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MODE I STRESS INTENSITY FACTORS OF SLANTED CRACKS

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ABSTRACT

The solutions of stress intensity factors of slanted cracks in plain strain plate are hard to find in open literature. There are some previous solutions of stress intensity factors available, however they are not studied completed except for the case of plain stress. The slanted cracks are modelled numerically using ANSYS finite element program. There are ten slanted angles and seven relative crack depths are used and the plate containing cracks is assumed to fulfil the plain strain condition. The plate is then forced uni-axially the stress intensity factors are determined according to the displacement extrapolation method. Based on the numerical analysis, it is found that slanted angles have inverse effects on the behaviour of stress intensity factors. Increasing such angles capable to reduce the mode I stress intensity factors. On the other hand, it is also enhanced the capability of mode II stress intensity factors at the crack tip. Due to difficulty of determining stress intensity factors numerically, a regression technique is used to formulate mathematical expressions which are capable to predict the stress intensity factors in reasonable accuracies.

Keywords: stress intensity factor, slanted cracks, mode I loading, singular elements.

INTRODUCTION

Stress intensity factor (SIF) is successfully used to characterize the cracks in elastic materials and structures [1]. It is also used to replace the concept of stress concentration factor leading to infinite stress especially when there is a sharp edge for example cracks. There are several other methods or techniques are used to investigate the defects and it can be found in [2]. On the other hand, the basic crack configurations are also available in [2]. However, the oblique or slanted cracks are rarely found in open literature [3-6]. In [3] present a closed form solution for the geometrical correction factors of slanted cracks in a plain stress plate. The closed form solution of SIFs comprised of three important parameters such as inclination angles, plate widths and crack lengths. They claimed that the results from such expression are well agreed with the analytical, numerical and experimental results.

In [4] also investigated the stress intensity factors of inclined cracks. However, the cracks are positioned between two different materials. Their work concentrated more on the comparison between their method and displacement extrapolation method. In [5] investigated the stress intensity factor of two crack conditions such as slanted single edge and central cracks using displacement extrapolation method. They only focused on the 45°crack inclination and concluded that the displacement extrapolation method gave accurate results even coarse finite element model is used. Three dimensional slanted cracks in round bars under mode I tension can be found in [6]. The surface cracks are assumed as elliptical shapes and different slanted angles, elliptical ratios and relative crack depth are used. It is found that when the inclination angles are introduced, the stress intensity factors increased gradually. On the other hand, slanted angles are also produced the mode II stress intensity factors.

This paper investigates the stress intensity factors of slanted edge crack in plain strain plate under mode I tension stress. The crack is modelled numerically using ANSYS finite element program. Displacement extrapolation method is then used to calculate the stress intensity factors and they are plotted against the relative crack depths. The effect of relative crack depths and the oblique angles on the stress intensity factors are discussed and analysed.

BACKGROUND OF STRESS INTENSITY FACTOR

The calculation of J-integral around the crack tip is based on the domain integral method firstly introduced by [7]. This formulation uses area integration for two-dimensional while volume integration for three-dimensional problems, which offer better accuracy compared with contour integral and additionally it is also much easier to implement numerically. Equation. (1) represents the J-integral formulation for two-dimensional condition taking accounts the absence of thermal strain, path dependent plastic strain, body forces occur within the integration area [8]:

$$J = \int A\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\delta_{1i}\right]\frac{\partial q}{\partial x_i}dA \qquad (1)$$

where q_{ij} is the stress tensor, u_j is the displacement vector, w is the strain energy density, δ_{ij} is the Kronecker delta, x_i is the coordinate axis and q is referred to as the crack extension vector. The direction of q is similar with x-axis of the local coordinate specified at the crack tip and it is normally chosen as zero at nodes along the contourΓ. It is also a unit vector for all nodes inside Γ except the mid-side nodes and known as virtual crack extension nodes. For three-dimensional problems, the principal is similar to the two-dimensional problems. However, domain integral representation of the J-integral becomes volume integration [9].

