



# A MODEL OF CONTINUOUS LINEAR ELECTRON ACCELERATOR

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## ABSTRACT

The scale of the human economic activity and the specificity of advanced technological processes used in industry, especially in the sectors such as chemistry and metallurgy, necessitate special measures to protect the environment. The environmental protection, the careful use and reproduction of its diverse resources, improvement of the human environment is a major part of the humanity survival program in the global world. The problem of protection of the environment and rational use of natural resources becomes more and more acute and urgent every year. Currently, linear electron accelerators operating in pulsed mode are used for practical purposes. Such accelerators have a small beam power and cannot be effectively used in industry and for the environmental protection. To overcome this drawback, one can create a continuous linear electron accelerator based on a biperiodic decelerating structure, which operates in the standing wave mode. The design of such an accelerator is simpler as compared with the resonant pulse accelerator, because the need is eliminated for a modulator of the HF energy source. The article presents the results of the conducted research and the calculations of a model of linear electron accelerator, which demonstrate the possibility of creating such a continuous accelerator for using in industry and for environmental protection

**Keywords:** electron linear accelerator, shunt resistance, biperiodic decelerating system, accelerating cell, connection cell.

## 1. INTRODUCTION

Currently, a whole series of linear electron accelerators (LEA) is created for various sectors of the economy: medicine [1], defectoscopy [2], chemical industry [3], and other industries [4]. Each of the accelerators, depending on the sphere of its applications, has its own features, but they all operate in a pulse mode and have a small beam power, so none of these accelerators can be profitably used for the environmental protection.

To overcome this shortcoming, one can create a continuous LEA based on a biperiodic decelerating structure (BDS). Such an accelerator for the energy of 2-3 MeV and the beam power of 100 kW will not exceed the following dimensions: 1m in length, 1m in height, 0.5m in width, which is much smaller than a transformer accelerator. Besides, the design of the continuous LEA is somewhat simplified compared with the resonant pulse accelerator, because the need is eliminated for a modulator of the HF energy source.

To create a continuous LEA, one needs to solve a number of typical problems as for any linear accelerator. First, the geometric dimensions of the decelerating system are determined, and the design and configuration of the LEA are performed. To determine the geometric dimensions of BDS, one should know the dependence of the basic electrodynamic characteristics of the decelerating system on its geometric dimensions.

In the studies [5], appeared earlier, in which the biperiodic structures are modeled by a chain of interconnected radio circuits, several characteristics of these systems have been identified [6]. Such representation of the biperiodic structure has a number of fundamental limitations; in particular, the use of a contour model presupposes setting the parameters of individual cells, such as the Q-factor, natural frequency, the effective

shunt resistance and the electromagnetic field distribution in them [7].

At the stage of adjustment of the models of structures, this model is the most appropriate, since all the characteristics for it are taken directly from the experiment. At the stage of design and the choice of geometry of the structure, the model of interconnected circuits is inferior compared with the model of interconnected resonators, since the latter model allows linking the main electrodynamic characteristics of the decelerating system with its geometrical dimensions. It is advisable to present the electrodynamic characteristics in a parameterized form that allows using the obtained data in any frequency range.

## 2. METHODS

Consider a resonator of the volume  $V$ , bounded by a metal closed surface  $S$ . In the surface  $S$ , the connecting holes with other resonators or supply waveguide channel are cut. The inner surface of the resonator may be quite complex, and the exact solving of Maxwell's equations is not possible.

We represent the sought-for solution as a sum of orthogonal oscillations and consider the problem of determining the electromagnetic fields that are excited in the resonator at a given frequency  $\omega$ . We restrict ourselves to the solenoidal component of the solutions of Maxwell's equations, which is generally regarded as the "radiation field."

