigation is extended from [19] work which considered the effect of porous medium in the fluid flow, and heat and mass transfer on mixed convection in square lid-driven cavity. An attention given in the present study on the effect of sinusoidal heating on the lid-driven while, the bottom wall kept cold and both vertical sides of the cavity kept insulated.

The problem will be solved numerically by using finite volume method with SIMPLE algorithm. The present study want to investigate the effect of sinusoidal heating on the lid-driven to movement of heat transfer and fluid flow inside the cavity with the presence of porous medium. The effect of Richardson number on the flow will be analyzed. The results will plotted inside the cavity.

FORMULATION OF PROBLEM

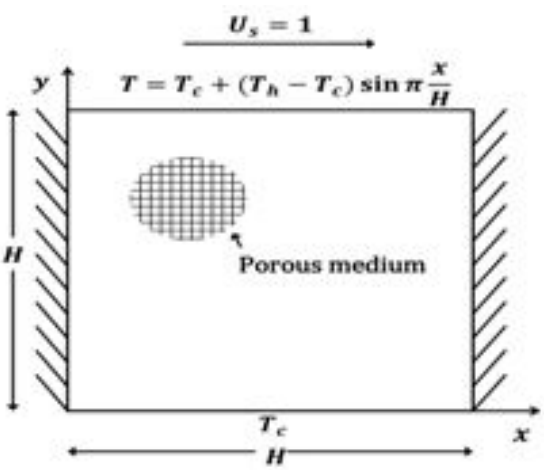

Figure-1. Schematic diagram of the physical model

The physical model of a two-dimensional square porous cavity of length and height H is shown in Figure-1. The side walls of the cavity are kept insulated while the bottom wall is maintained at cold temperature, T_c. Meanwhile, the top lid is moving from left to right at a uniform speed, U_s with a sinusoidal temperature, $T = T_c + (T_h - T_c)\sin(\pi x/H)$ where$T_h > T_c$.

The flow is assumed to be laminar and the binary fluid is assumed to be Newtonian and incompressible. Here, the dimensionless variables as used by [19] are introduced

$$X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad t = \frac{\tau U_s}{H}, \quad \theta = \frac{T - T_c}{T_h - T_c},$$
$$U = \frac{u}{U_s}, \quad V = \frac{v}{U_s}, \quad P = \frac{p}{\rho U_s^2}, \quad Re = \frac{U_s H}{\nu},$$
$$Da = \frac{\kappa}{H^2}, \quad Gr = g\beta\frac{T_h - T_c}{\nu^2}H^3, \quad Pr = \frac{\nu}{\alpha},$$
$$\nu = \frac{\mu}{\rho}, \quad \alpha = \frac{K}{\rho C_\rho}, \tag{1}$$

where the u and v are the velocity components in the x and y directions, respectively. t is the time, τ is the dimensionless time and θ is the dimensionless of temperature. The gravitational acceleration g is acting downward and β is the coefficient of thermal expansion, K is the thermal conductivity, ν is the reference kinematic viscosity, κ is the permeability of the porous medium, ρ is the fluid density and C_ρ is the specific heat of constant pressure, μ is the fluid density and P is the fluid pressure. Gr, Re, Da, Pr are the Grashof number, the Reynolds number, the Darcy number and the Prandtl number, respectively.

The full governing equations are formulated based on the laws of mass, linear momentum and thermal energy by considered unsteady problems as used [19] were transformed into nondimensional form by using the nondimensional variables from equation (1). The resulting nondimensional form of the governing equations are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{U}{DaRe}, \tag{3}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Gr}{Re^2}\theta - \frac{V}{DaRe}, \tag{4}$$

$$\frac{\partial \theta}{\partial t} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{PrRe}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right). \tag{5}$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$U = V = \theta = 0, \quad \text{at } t = 0,$$
$$U = V = 0, \partial\theta/\partial Y = 0 \quad \text{at } X = 0,$$

$U = V = 0\,, \partial\theta/\partial Y = 0$ at $X = 1$,
$U = V = 0\,, \theta = 0$ at $Y = 0$,
$U = V = 0\,, \theta = \sin \pi X$ at $Y = 1$. (6)

