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APPLICATION OF A PREDICTIVE CONTROLLER WITH VARIABLE TIME DELAY INGENERAL ANESTHESIA

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ABSTRACT

The purpose of this work is to use an online time delay estimation obtained from artificial data in order to update the prediction model of the model-based controller algorithm. The performance of the closed-loop system to track a reference is evaluated. The disturbance rejection is analyzed when some step disturbances are applied to the closed-loop system output.

Keywords: anesthesia control, dead-time compensation, intensive care unit, model-based predictive control.

1. INTRODUCTION

Model-Based Predictive Control (MBPC) is conceptually a method for generating feedback control actions for linear and nonlinear systems subject to point wise-in-time input and/or state-related constraints.

For instance, while a human is driving a vehicle, generates steering-wheel commands by forecasting or predicting over a finite time-horizon the (possible) vehicle state-evolutions on the basis of vehicle current state and dynamics, and a virtual or potential steering-wheel command sequence. Then, one, among such sequences, is sorted out, which fulfils safety constraints and meets performance requirements. Only a short initial portion of such a sequence is applied by the driver to the steering-wheel, while its remaining part is discarded. After such an initial portion is applied, the driver repeats the whole operation by restarting predictions over a moved-ahead or receded time-horizon from the updated vehicle state as determined by the applied command.

MBPC complies with the same logical scheme: the control sequence is computed by solving online, over a finite control horizon, an open-loop optimal control problem, given the plant dynamical model and current state. Though this computation hinges upon an open-loop control problem, MBPC yields a feedback-control action. Indeed, similarly to the driver behaviour, in a discrete-time setting only the first control of the open-loop control sequence is applied to the plant, and, according to the receding horizon control concept, the whole optimization cycle is repeated at the subsequent time-instant based on the new plant-state. Because it involves a control horizon made up by only a finite number of time-steps, MBPC can be often calculated on-line by existing optimization routines so as to minimize a performance index in the presence of hard constraints on the time evolutions of input and/or state. MBPC ability of handling constraints is of paramount importance whenever constraints are part of the control design specifications. In fact, constraints are typically present in applications, as they stem from actuators saturations and/or physical, safety or economical requirements. Despite the importance of constraints, there is a shortage of control methods for handling them effectively. The main reason for the interest of control engineers in MBPC is therefore its ability to systematically and effectively handle hard constraints.

The MBPC strategy can be visualized in the next block scheme:

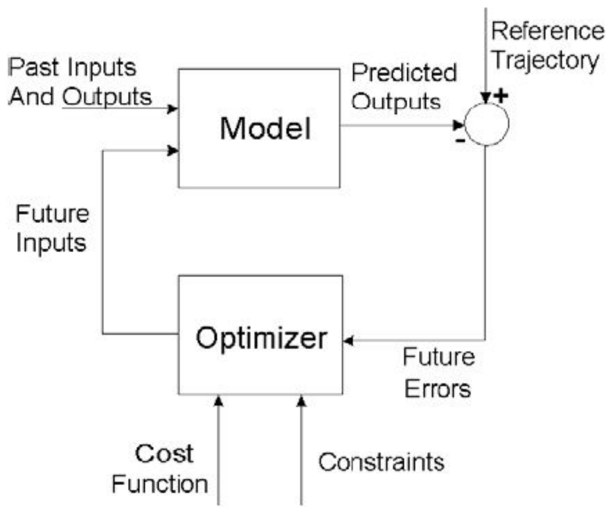

Figure-1. MBPC strategy as a block scheme.