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There are two approaches for calculating stress intensity factor which are available in ANSYS finite element program such as interaction integral and displacement extrapolation methods. The first method is used because much easier to implement numerically and it is also offers better accuracy and fewer mesh requirement. This method is similar to the domain integral method for J-integral evaluation describe previously. The discussion on the domain integral methods can be found elsewhere [9]. The interaction integral is defined as in Equation. (2):

$$I = -\int_V q_{ij}\left[\sigma_{k,l}\varepsilon_{k,l}^{aux}\delta_{i,j} - \sigma_{k,j}^{aux}u_{k,i} - \sigma_{k,j}u_{k,i}^{aux}\right]dv \Big/ \int_s \delta q_n ds \quad (2)$$

where σ_{ij}, ε_{ij} and u_i are the stress, strain and displacement and $\sigma_{k,j}^{aux}$, $\varepsilon_{k,l}^{aux}$ and $u_{k,i}^{aux}$ are the stress, strain and displacement of the auxiliary field and q_i is the crack extension vector.

$$\sigma_{ij}^{aux} = \frac{K_I^{aux}}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}^{aux}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + \frac{K_{III}^{aux}}{\sqrt{2\pi r}} f_{ij}^{III}(\theta) \quad (3)$$

$$u_{ij}^{aux} = \frac{K_I^{aux}}{2\mu}\sqrt{\frac{r}{2\pi}} g_j^{I}(\theta,\nu) + \frac{K_{II}^{aux}}{2\mu}\sqrt{\frac{r}{2\pi}} g_j^{II}(\theta,\nu) + \frac{2K_{III}^{aux}}{\mu}\sqrt{\frac{r}{2\pi}} g_j^{III}(\theta,\nu) \quad (4)$$

$$\varepsilon_{i,j}^{aux} = \frac{1}{2}\left(u_{i,j}^{aux} + u_{j,i}^{aux}\right) \quad (5)$$

An expression for the energy release rate in terms of mixed-mode stress intensity factor is defined as for plane strain condition as in Equations. (6)-(8):

$$J = \frac{\left(K_I^2 + K_{II}^2\right)\left(1-\upsilon^2\right)}{E} + \frac{K_{III}^2\left(1+\upsilon\right)}{E} \quad (6)$$

$$J = \frac{\left[\left(K_I + K_I^{aux}\right)^2 + \left(K_{II} + K_{II}^{aux}\right)^2\right]\left(1-\upsilon^2\right)}{E} + \frac{\left(K_{III}^2 + K_{III}^{aux}\right)\left(1+\upsilon\right)}{E} \quad (7)$$

$$J = J + J^{aux} + I \quad (8)$$

The interaction integral can be associated with the stress intensity factors as Equation. (9):

$$I = \frac{2(1-\upsilon^2)}{E}\left(K_I K_I^{aux} + K_{II} K_{II}^{aux}\right) + \frac{1}{\mu} K_{III} K_{III}^{aux} \quad (9)$$

By setting $K_{II}^{aux} = 1$ and $K_I^{aux} = K_{III}^{aux} = 0$,

$$K_I = \frac{E}{2(1-\upsilon^2)} I \quad (10)$$

By setting $K_{II}^{aux} = 1$ and $K_I^{aux} = K_{III}^{aux} = 0$ and $K_{III}^{aux} = 1$ and $K_I^{aux} = K_{II}^{aux} = 0$ leads to the relationship between modes II and III stress intensity factors with I respectively.

$$K_{II} = \frac{E}{2(1-\upsilon^2)} I \quad (11)$$

$$K_{III} = \mu I \quad (12)$$

where K_i is a stress intensity factor with i is a loading mode, i = 1, 2 and 3. J-integral can be represented as J and I is an interaction domain integral. While, E and μ is a modulus of elasticity and modulus of rigidity respectively.

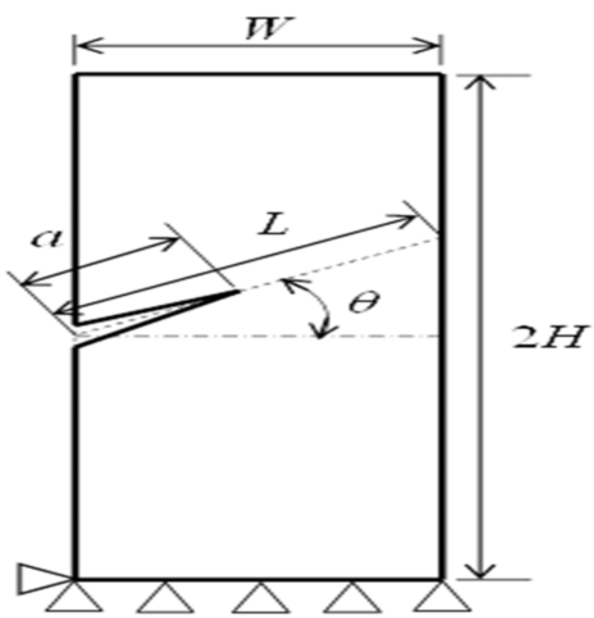

Figure-1. Schematic diagram of the slanted crack.