The average Nusselt number, $\overline{Nu}$ at the top wall is calculated by integrating the local Nusselt number, Nu_xalong the top wall and could be written in dimensionless forms:

$$\overline{Nu} = \frac{1}{1 \times H} \int_0^H Nu_x \, dX \,, \tag{7}$$

where, the local Nusselt number is defined as

$$Nu_x = -\left.\frac{\partial\theta}{\partial Y}\right|_{Y=1}. \tag{8}$$

The stream function is calculated from its definition

$$U = \frac{\partial\psi}{\partial Y}, \qquad V = -\frac{\partial\psi}{\partial X}. \tag{9}$$

NUMERICAL PROCEDURE

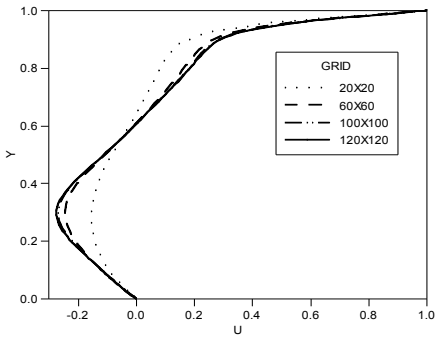

Figure-2. Velocity profiles for various mesh sizes at Da = 0.1.

The governing equations (2)-(5) subject to the boundary conditions (6) are solved numerically by using finite volume method. The SIMPLE algorithm is used to solve the couple system of the governing equations with approach the power law scheme as discussed by [21], and a program code in Fortran is developed. In order to check the grid independency of the solution, the numerical experiment has been conducted for different grid resolutions. The grid independence test is performed using successively sized uniform grids, 20×20, 60×60, 100×100 and 120×120 with the values of $Re = 400$, $Gr = 100$, $Da = 0.1$ and $Pr = 0.71$ as demonstrated in Figure-2. It's showed that the line overlap with each other for 100×100 and 120×120 thus, a grid point 100×100 is fine enough to obtain accurate result and was chosen for all the computations in the present study.

Table-1. Comparisons of the maximum and minimum values of the horizontal and vertical velocities at the mid-sections of the cavity between the present solution and previous works.

	$Re = 400$		
	(Iwatsu *et al.*, 1993)	**(Khanafer and Chamkha, 1999)**	**Present**
U_{min}	-0.3197	-0.3099	-0.3023
U_{max}	1.0000	1.0000	1.0000
V_{min}	-0.4459	-0.4363	-0.4219
V_{max}	0.2955	0.2866	0.2802
	$Re = 100$		
U_{min}	-0.2122	-0.2037	-0.2049
U_{max}	1.0000	1.0000	1.0000
V_{min}	-0.2506	-0.2448	-0.2328
V_{max}	0.1765	0.1699	0.1673

In order to validate of the present numerical procedure, the present horizontal and vertical velocity profiles was performed to compared with the work by (Iwatsu *et al.*, 1993) and (Khanafer and Chamkha, 1999) in the absence of porous medium for both $Re = 100$ and $Re = 400$ as shown in Table 1. From the Table 1, it's showed the minimum and maximum values of the velocity profiles results are in good agreement with the two previous results.

RESULTS AND DISCUSSIONS

The numerical results for the effect of sinusoidal heating on mixed convection lid-driven flow and heat transfer filled with porous medium are discussed. The non-dimensional governing parameters for this investigation are $Ri = Gr/Re^2$, Re and Da. The Ri is to signifying the relative dominance of buoyancy to forced convection and the Da that inversely accounting for the intensity of porous medium. The Pr used in this investigation is $Pr = 0.71$ while the Ri is considered to vary from 10^{-4} , 6.25×10^{-4}, 10^{-2} with fixed the $Gr = 10^2$ and the

corresponding Re is varied. Four different values on non-dimensional Da have been considered as 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}. The characteristic of flow, streamline distribution and isotherms for the effect of Ri and Da has been demonstrated in graph within the enclosure.