MBPC reflects human behaviour when the control actions presumed to lead to the best predicted output over some limited horizon are selected. To make this selection an internal model of the process in question is used. The decisions are constantly updated as new observations become available. The MBPC principle is characterized by the following strategy (De Keyser, 2003):

- at each present moment t, the process output $y(t+k)$ is predicted over a time horizon $k = 1 \dots N_2$. The predicted values are indicated by $y(t+k|t)$ and the value N_2 is called the prediction horizon. The prediction depends on the past inputs and outputs, but also on the future control scenario (the control actions that are intended to be applied from the present moment $t$on);
- a reference trajectory $r(t+k|t)$, $k = 1 \dots N_2$, starting at $r(t|t) = y(t)$ and evolving towards the set point w, is defined over the prediction horizon, describing

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how the process output is desired to be guided from its current value $y(t)$ to its set point w; if the process has a time delay (dead-time), it is reasonable to start the reference trajectory after the time delay;

- the control vector $u(t+k|t), k = 0 \ldots N_2 - 1$, is calculated in order to minimize a specified cost function, depending on the predicted control errors $r(t+k|t) - y(t+k|t),\ k = 1 \ldots N_2$; also, in most methods there is some structuring of the future control law, $u(t+k|t), k = 0 \ldots N_2 - 1$, there might also be constraints on the process variables;
- the first element $u(t|t)$ of the optimal control vector $u(t+k|t), k = 0 \ldots N_2 - 1$ is actually applied to the real process. All other elements of the calculated control vector can beforgotten, because at the next sampling instant all time-sequences are shifted, a new output measurement $y(t+1)$ is obtained and the whole procedure is repeated; this leads to a newcontrol input $u(t+1|t+1)$, which is generally different from the previously calculated $u(t+1|t)$.

2. MATERIALS AND METHODS

The beginning of model-based predictive control was around 1980, when some pioneering institutions started to develop the main ideas and computer algorithms. The Ghent University impact on the early development of MBPC has been internationally appreciated via an invited paper in the prestigious UNESCO Encyclopaedia of Life Support Systems (De Keyser, 2003).

Being one of the earlier predictive controllers, Extended Prediction Self-Adaptive Control - EPSAC is based on a generic process model:

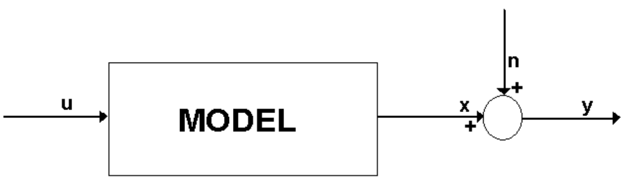

Figure-2. Generic process model.

The process is modelled as

$$y(t) = x(t) + n(t)$$

with, $y(t)$: (measured) process output; $u(t)$: process input; $x(t)$: model output; $n(t)$: process/model disturbance; t: discrete time index. The disturbance $n(t)$ includes the effects in the measured output $y(t)$ which do not come from the model input $u(t)$ via the available model. These non-measurable disturbances have a stochastic character with non-zero average value, which can be modelled by a coloured noise process:

$$n(t) = \frac{C(q^{-1})}{D(q^{-1})} e(t)$$

where, $e(t)$: uncorrelated (white) noise with zero mean value; $C(q^{-1})$, $D(q^{-1})$: monic polynomials in the backward shift operator q^{-1} of orders n_c and n_d.

It is common practice in the MBPC approach to consider this filter as a design filter. It can be used – in order to improve the quality of the control performance – to 'supply' information to the controller about the type of disturbances that can be expected. The simplest way to design this filter is to neglect it, thus make it equal to 1. In doing this, not any information about the disturbance is given to the controller. In fact, this results in 'telling' to the MBPC-controller that the disturbance $n(t) = e(t)$, defined as uncorrelated noise with zero-mean average value. As a consequence then, the controller will not take any specific action to remove non-zero-mean disturbances. Usually, the disturbance has in practice a non-zero average component, and a steady-state control error can thus be expected as the result of a permanent disturbance.

A better choice for the disturbance model might be:

$$\frac{C(q^{-1})}{D(q^{-1})} = \frac{1}{1 - q^{-1}}$$

resulting in a disturbance signal $n(t)$ with non-zero average value (coloured noise). In this case the MBPC-controller will intrinsically take action to remove steady-state errors, similar to the effect of the integrator in a PID-type controller. Notice that it is the 'default' disturbance model that is usually applied in practice.