CRACK MODELLING

ANSYS finite element program is used to model and calculate the stress intensity factors. There are two kind of cracks are modelled and the first is a normal crack. It is used for validation purposes and the second type is slanted crack. The crack mounts is placed in the middle of plate height where the plate is also assumed to fulfil the plain strain condition. There are nine slanted angles are used including normal cracks and there are normalized against angle of 90°. The relative angles are 0.00, 0.056, 0.111, 0.167, 0.222, 0.278, 0.333, 0.389, 0.444 and 0.500. The relative crack depth, a/L used as follows 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7. It is also assumed that the plate width is 50mm and the height, 2H is 100mm. The schematic diagram of the crack in a plate is shown in Figure-1.

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Displacement extrapolation method [10] is used to determine the stress intensity factor at the crack tip. Since the crack faces are oblique, two types of SIFs are produced such as modes I and II. The SIFs are also normalized as below:

$$F_{I,a} = \frac{K_I}{\sigma\sqrt{\pi a}} \tag{13}$$

$$F_{II,a} = \frac{K_{II}}{\sigma\sqrt{\pi a}} \tag{14}$$

where K_I and K_{II} are the modes I and III stress intensity factors respectively while F_I and F_{II} are their corresponding geometrical correction factors or normalized stress intensity factors, σ is an axial stress and a is a slanted crack length. Before the model is further used, it is a compulsory work to have the validation by comparing with the existing model [10]. Since there is lack of number of works on the slanted cracks, then the normal cracks are used for such purposes. Based on the Figure-3, it is revealed that the present model is well agreed with the existing model.

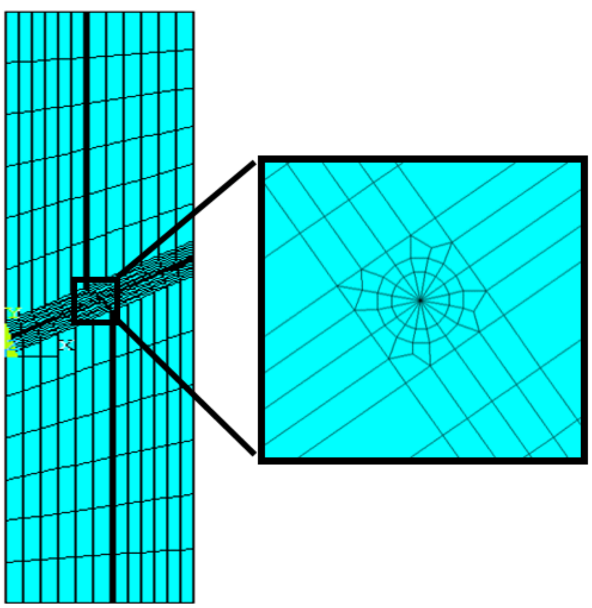

Figure-2. (a) FE model and (b) An associated crack tip.

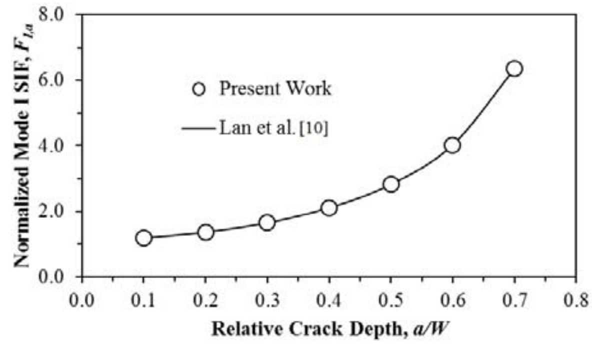

Figure-3. Validation of the present model.

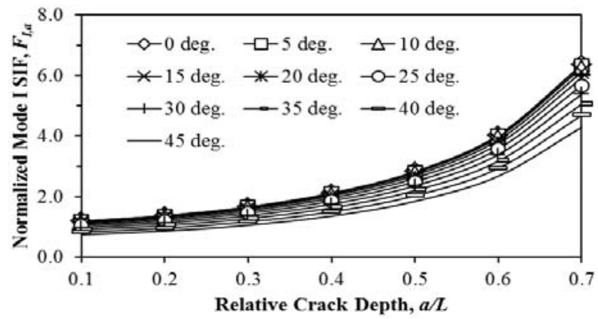

Figure-4. The effect of a/L on mode I SIFs.

RESULTS AND DISCUSSIONS

Both modes I and II stress intensity factors are plotted against relative crack depth, a/L as shown in Figure-4. It is revealed in general that the introduction of oblique angles capable to reduce the SIFs. However, increasing the mode II stress intensity factors. Figure-4 also indicates that slightly increased the slanted angles from 0° to 15° produced the stress intensity factors almost similar with the normal cracks. This is indicated that the current model is verified and capable to produce accurate results. It is observed that the stress intensity factors increased as the ratio a/L increased [11-12].

