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MIXED CONVECTION IN AN INCLINED LID-DRIVEN SQUARE CAVITY WITH SINUSOIDAL HEATING ON TOP LID

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ABSTRACT

Numerical study on the effect of inclination angle with sinusoidal heating on top moving lid in two-dimensional square cavity is investigated. The top lid is heated sinusoidally while the bottom wall is maintained at cold temperature. The vertical walls are insulated and the cavity is filled with water. Finite volume method and SIMPLE algorithm are employed to solve the dimensionless governing equations. The effect of Richardson number, ranging from 0.1 to 10.0 and inclination angle ranging from 0° to 60° on heat and fluid flow are investigated by utilizing the discretized equations in FORTRAN programming language. The Reynolds number and Prandtl number are fixed. Finally the solutions are discussed using a graphical approach. The results demonstrate that for the case of forced convection and mixed convection dominated regime, heat transfer rate increases with the increase of cavity inclination.

Keywords: finite volume method, lid-driven,mixed convection, sinusoidal temperature, square cavity.

INTRODUCTION

A variety of heat and mass transfer problems in a cavity with various boundary condition and configurations has been studied by diverse researchers due to wide variety of applications in science and engineering industries. [4], [16] and [17] investigated natural convection heat transfer in a cavity with the effect of thermal boundary conditions, magnetic field and inclination angle, respectively. Meanwhile, [13] studied the effect of magnetic field and sinusoidal temperature distribution to the natural convection flow in a cavity filled with nanofluid. Then, [6] conducted the heat line approach to study mixed convection in a porous square cavity with various thermal boundary conditions. Later, [19] extended the work of [6] with various fluid in a square cavity based on thermal aspect ratio. [8] investigated on mixed convection problem in air filled cavity with sinusoidally heated moving lid. Two cases were conducted; uniformly heated walls and cooled side walls. On the other hand, [5] analyzed mixed convection flows in a square cavity with linearly heated side walls. [21] investigated the hydro-magnetic mixed convection problem with sinusoidally heated side walls. The effect of magnetic field to the mixed convection flow in lid-driven cavity with linearly heated wall was studied by [12] and [2]. [3] and [14] conducted the numerical investigation on lid-driven cavity filled with nanofluid with sinusoidal heating on the side walls. Then, [15] carried out the investigation on mixed convection in a nanofluid-filled lid-driven cavity in the presence of internal heat generation and non-uniform heating of bottom wall. Inclined cavity filled with nanofluid was conducted by [9]. They reported that heat transfer increased with the increased of nanoparticles while cavity inclination is insignificant to the isotherms pattern for constant Reynolds and Richardson numbers. Recently, [11] studied the effect of inclination angle to the mixed convection flow in a nanofluid-filled shallow cavity. Effect of non-uniform heating on both side walls on an inclined cavity together with the presence of magnetic field was investigated by [1]. [20] reported on laminar mixed convection in shallow inclined cavity with hot moving lid. He found that the rate of heat transfer increased with cavity inclination. Then, [10] extended the work of [20]. They conducted the unsteady flow for shallow inclined driven cavity with aspect ratio of five. The heat transfer rate increased with cavity inclination for mixed and natural convection dominated regime but decreased mildly for forced convection dominated regime. [22] investigated numerically the problem of mixed convection in an inclined cavity with discrete heating. Recently, [7] reported the effect of inclination angle to three aspect ratios of lid-driven cavity. It was found that increase of cavity inclination does not affect the flow pattern and heat transfer for forced convection regime.

The aim of this paper is to extend the work of [10] such that the effect of cavity inclination, Richardson number and sinusoidal heating of top sliding lid on the flow distributions and heat transfer in a square cavity is investigated and compare the results with [7].

Mathematical modelling

A two-dimensional square lid-driven cavity of height and width H filled with incompressible fluid is considered in this study. The upper wall is heated sinusoidally and allowed to move at a constant speed U_0 in x-direction. The bottom wall is kept at a constant cold temperature $T_c$while the vertical walls are adiabatic. The physical model and boundary conditions considered are shown in Figure-1.The effect of cavity inclination ranges from 0° to 60° with an interval of 30° for Richardson number, Ri = 0.1, 1.0 and 10.0 are analysed.The present flow is unsteady and laminar. The thermophysical properties of considered fluid are assumed to be constant except for the density variation which is approximated by Boussinesq model.