Figures 3(a) and 3(b) represent the streamline and temperature in contours within the square enclosure respectively, at $Ri = 10^{-4}$ and $Re = 10^{3}$ for different Da. Figure 3(a) shows that the strength of circulations for $Da = 10^{-4}$ is seen to very weak as the maximum value of streamline of circulation is found to be 0.0064 where the permeability of the medium at this Da approaches to zero. This causes the flow prevented from flowing in the bulk of cavity. As the Da increases to $Da = 10^{-3}$ followed by $Da = 10^{-2}$, the circulations increases with maximum value of streamline is 0.0183 and 0.0527, respectively. The boundary layers are formed on the right side at top wall due to drag force created by the movement of the upper lid. When the Da increases to 10^{-1}, the circulations is seen to be very strength with the maximum value streamline 0.0827. Its indicates that as Da increase, the fluid flow become fast due to the increase in permeability of porous medium.

Besides that, the effect of Da on isotherm can be observed in Figure 3(b). The isotherm plots at $Da = 10^{-4}$ show the temperature lines are almost parallel to the horizontal walls and pushed towards upper wall which indicate that a quasi-conduction regime is reached. As Da increases at $Da = 10^{-3}$, the isotherms attempt to circulate through the center of the cavity and occupie most of the cavity. The circulation does appear inside the cavity when $Da = 10^{-2}$ with the maximum values0.4071. The thermal boundary layers are formed on the right and left sides on the top wall. The stronger circulation and also the formation of thermal boundary layers can be seen clearly as $Da = 10^{-1}$ where the maximum values are 0.4215. Therefore, as Da increases, the isotherms are significant changes in the stagnant bulk of interior region that shows the overall of heat is transferred by conduction in the middle and bottom part of the cavity.

The effect of the variation of Ri by using different Re with fixed the $Gr = 100$ on the streamlines and isotherms are illustrated in Figures 3-5 for $Ri < 1$ and Figure 6 for $Ri \geq 1$ at $Da = 0.1$. With the decrease of Re where the Ri is increases, not much of change is observed in streamlines however, the maximum value is decreases. Otherwise, the isotherms show a significant change due to the buoyancy parameter which is Richardson number for $Ri < 1$. At large Re causes the rotating cell near the upper moving wall becomes larger due to the mechanical effect of the top moving wall and the forced convection becomes dominant over natural convection. Moreover, for $Ri \geq 1$, the Reynolds number at small values thus, the strength of circulation in streamlines for $Ri = 1$, and $Ri = 10$ is seen to be weak with the maximum values decreases to $0.0688$and 0.0685, respectively. While, The isotherms lie symmetrically along the mid-vertical of cavity and pulled towards the middle of bottom wall. The isotherms at the bottom part of the cavity are almost to parallel to the horizontal wall showing that the natural convection is the dominant heat transfer. The effect of sinusoidal heating could be seen as in Figure-3-6. At large Da for $Ri = 10^{-4}, 6.25 \times 10^{-4}$ and 10^{-2}, the strength of circulations of streamline and isotherm and boundary layer clearly seen with occupies the region of the cavity. The streamline and isotherms in this study increases compared to (Khanafer and Chamkha, 1999) work which considered uniform heating on the lid-driven. Other than that, the vortex shapes were formed in the present streamline and the flow patterns look strength than the previous study.