2.1 Prediction algorithm

The model output $x(t)$ represents the effect of the control input $u(t)$ on the process output $y(t)$ and is also a non-measurable signal. The relationship between $u(t)$ and $x(t)$ is given by the generic dynamic system model:

$$x(t) = f[x(t-1), x(t-2), \ldots, u(t-1), u(t-2), \ldots]$$

The fundamental step in MBPC methodology consists in prediction of the process output $y(t+k)$ at time instant t, indicated by $\{y(t+k|t), k = 1 \ldots N_2\}$, over the prediction horizon N_2, and based on:

- measurements available at sampling time instant t: $\{y(t), y(t-1), \ldots, u(t-1), u(t-2), \ldots\}$;
- future values of the input signal (postulated at time t): $\{u(t|t), u(t+1|t), \ldots\}$.

Using the generic process model, the predicted values of the output are:

$$y(t+k|t) = x(t+k|t) + n(t+k|t)$$

The prediction of $x(t+k|t)$ and of $n(t+k|t)$ can be done respectively by recursion of the process model and by using filtering techniques on the noise model.

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2.2 Control algorithm

In EPSAC for linear models, the future response is considered as being the cumulative result of two effects:

$$y(t+k|t) = y_{base}(t+k|t) + y_{opt}(t+k|t)$$

The two contributions have the following origins:

➔$y_{base}(t+k|t)$:

- effect of past control $\{u(t-1), u(t-2), \ldots\}$ (initial conditions at time t);
- effect of a *base* future control scenario, called $u_{base}(t+k|t), k \geq 0$, which is defined *a priori*; for linear systems the choice of u_{base} is irrelevant, a simple choice being $\{u_{base}(t+k|t) = 0, k \geq 0\}$ will lead to the same control scenario;
- effect of future (predicted) disturbances $n(t+k|t)$.

➔$y_{opt}(t+k|t)$:

- effect of the *optimizing* future control actions $\{\delta u(t|t), \delta u(t+1|t), \ldots, \delta u(t+N_u-1|t)\}$with $\delta u(t+k|t) = u(t+k|t) - u_{base}(t+k|t)$.

Notice that $u(t+k|t)$ is constrained to be constant from $k = N_u$ on (and this is realized by *selecting*$u_{base}(t+k|t)$ constant from $k = N_u$ on and by *imposing* that $\delta u(t+k|t)$ should be constant from $k = N_u$ on). The *design* parameter N_u is called the *control horizon* (a well-known concept in MBPC-literature).

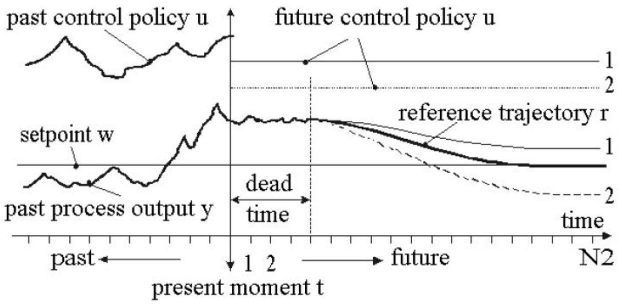

Figure-3. The EPSAC concept of base/ optimizing controls.

The concepts of base and optimizing controls are referred in the figure above. It is obvious that the component $y_{opt}(t+k|t)$ is the cumulative effect of a series of impulse inputs and a step input:

- an impulse with amplitude $\delta u(t|t)$ occurring at time t, resulting in a contribution$h_k \delta u(t|t)$ to the process output at time $t+k$ (k sampling periods later);
- an impulse with amplitude $\delta u(t+1|t)$ occurring at time $t+1$, resulting in a contribution $h_{k-1}\delta u(t+1|t)$ to the predicted process output at time $t+k$($k-1$ sampling periods later);
- etc;
- finally, a step with amplitude $\delta u(t+N_u-1|t)$ at time $t+N_u-1$, resulting in a contribution $g_{k-N_u+1}\delta u(t+N_u-1|t)$ to the predicted process output at time $t+k$. ($k-N_u+1$ sampling periods later).