Based on these assumptions, the dimensionless governing equations of mass, momentum and energy can be written as follows:

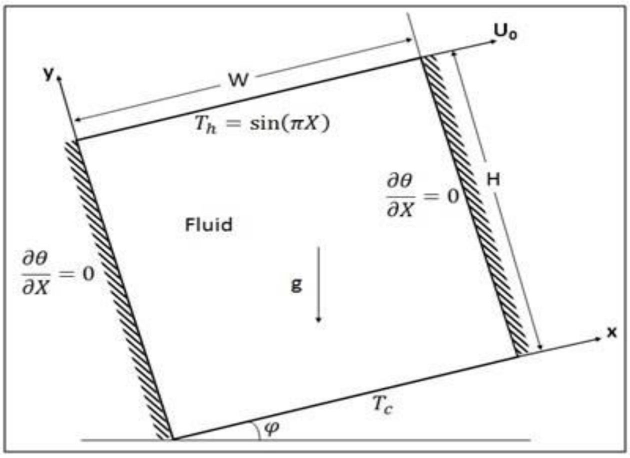

Figure-1. Schematic configuration of present study.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + Ri\theta \times \sin\varphi, \tag{2}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ri\theta \times \cos\varphi, \tag{3}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right), \tag{4}$$

$$\begin{array}{ll} \text{Top wall} & : U = 1,\ V = 0,\ \theta = \sin(\pi X), \\ \text{Bottom wall} & : U = 0,\ V = 0,\ \theta = 0, \\ \text{Right and left walls} & : U = 0, V = 0, \frac{\partial\theta}{\partial X} = 0. \end{array} \tag{5}$$

Here, the dimensionless variables are: U and V are the X and Y-component velocity respectively, τ is the time, P is the pressure, θ is the temperature and φ is the inclination angle. Re is the Reynolds number, Ri is the Richardson number$(Ri = Gr/Re^2)$ where Gr is the Grashof number and Pr is the Prandtl number. In present analysis, the dimensionless variables used are defined accordingly to:

$$X = \frac{x}{H}, Y = \frac{y}{H}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \theta = \frac{T - T_c}{T_h - T_c}, Pr = \frac{\nu}{\alpha},$$

$$Gr = \frac{g\beta(T_h - T_c)H^3}{\nu^2}, \tau = \frac{tU_0}{H}, P = \frac{p}{\rho U_0^2}, Re = \frac{U_0 H}{\nu}$$

where u and v are the dimensional velocity components along the x and y axes respectively, H is the height and width of the cavity, ν is the kinematic viscosity, t is the time, U_0 is the top wall driven velocity, $\alpha = k/\rho c$ is the thermal diffusivity with k as thermal conductivity and c as heat capacity, ρ is the density, β is the coefficient of thermal expansion, g is the gravity, p is the pressure and T is the temperature such that subscript h and c indicate the hot and cold wall respectively. The stream function is calculated from the definition $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$.

Numerical approach and code validation

In order to solve numerically the governing Equation. (1) – (4) subject to the boundary conditions (5), the finite volume method is employed. Power law scheme is used to discretize the convection and diffusion terms. The pressure and velocities are coupled by using the SIMPLE algorithm. An unstructured, cell centered with staggered grid system is used. The details on finite volume method, SIMPLE algorithm, and unstructured and staggered grid system were given by [18]. Then, the resulting discretized equations are solved iteratively by Tri-Diagonal Matrix Algorithm (TDMA). The procedure is then adapted into the FORTRAN90 programming language. The iterative procedure is terminated when the following convergence criterion is fulfilled for each variable:

$$\text{Error} = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left|\xi_{i,j}^{k+1} - \xi_{i,j}^{k}\right|}{\sum_{j=1}^{m}\sum_{i=1}^{n}\left|\xi_{i,j}^{k+1}\right|} \leq 10^7$$

and k is the number of iteration. The local Nusselt number measured the rate of heat transfer and is formulated as $Nu_x = -(\partial\theta/\partial Y)_{Y=0}$. The average Nusselt number is obtained by integrating the local Nusselt number and is determined from the following formula:

$$\overline{Nu} = \int_0^1 Nu_x\, dX.$$

A grid independency test is conducted for $Ri = 1.0$ and $\varphi = 0°$ to determine the appropriate grid density so that the numerical results is accurate. Figure-2 shows the optimum grid density according to the variation of Nusselt number along the sinusoidally heated top wall. Thus, the grid 81 × 81 grid density is selected.