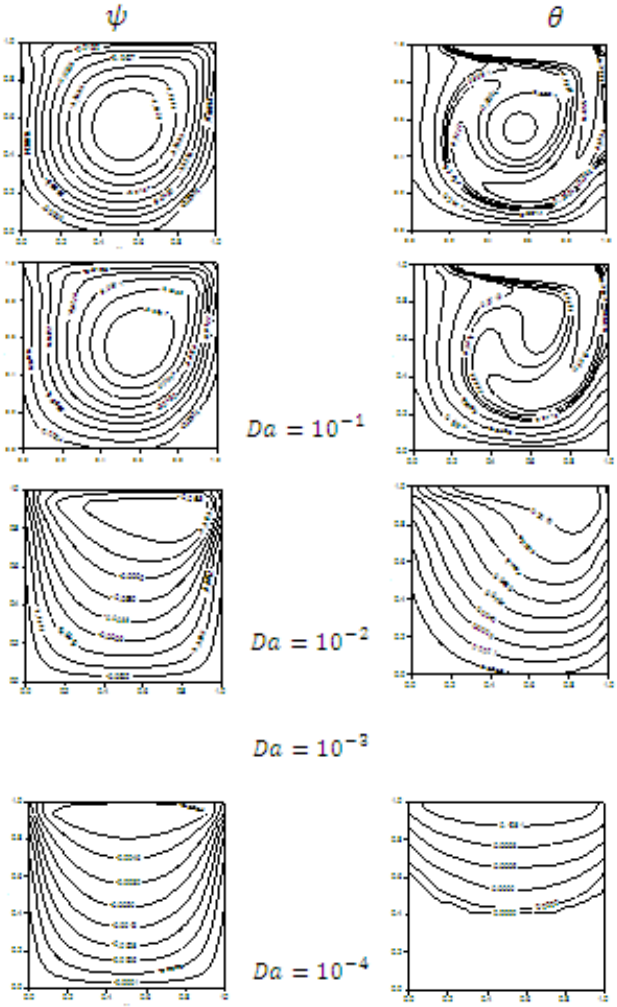

Figure-3. Streamlines and isotherms for various Darcy number at $Ri = 10^{-4}$, and $Re = 10^{3}$.

To vary the Richardson number by fixed the Reynolds number $Re = 10^{3}$, the corresponding of Gr are $10^{2}, 10^{6}, 10^{7}$ hence, the effect of Gr results areillustrated in Figure-7. The increase in Gr causes the formation of the circulation in streamlines. At $Gr = 10^{6}$, a secondary of circulation is appear near the left side of the bottom wall. While, at large Gr, where $Gr = 10^{7}$ the circulations split into four part indicating the buoyancy effect is dominant thus, the convection dominated the flow of heat transfer. The pattern lines in isotherms are found to be weak as Gr increases. The circulation cell attempts to disappear at

$Gr = 10^6$ and become nearly parallel to the horizontal walls showing natural convection dominant mode. Besides that, the maximum value in the isotherms illustrating the top region is well mixed and at lower temperature.

Figure-8 represents the effect of Da on the V-velocity distributions at mid-section of vertical walls along the X-direction with $Re = 1000$ and $Gr = 100$ for different Darcy numbers. As Da increases, the velocity increases due to the increase in permeability of porous medium causes the velocity of fluid flow increases. At large Darcy number, the presence of a porous medium makes a strong opposite to the flow direction that tends to resist the flow. The effect of Ri on the local heat transfer or the local Nusselt number at $Da = 10^{-2}$ and $Gr = 10^2$ has been shown in Figure-9. It is observed that as the Ri increases, the local Nusselt number decreases and the magnitude depends on the temperature of the top lid which is heating sinusoidal. At small Ri, the local Nusselt number is oscillated very high showing that the convection is dominant and at large Ri, the opposite situation is observed.