The cumulative effect of all impulses and the step is:

$$y_{opt}(t+k|t) = h_k \delta u(t|t) + h_{k-1}\delta u(t+1|t) + \cdots + g_{k-N_u+1}\delta u(t+N_u-1|t)$$

The parameters $g_1, g_2, \ldots, g_k, \ldots, g_{N_2}$ are the coefficients of the unit step response of the system, i.e. the response of the system for a stepwise change of the input (with amplitude 1). The parameters $h_1, h_2, \ldots, h_k, \ldots, h_{N_2}$ are the coefficients of the unit impulse response of the system and can be easily calculated from the step response coefficients and viceversa: $h_k = g_k - g_{k-1}$ (and $h_0 = h_{-1} = \cdots = g_0 = g_{-1} = \cdots = 0$).

Thus, the key EPSAC-MBPC equation is:

$$Y = \overline{Y} + GU$$

where,

$$\mathbf{Y} = [y(t+N_1|t), \ldots, y(t+N_2|t)]^T$$

$$\overline{\mathbf{Y}} = [y_{base}(t+N_1|t), \ldots, y_{base}(t+N_2|t)]^T$$

$$\mathbf{U} = [\delta u(t|t), \ldots, \delta u(t+N_u-1|t)]^T$$

$$\mathbf{G} = \left[\begin{array}{cccc|c} h_{N_1} & h_{N_1-1} & \cdots & h_{N_1-N_u+2} & g_{N_1-N_u+1} \\ h_{N_1+1} & h_{N_1} & \cdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ h_{N_2} & h_{N_2-1} & \cdots & h_{N_2-N_u+2} & g_{N_2-N_u+1} \end{array}\right]$$

The controller task is to find the control vector $u(t+k|t), k = 0 \ldots N_2 - 1$which minimize the cost function:

$$J(U) = \sum_{k=N_1}^{N_2} [r(t+k|t) - y(t+k|t)]^2$$

with $r(t+k|t)$ the desired reference trajectory and the horizonsN_1, N_2 being design parameters. It is now straightforward to derive the solution. The cost function is a quadratic form in **U**, having the following structure using the matrix notation and with **R** defined similarly to **Y**:

$$J(U) = [(R - \overline{Y}) - GU]^T [(R - \overline{Y}) - GU]$$

which leads after minimization w.r.t. **U** to the optimal solution:

$$U^* = (G^T G)^{-1} G^T (R - \overline{Y})$$

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The matrix $G^T G$, which has to be inverted, has dimensions $N_u \times N_u$. For the default case $N_u = 1$, this results in a simple *scalar* control law. Only the first element $\delta u(t|t)$ in U^* is required in order to compute the actual control input applied to the process:

$$u^*(t) = u_{base}(t|t) + \delta u(t|t) = u_{base}(t|t) + \mathrm{U}^*(1)$$

At the next sampling instant $t+1$, the whole procedure is repeated taking into account the new measurement information $y(t+1)$. This is called the principle of receding horizon control, another well-known MBPC-concept.

2.3 Dead-time compensation

The complexity of the prediction procedure is of a higher order for systems with variable time delay than for those with constant time delay. For a system with time delay, changes in the controlled variable are noticeable once the time delay has passed. Therefore, in order to find the optimal control sequence only output predictions occurring after the time delay should be taken in the cost function. This means that the minimum costing horizon N_1 should be equal to the time delay. For systems with constant time delay this is easy to do. Then the maximum prediction horizon N_2 can be set to an appropriate value that ensures a stable and robust response and the control loop can be operated with fixed controller parameters.

For a variable time delay however, the value of N_1 (and thus also N_2) varies with the dead-time index. An alternative solution for controlling processes that present significant and varying dead-times is to make use of a Dead-Time Compensator (DTC). The Smith Predictor (SP) was the first control system proposed in the literature that included a DTC and is perhaps one of the best known in industrial applications (Normey-Rico and Camacho, 2007).

The combination of MBPC with DTC is an idea presented in (Normey-Rico, 1999), where a filtered SP combined with the GPC algorithm - a linear MPC strategy - is studied in detail. The combination of the filtered SP with the EPSAC, namely Smith Predictor based EPSAC (SP-EPSAC), provides a control strategy for linear processes with variable time delay. The resulting structure is presented below.