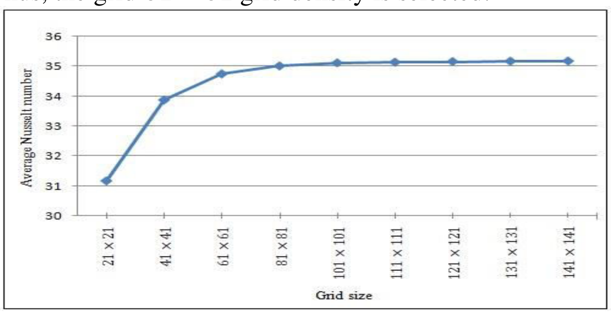

Figure-2. Average Nusselt number variation along the hot wall with grid sizes for $Ri = 1.0$ and $\varphi = 0°$.

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Present code is validated with previous study in the literature in order to check the accuracy of the numerical results. Figure-3 presented the comparison of streamlines and isotherms with the work of [7] for $Ri = 1.0$, aspect ratio =1.0 and inclination angle 0°. It is clear that good agreement exists between present numerical results with the numerical results of [7].

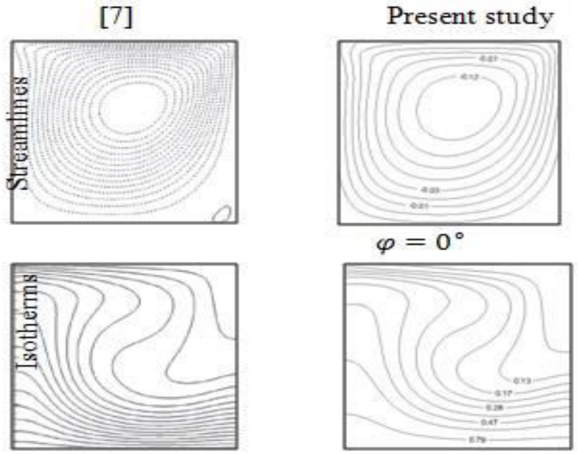

Figure-3. Comparison of present study with [7] for $Ri = 1.0$ and $\varphi = 0°$.

RESULTS AND DISCUSSION

Mixed convection heat transfer regime in an inclined square cavity with sinusoidally heated top wall is investigated in this study. The bottom wall is kept at constant cold temperature while the sidewalls are perfectly insulated. The working fluid is water with Pr = 6.0. The Reynolds number is fixed at Re = 408.21 while the Richardson number is considered to vary from 0.1 to 10. The cavity inclination is ranging from $0°$ to$60°$. However, in present study only three different values of inclination that is $\varphi = 0°, 30°$ and $60°$ is discussed such that the cavity is tilted in counter-clockwise direction.

Figure-4 showed the streamlines and isotherms of forced convection dominated regime with $Ri = 0.1$. It is observed that the flow field is characterized by a primary clockwise recirculating cell that occupied the entire cavity. This is due to the flow driven by the movement of the top lid. When the cavity is inclined to $\varphi = 30°$, two small cells at the left and right bottom wall existed. However, when $\varphi = 60°$, the two cells at the bottom are vanished. The inclination of the cavity has no significant effect on the flow behaviour and isotherms.

For mixed convection regime with $Ri = 1.0$, the streamlines and isotherms are presented in Figure-5. A primary clockwise recirculating vortex and a smaller counter-clockwise recirculating vortex at left bottom wall are existed when $\varphi = 0°$. As the cavity is inclined to 30°, the small counter-clockwise recirculating vortex has developed into a primary vortex. Further increase of inclination angle to $\varphi = 60°$, two major recirculating vortices occupied the cavity.The top one is clockwise recirculating vortex due to the flow driven by top lid that move from left to right while the bottom counter-clockwise recirculating vortex is induced by the buoyancy force.