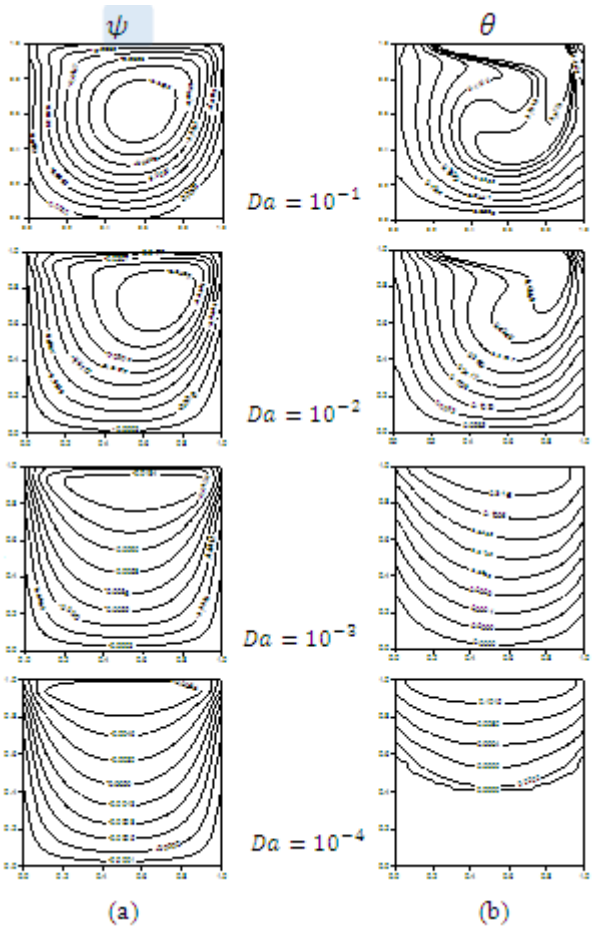

Figure-4. Streamlines and isotherms for various Darcy number at $Ri = 6.25 \times 10^{-4}$and $Re = 4 \times 10^2$.

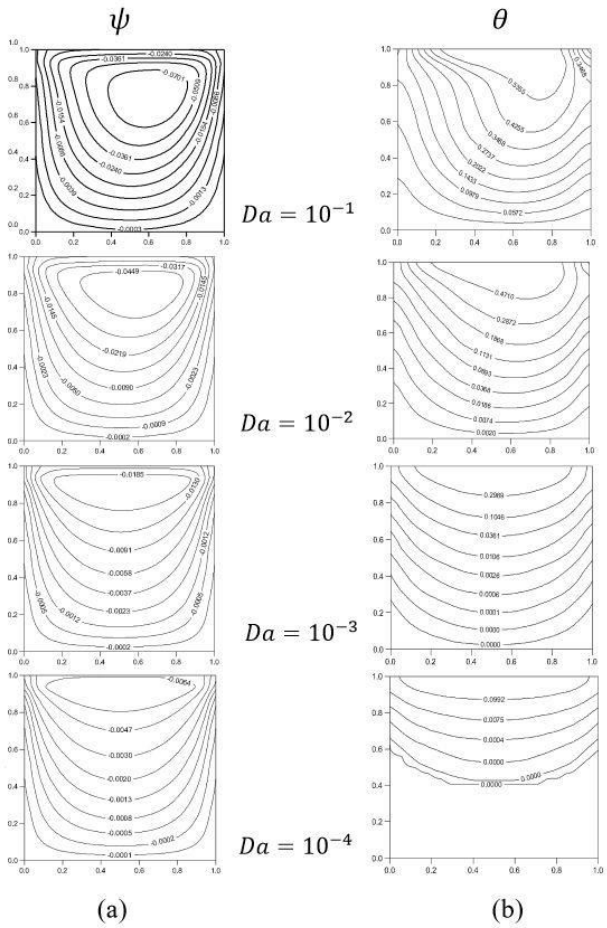

Figure-5. Streamlines and isotherms for various Darcy number at Ri = 10^{-2} and Re = 10^2.

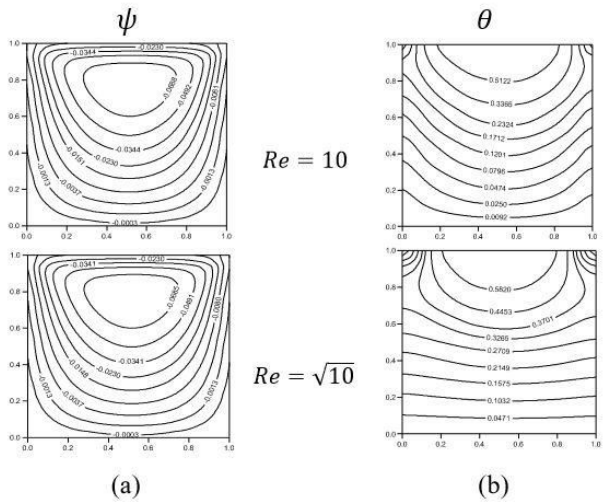

Figure-6. Streamlines and isotherms for various Richardson number Ri = 1 and Ri = 10 at Da = 10^{-1} and Gr = 10^2.