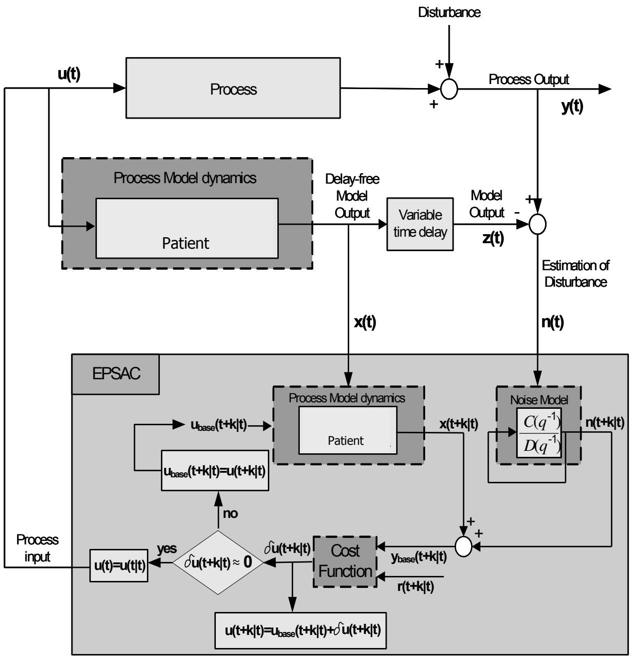

Figure-4. SP-EPSAC control scheme.

Since the process under consideration is stable, a parallel structure is used for output estimation. The model of the process consists of the patient dynamics and the variable time delay introduced by the BIS monitor, which are separated from the variable time delay model. At each sampling instant, the delay-free model output $x(t)$, resulting from the process dynamics only, is calculated using the stored values $[x(t-1), \ldots, u(t-1), \ldots]$. At the same sampling instant, the variable time delay is computed using the online TDE algorithm. Once time delay (τ_d) is known, $x(t-\tau_d)$ can be selected out of the stored x-values, such that $z(t) = x(t-\tau_d)$.

The feedback signal is equal to the response of the linear model without the output dead-time, plus the filtered model error. Therefore, the advantage is that prediction horizons N_1, N_2 are no longer dependent on the value of the dead-time, being $N_1 = 1$ and $N_2 = N$. Especially for longer and varying dead-times, this reduces considerably the computational load of a predictive controller.

In this way, the prediction procedure is thoroughly simplified, resulting in a Smith predictor-like scheme, with separation of the patient dynamics on one hand and the varying time delay on the other hand. In such approach the minimum prediction horizon is no longer varying and obviously equal to one. Hence, the maximum prediction horizon is also constant.

3. RESULTS AND DISCUSSIONS

The estimated time delay is used to update the prediction model of the EPSAC algorithm. The control horizon has been set to $N_u = 1$. Generally, the value of N_2 should be selected with respect to the time constant of the system. This favors both stability and performance. For systems with variable time constant as the one under

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consideration, it is reasonable to take the conservative choice corresponding to the largest time constant. In this case, $N_2 = 10$ is a good compromise?

The performance of the closed-loop system to track a reference is evaluated when the artificial data are used. The disturbance rejection is analyzed when some step disturbances are applied to the closed-loop system output.

The estimated time delay found with the three Time Delay Estimation (TDE) methods is used for evaluating the performance of the closed-loop system for reference tracking. The biometric values used in the EPSAC prediction model as well as the results shown are for one of the patients. The reference signal and the closed-loop response are shown in detail as follows:

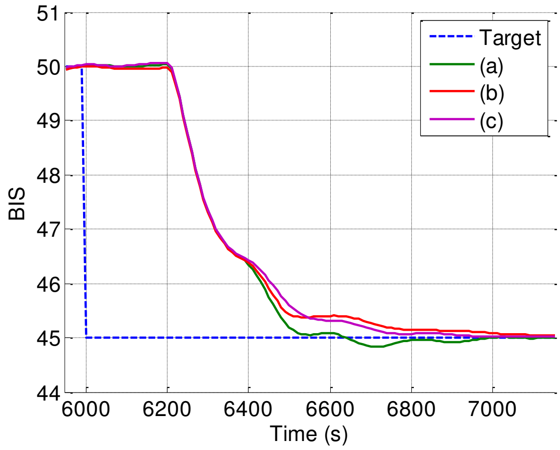

Figure-5. Performance of the closed-loop system for tracking when (a) Offline TDE, (b) Semi-Online, (c) Online TDE are used.