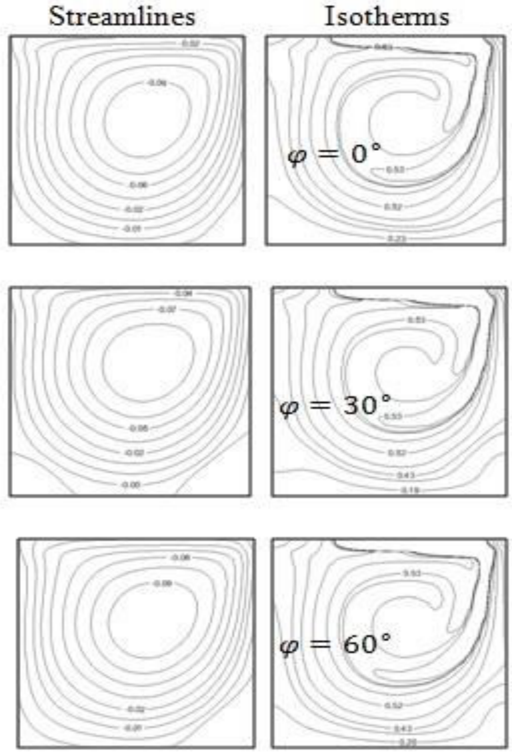

Figure-4. Streamlines and isotherms in the cavity with increasing cavity inclination for $Ri = 0.1$.

This indicates that a steep temperature gradient existed in this region due to sinusoidally heated top wall and sliding lid. The isotherms near the bottom are thermally stratified in the horizontal direction. As the cavity is inclined to $\varphi = 30°$, the steep temperature gradient on the right wall moved to a slightly lower position near the right wall and in opposite direction. Another high temperature gradient region existed at the left bottom corner. A thermally stratified isotherm in vertical direction occupied in the bulk centre region of the cavity. Increasing the cavity inclination to 60° causes the existence of high temperature gradient near the upper right wall and bottom left corner of the cavity.

The effect of inclination angle on streamlines and isotherms for natural convection dominated regime with $Ri = 10.0$ are presented in Figure-6. The effect of inclination angle to the streamline and isotherm patterns is clearly visible. When φ=0°, the lower half of the cavity is occupied with large horizontally elongated vortex and a small vortex in lower right corner. The upper half of the cavity is occupied with two vortices of almost the same size but in opposite recirculating direction. When the inclination angle is 30°, the whole cavity is filled with a major counter-clockwise recirculating vortex caused by the buoyancy forces. Further increase of inclination angle to 60° again showed the cavity is occupied with a major recirculating vortex. However, there is a clockwise recirculating vortex near the top wall. The isotherms showed a quasi-conduction domain that is the heat is

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transferred by conduction except at the region near the sliding lid. When φ=30°, high temperature gradient occurs at the right top corner and left bottom corner. The temperature gradient became steeper as the inclination angle increased.

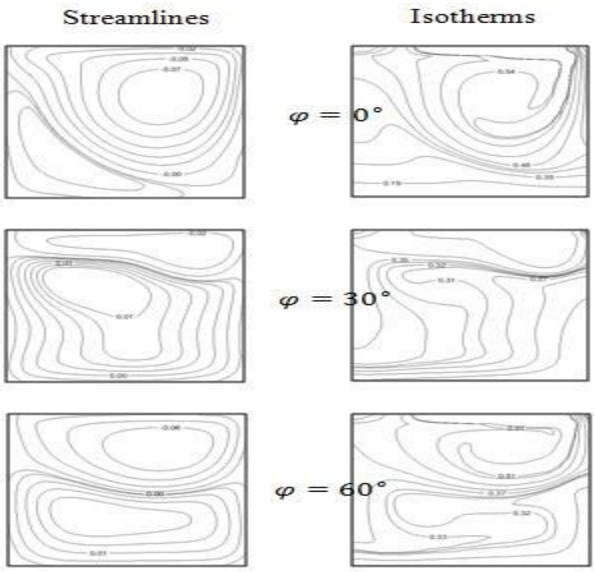

Figure-5. Streamlines and isotherms in the cavity with increasing cavity inclination for $Ri = 1.0$.