CONCLUSIONS

The study on unsteady mixed convection heat transfer in two-dimensional square enclosure lid-driven filled with porous medium where the top lid is sinusoidally

heating was solved numerically by finite volume method with employed the SIMPLE algorithm. Comparisons with previously published work on special cases of the problem are in good agreement. The effect of the Darcy number at different Richardson number on both stream function and isotherms were presented and discussed. It was found that the presence of a porous medium has significant influence to the heat transfer and flow characteristic inside the cavity. Other than that, the effect of Richardson number by increasing of Reynolds number or decreasing Grashof number causes the streamlines and isotherms increase. The local Nusselt number distribution along the top wall is decrease as the Richardson number increase. With the increase in Richardson number, the local Nusselt number signifies the conduction is dominant. Furthermore, the effect of non-uniformly or sinusoidal heating to the flow inside the cavity is increases and look strength compared to uniform heating.

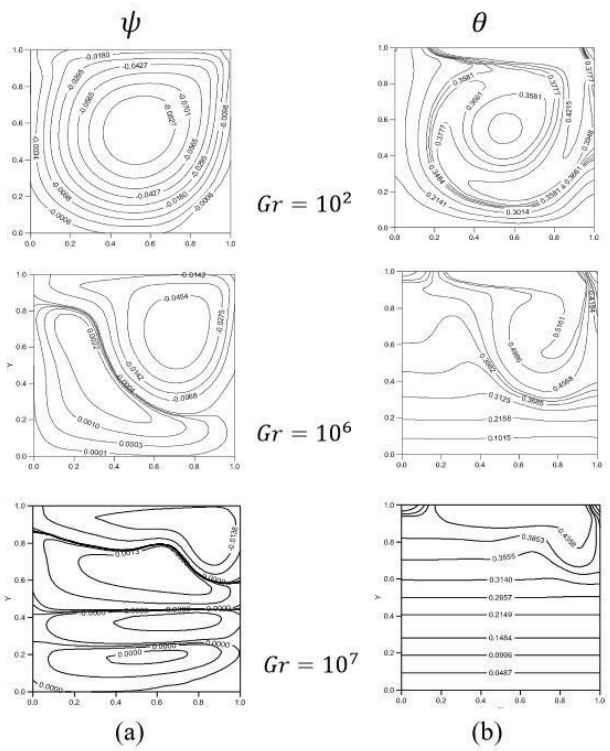

Figure-7. Streamlines and isotherms for various Richardson number Ri = 10^{-4}, Ri = 1, Ri = 10, at Da = 10^{-1} and Re = 10^3.

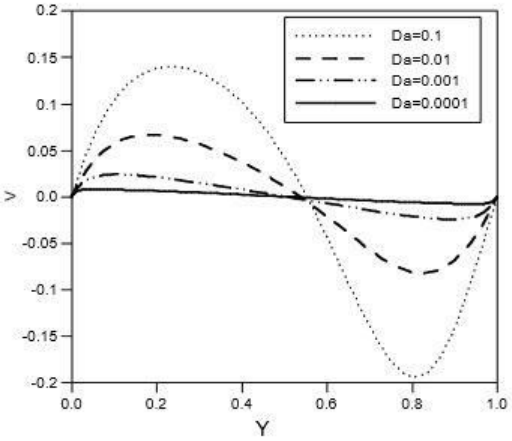

Figure-8. Velocity profiles for various Da at Ri = 10^{-2} and Gr = 10^2.

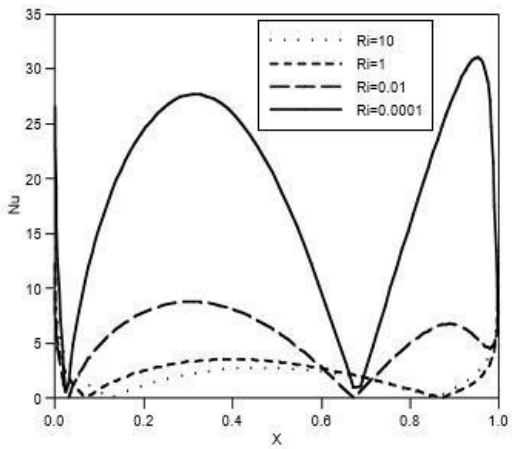

Figure-9. Local Nusselt number for various Ri at Da = 10^{-1} and Gr = 10^2.

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