When the estimated time delay by the offline algorithm is used in the EPSAC prediction model, the undershoot is 0.36% and the setting time is 240 seconds. When the semi-online and the online algorithms are used the closed-loop response does not have undershoot, but the settling time is increased to 250 seconds.

This means that estimating a time delay closer to the real value reduces the modeling errors in the EPSAC and therefore the closed-loop system performance will be higher. In order to evaluate the performance of the system for disturbance rejection a positive step with amplitude 5 is applied. The result can be observed in Figure-6.

It can be observed that the use of the online TDE algorithm reduces the time to reject the disturbances, thus with fewer errors of modeling the disturbance rejection is performed more effectively and the robustness is higher.

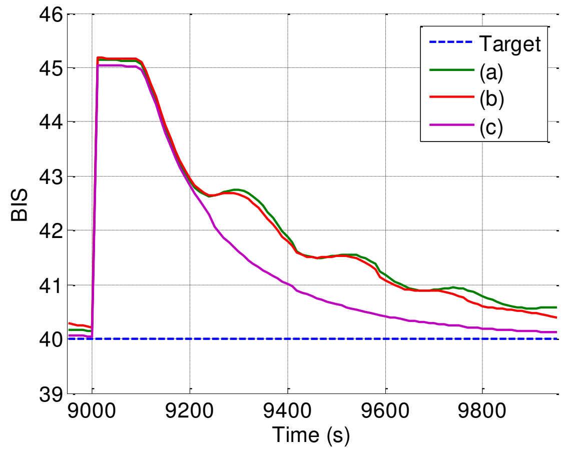

Figure-6. Performance of the closed-loop system for disturbance rejection when (a) Offline TDE, (b) Semi-Online, (c) Online TDE are used.

The error between the artificial BIS signal with variable time delay and the simulated BIS signal with fixed time delay estimated by the offline algorithm is calculated with the Mean Squared Error (MSE) formula:

$$MSE = \frac{1}{N}\sum_{k=1}^{N} |y(k) - \hat{y}(k)|^2$$

where $y(k)$is the reference BIS signal and $\hat{y}(k)$is the closed-loop system output.

The reference and the disturbance signals are applied to the other patient models in order to validate the performance of the closed-loop system. The MSE was evaluated in each patient and Table-1 presents the results.

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Table-1. MSE values calculated to validate the performance of the closed-loop system for tracking and disturbance rejection for each patient.

Patient	MSE for offline TDE	MSE for semi-online TDE	MSE for online TDE
1	30.73	18.48	17.95
2	29.14	18.35	17.90
3	30.66	18.16	17.61
4	31.90	17.98	17.77
5	30.49	18.29	17.73
6	32.80	18.01	17.48
7	27.99	18.72	18.41
8	29.32	18.37	17.85
9	30.68	18.33	18.00
10	31.32	18.12	17.95
11	32.41	18.01	17.75
12	32.14	18.00	17.69
13	32.26	18.10	17.77
15	31.10	18.45	18.03

It can be observed that the use of a constant time delay value in the prediction model produces a MSE with an average value of 30.92 and a standard deviation of ±1.38. If the time delay obtained with the semi-online algorithm is used to update the EPSAC prediction model, then the average MSE value decreases to 18.24 with a standard deviation of ±0.22. Finally, if the delay obtained with the online algorithm is used in the EPSAC prediction model, then the average MSE value decreases to 17.85 with a standard deviation of ±0.19.

The present work has presented a combination of a dead-time compensator with a linear predictive control algorithm, which resulted in a SP-EPSAC. The latter proved to be a control strategy that can handle both significant linear dynamics and variable dead-times. The algorithm proposed for online TDE may be used to update the time delay in the prediction model of the EPSAC using a Smith predictor to reduce the complexity of programming. The conclusion is that an online time delay estimation of the BIS monitor is able to improve the performance of the closed-loop system for tracking and disturbance rejection.

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