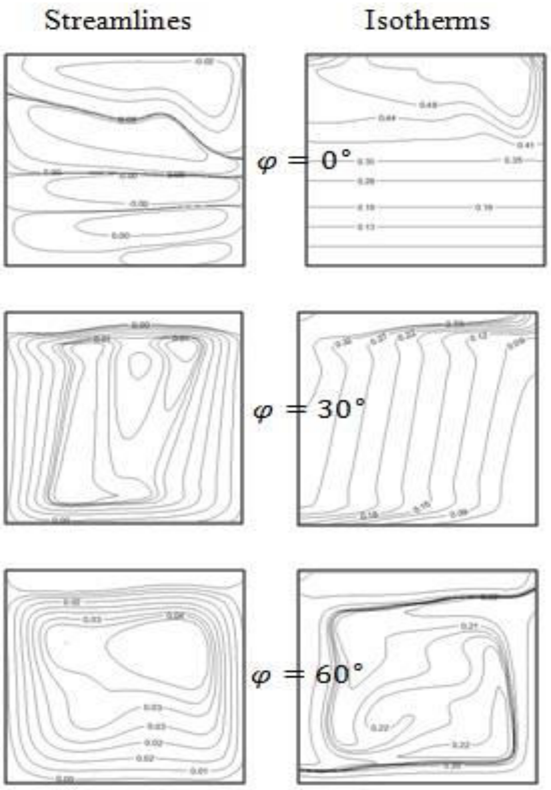

Figure-6. Streamlines and isotherms in the cavity with increasing cavity inclination for $Ri = 10.0$.

Figure-7 shows a graph of the variation of average Nusselt number that measured the heat transfer rate against the inclination angle for different Richardson numbers. From the graph, it can be seen that cavity inclination almost has no effect on the overall heat transfer process for $Ri = 0.1$ and 1.0. However, for $Ri = 10.0$ cavity inclination has significant effect on the overall heat transfer rate. The maximum average Nusselt number occurs when $\varphi = 60°$ for natural convection dominated regime. The overall heat transfer rate decreased drastically when the inclination angle is 30°.

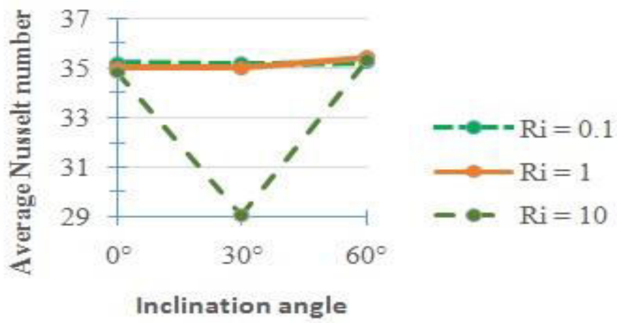

Figure-7. Variation of average Nusselt number along the sinusoidally heated wall with inclination angle and Richardson number.

CONCLUSIONS

In present paper, a problem on mixed convection flow of water as the working fluid in a square cavity with the effect of inclination angle has been conducted numerically. The vertical walls are adiabatic. The top wall is sliding from left to right and sinusoidally heated while the bottom wall is kept at a constant cold temperature. The governing equations are discretized by using finite volume method. The numerical results has been presented graphically in streamlines and isotherms plot and discussed. It was found that for the case of $Ri = 0.1$, inclination angle has no effect on the flow field, isotherms pattern as well as the average heat transfer rate. For the case of $Ri = 10.0$, the streamlines, isotherms and average Nusselt number is strongly affected by the inclination angle. It is found that at $\varphi = 30°$, the overall heat transfer rate decreased drastically for $Ri = 10.0$ and increased as the inclination angle increased to 60°. For the case of $Ri = 1.0$, mild effect of cavity inclination is found on the flow field, temperature gradient and average heat transfer rate. Finally, heat transfer rate increase mildly with the increase of cavity inclination for the case of forced convection and mixed convection dominated regime. However, for natural convection dominated regime, cavity inclination plays an important role to control the heat transfer rate and fluid flow.

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ARPN Journal of Engineering and Applied Sciences

NOMENCLATURE

c	Specific heat capacity, J/kg K
g	Acceleration due to gravity, m/s^2
Gr	Grashof number
H	Height and width of the cavity, m
k	Thermal conductivity of fluid, W/m K
$\overline{Nu}$	Average Nusselt number
Nu_x	Local Nusselt number
p	Pressure
P	Dimensionless pressure
Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
t	Time
T	Temperature, °C
T_c	Temperature at cold wall, °C
T_h	Temperature at hot wall, °C
u, v	Velocities in the x- and y-direction respectively
U, V	Dimensionless velocity in X- and Y-direction respectively
U_0	Lid velocity, m/s
ν	Kinematic viscosity, m^2/s
x, y	Cartesian coordinates
X,Y	Dimensionless Cartesian coordinates

Greek symbols

α	Thermal diffusivity, m^2/s
β	Coefficient of thermal expansion of fluid, K^{-1}
θ	Dimensionless temperature
ρ	Fluid density, kg/m^3
φ	Inclination angle
τ	Dimensionless time
ψ	Stream function

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