Let us choose a system of solenoidal functions  $(\mathbf{E}_m, \mathbf{H}_m)$ , where  $m = 1, 2, \dots$ , with respect to which we decompose the desired  $\mathbf{E}$  and  $\mathbf{H}$  fields in the resonator. This system of functions satisfies the wave equation and the homogeneous boundary conditions. Thus, the system of solenoidal functions  $(\mathbf{E}_m, \mathbf{H}_m)$  is a solution of Maxwell's equations in the resonator, which surface is an



ideal conductor. This enables us to claim that, under the made assumptions, the selected system forms a complete orthonormal system of functions [13]:

$$\int_V \mathbf{E}_k \mathbf{E}_m dV = \int_V \mathbf{H}_k \mathbf{H}_m dV = \delta_{km} = \begin{cases} 0, k \neq m \\ 1, k = m \end{cases} \quad (1)$$

In this case, one can obtain a system of equations for the desired electric field of the considered resonator:

$$\begin{aligned} \frac{d^2}{dt^2} \int_V \mathbf{E} \mathbf{E}_m dV + \omega_m^2 \int_V \mathbf{E} \mathbf{E}_m dV \\ = -\frac{\omega_m}{\sqrt{\varepsilon\mu}} \int_S [\mathbf{nE}] \mathbf{H}_m dS - \frac{1}{\varepsilon} \frac{d}{dt} \int_V \mathbf{J} \mathbf{E}_m dV - \frac{\omega_m}{\sqrt{\varepsilon\mu}} \int_{S_{hc}} [\mathbf{nE}] \mathbf{H}_m dS. \end{aligned} \quad (2)$$

Here,  $\mathbf{J}$  is the current density vector at an arbitrary point of the resonator volume;  $\mathbf{E}_m, \mathbf{H}_m, \omega_m$  are the eigen functions and natural frequencies of the resonator;  $t$  is time;  $S_{hc}$  is the total surface of the

connection holes of the resonator;  $\varepsilon, \mu$  are the absolute dielectric and magnetic permeability.

The obtained system (2) can be generalized to the case of a chain of interconnected resonators forming the BDS; in this case, the corresponding quantities will get the index  $n$ , where  $n$  is the number of the resonator:

$$\frac{d^2}{dt^2} \int_{V_n} \mathbf{E}_n \mathbf{E}_{nm} dV + \omega_{nm}^2 \int_{V_n} \mathbf{E}_n \mathbf{E}_{nm} dV = -\frac{\omega_{nm}}{\sqrt{\varepsilon\mu}} \int_S [\mathbf{nE}_n] \mathbf{H}_{nm} dS - \frac{1}{\varepsilon} \frac{d}{dt} \int_{V_n} \mathbf{J}_n \mathbf{E}_{nm} dV - \frac{\omega_{nm}}{\sqrt{\varepsilon\mu}} \int_{S_{hc}} [\mathbf{nE}_n] \mathbf{H}_{nm} dS. \quad (3)$$

The order of the system is equal to the product of the number of resonators in the chain and the number of the considered functions of the system ( $\mathbf{E}_m, \mathbf{H}_m$ ) in the expansion of the fields  $\mathbf{E}, \mathbf{H}$ .

The system of equations (3) is the desired one, and solving this system allows determining electrodynamic characteristics of the decelerating structure in the general case: the dispersion, the distribution of fields with respect to structure, as well as the input impedance, the reflection coefficient, etc.

If we assume that the decelerating structure is not connected to the supply waveguide channel and does not contain the particles being accelerated, then in this case the second term in the right-hand side of equation (3), reflecting the interaction of the beam of accelerated particles with the field of the resonator, is equal to zero, whereas the value  $S_{hc}$  is just the surface of the holes of the resonators connection between one another. In this case, the fields in the  $n$ -resonator of the decelerating structure can be represented as:

$$\mathbf{E}_n(\mathbf{r}, t) = \text{Im}[\mathbf{E}_n(\mathbf{r})e^{-i\omega t}] = \sum_{m=1}^{\infty} \text{Im}[V_{nm} \mathbf{E}_{nm}(\mathbf{r})e^{-i\omega t}], \quad (4)$$

$$\mathbf{H}_n(\mathbf{r}, t) = \text{Im}[\mathbf{H}_n(\mathbf{r})e^{-i\omega t}] = \sum_{m=1}^{\infty} \text{Im}[I_{nm} \mathbf{H}_{nm}(\mathbf{r})e^{-i\omega t}], \quad (5)$$

where  $V_{nm}, I_{nm}$  are unknown amplitude coefficients of the expansion of the fields  $\mathbf{E}$  and  $\mathbf{H}$ ;  $\mathbf{r}$  is the radius vector of the considered point in the  $n$ -resonator.