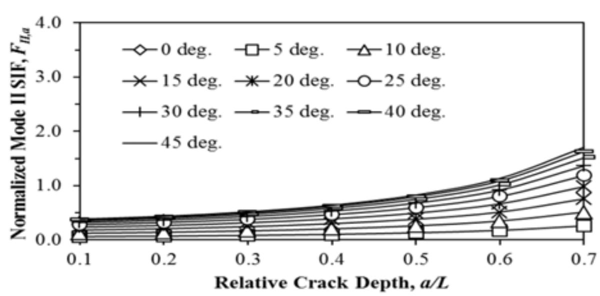

Figure-5. The effect a/L on mode I stress intensity factors.

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Table-1. Mode I normalized stress intensity factors of slanted cracks.

θ	a/L						
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**
0	1.189	1.367	1.656	2.111	2.824	4.031	6.352
5	1.182	1.360	1.652	2.102	2.814	4.019	6.336
10	1.163	1.338	1.626	2.070	2.772	3.962	6.250
15	1.132	1.303	1.583	2.017	2.703	3.867	6.108
20	1.089	1.254	1.525	1.944	2.608	3.738	5.913
25	1.035	1.193	1.452	1.853	2.491	3.576	5.669
30	0.972	1.121	1.366	1.747	2.352	3.385	5.380
35	0.900	1.040	1.269	1.627	2.196	3.169	5.051
40	0.821	0.951	1.164	1.495	2.024	2.930	4.686
45	0.737	0.857	1.052	1.355	1.840	2.673	4.290

Table-2. Mode II normalized stress intensity factors of slanted cracks.

θ	a/L						
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**
0	0.063	0.072	0.085	0.104	0.132	0.177	0.260
5	0.124	0.141	0.167	0.205	0.261	0.350	0.514
10	0.182	0.207	0.245	0.301	0.383	0.514	0.756
15	0.235	0.267	0.317	0.389	0.495	0.666	0.983
20	0.282	0.320	0.380	0.466	0.595	0.802	1.187
25	0.321	0.365	0.433	0.531	0.679	0.919	1.364
30	0.352	0.399	0.474	0.583	0.747	1.015	1.512
35	0.373	0.423	0.502	0.619	0.797	1.087	1.628
40	0.384	0.435	0.518	0.640	0.827	1.134	1.708
45	0.063	0.072	0.085	0.104	0.132	0.177	0.260

The determinations of both modes I and II stress intensity factors are difficult, especially when calculating the SIFs values between two angles for an example. Therefore, it is an important thing to formulate a mathematical expression to represent the stress intensity factors tabulated in Table-1 and Table-2. Then, the regression method is used to establish the mathematical model. Based on the simple analysis, thirteen parameters or variables is used to results the regression statistics as listed in Table-3 using Microsoft EXCEL.

Table-3. Regression statistics for modes I and II stress intensity factors.

Modes	Multiple R	R Square	Adjusted R square	Standard error
I	0.999	0.999	0.999	0.063
II	0.999	0.998	0.998	0.018

The formulations of the stress intensity factors prediction models for both stress intensity factors can be expressed as Equations. (3) and (4):

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$$
\begin{aligned}
F_I &= 0.6111 + 7.5689\left(\frac{a}{L}\right) + 0.2638\left(\frac{2\theta}{\pi}\right) \\
&-24.075\left(\frac{a}{L}\right)^2 + 35.7494\left(\frac{a}{L}\right)^3 \\
&-4.6469\left(\frac{2\theta}{\pi}\right)^2 + 6.0033\left(\frac{2\theta}{\pi}\right)^3 \\
&-2.8684\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right) - 15.0496\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)\right)^2 \\
&-61.1228\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)\right)^3 + 26.3601\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)^2 \\
&+167.042\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)^2\right)^2 - 173.548\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)^2\right)^3 \\
&-59.1111\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)^3
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
F_{II} &= -0.1469 + 2.0482\left(\frac{a}{L}\right) + 1.1526\left(\frac{2\theta}{\pi}\right) \\
&-6.0083\left(\frac{a}{L}\right)^2 + 1.5241\left(\frac{2\theta}{\pi}\right)^2 + 4.6929\left(\frac{a}{L}\right)^3 \\
&-4.2139\left(\frac{2\theta}{\pi}\right)^3 - 6.0567\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right) \\
&-28.1157\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)\right)^2 + 19.7386\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)\right)^3 \\
&+8.9143\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)^2 + 18.2864\left(\frac{a}{L}\right)^2\left(\frac{2\theta}{\pi}\right) \\
&-173.548\left(\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)^2\right)^3 - 59.1111\left(\frac{a}{L}\right)\left(\frac{2\theta}{\pi}\right)^3
\end{aligned}
\tag{16}
$$

These expressions can be used to predict the modes I and II stress intensity factors in a reasonable accuracy where θ is slanted angles in radian. For an example, a case of 10^o slanted crack under tensile stress is used and substituted into Equation. (1) as shown in Figure-6. It is revealed that both approached are well agreed where Equation. (1) can be used to predicted the modes I and II. Since, the modelling of slanted crack is quite difficult, then it is suggested to utilize these equations in order to determine the modes I and II stress intensity factors.

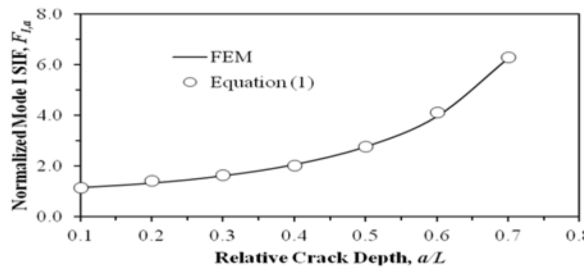

Figure-6. Comparison between two approaches.

CONCLUSIONS

Based on the investigations conducted numerically using ANSYS finite element program on the slanted crack subjected to tension stress, the behaviour of SIFs can be concluded as below:

1. As expected, higher relative crack depths produced higher SIFs for both type of stress intensity factors. However, mode I stress intensity factors increased gradually when a/L > 0.5 compared with mode II stress intensity factors.
2. The introductions of slanted cracks have affected both SIFs inversely where the mode I stress intensity factors reduced and on the other hand increased the mode II stress intensity factors.
3. The regression technique is used to formulate mathematical expressions for modes I and II. They are capable to calculate the stress intensity factors with reasonable accuracies.

REFERENCES

[1] A. E. Ismail, A. K. Ariffin, S. Abdullah and M. J. Ghazali. 2012. Off-set crack propagation analysis under mixed mode loadings. International Journal of Automotive Technology. 12(2): 225-232.

[2] J. C. Newman, M. A. Jamesand U. Zerbst. 2003. A review of the CTOA/CTOD fracture criterion. Engineering Fracture Mechanics. 70(3): 371-385.

[3] J. Albinmousa, N. Merah and S. M. A. Khan.2011. A model for calculating geometrical factors for a mixed-mode I-II single edge notched tension specimen. Engineering Fracture Mechanics. 78(18): 3300-3307.

[4] T. Matsumto, M. Tanaka and R. Obara. 2000. Computation of stress intensity factors of interface cracks based on interaction energy release rates and BEM sensitivity analysis. Engineering Fracture Mechanics. 65(6): 683-702.

[5] J. H. Kuangand L. S. Chen. 1993. A displacement extrapolation method for two dimensional mixed mode crack problems. Engineering Fracture Mechanics. 46(5): 735-741.

ARPN Journal of Engineering and Applied Sciences

[6] Ismail A.E., Mohd T. A.L. and Mohd N. N.H. 2015. Stress intensity factors of slanted cracks in round bars subjected to mode I tension loading. In: AIP Conference Proceedings. 1660(1): 1-6.

[7] S. Courtin, C. Gardin, G. Benzine and H. B. H. Hamouda. 2005. Advantages of the J-integral approach for calculating stress intensity factors when using the commercial finite element software ABAQUS. Engineering Fracture Mechanics. 72(14): 2174-2185.

[8] C. F. Shih,B. Moran and T. Nakamura. 1986. Energy release rate along a three-dimensional crack front in a thermally stressed body. International Journal of Fracture. 30(2): 79-102.

[9] M. C. Walters, G. H. Pauline and R. H. Dodds. 2005. Interaction integral procedures for 3D curved cracks including surface tractions. Engineering Fracture Mechanics. 72(11): 1635-1663.

[10] X. Lan, N. A. Noda, Y. Zhangand K. Michinaka. 2012. Single and double edge interface crack solutions for arbitrary forms of material combination. Acta Mechanica Solida Sinica. 25(4): 404-416.

[11] Ismail A.E. 2015. Stress intensity factors of eccentric cracks in bi-materials plate under mode I loading. In: AIP Conference Proceedings. 1660(1): 1-6.

[12] A. E. Ismail. 2014. Multiple crack interactions in bi-material plates under mode I tension loading. Applied Mechanics and Materials. 629: 57-61.