In view of (1), (4) and (5), the left-hand side of equation (3) can be written as:

$$\frac{d^2}{dt^2} \int_{V_n} \mathbf{E}_n \mathbf{E}_{nm} dV + \omega_{nm}^2 \int_{V_n} \mathbf{E}_n \mathbf{E}_{nm} dV = \text{Im}[(\omega_{nm}^2 - \omega^2)(V_{nm} e^{-i\omega t})]. \quad (6)$$

The first term in the right-hand side of equation (3) reflects the losses in the resonator walls and can be expressed in terms of its own quality factor  $Q_{nm}$

$$\frac{\omega_{nm}}{\sqrt{\varepsilon\mu}} \int_{S_n} [\mathbf{E}_n(\mathbf{r}) \mathbf{H}_{nm}] \mathbf{n} dS = \omega^2(i-1) \frac{1}{Q_{nm}} V_{nm}. \quad (7)$$

The integrals over the surface  $S_{hc}$  of the connection holes in the equation (3) reflect the excitation of the  $n$ -resonator by the electromagnetic field on the connection windows with neighboring resonators or the

supply waveguide channel if it is brought to the  $n$ -cell. In this case, the electrical connection between the resonators can be neglected, and the integrals over the surface of the connection windows can be represented as follows:

$$\int_{S_{hc}} [\mathbf{nE}_n] \mathbf{H}_{nm} dS = \int_{S_n} [\mathbf{E}_{tgn} \mathbf{H}_{nm}] \mathbf{n} dS. \quad (8)$$

Substituting (6), (7) and (8) to (3), we obtain a system of equations for the coefficients  $V_{nm}$



$$(\omega_{nm}^2 - \omega^2)V_{nm} = \omega^2(1 - i)\frac{1}{Q_{nm}}V_{nm} - \frac{\omega_{nm}}{\sqrt{\epsilon\mu}} \int_{S_n} [E_{tgn}H_{nm}] ndS. \quad (9)$$

In solving the system (9), it is assumed that the eigen parameters of the resonators such as the eigen frequencies  $\omega_n$ , the eigen Q-factors  $Q_n$ , the eigen electric  $E_n$  and magnetic fields  $H_n$  are known. To determine them, we consider an axially symmetric resonator with perfectly conducting surface.

Since the solution of this problem is reduced to finding not only the eigen functions but the eigen value, we use a double iterative process: a certain initial approximation for the eigen value is set and, using the method of successive over-relaxation, the distribution of the eigen-function in the volume of the resonator is determined. Then the new eigenvalue is calculated, and the procedure is continued until a given accuracy is achieved. In [8] it is shown that the process converges and gives the lowest Eigen value.

### 3. DISCUSSION AND RESULTS

After determining the value of the wave number and the distribution of the magnetic field over the volume of the cell, it is possible to calculate the electromagnetic characteristics of accelerating cavities by direct integration. In particular, the power of losses is given by the relation

$$P = \frac{1}{2} R_s \iint_S H_\phi^2 ds \approx 2\pi R_s h \sum_{i=1}^n \frac{F_i^2}{\rho_i},$$

where  $R_s$  is the surface resistance of copper;  $\rho_i$  is the radius by which the considered point is spaced from the axis. The summation is performed over all the boundary points.

An important parameter of the accelerating structures operating in the standing wave mode, which characterizes their sensitivity to perturbing factors, is the dispersion characteristic. The dispersion characteristic corresponds to the fundamental harmonic of BDS and is determined as the phase shift of the electromagnetic field oscillations by the element of the structure periodicity

$$\varphi = \frac{2L\pi}{\lambda\beta},$$

where  $L$  is the length of the accelerating structure,  $\lambda$  is the wavelength,  $\beta$  is the change rate of the phase of the high-frequency field in the resonator in the units of the speed of light.

To find the dispersion characteristic, one needs to use the values of Q-factor and the magnetic field on the level of connection gap and equate to zero the determinant of the resulting system [9]. The real parts of the roots will correspond to the frequencies of the oscillation types 0 and  $\pi$ .

The BDS of the accelerator operating in the standing wave mode is a chain of interconnected resonators, which can be excited only at certain frequencies. The dependence of the excitation frequency

values on the phase shift by the periodicity element is called the dispersion dependence, whereas its graphical representation is called the dispersion curve. The dispersion curve of the accelerating section operating on the basis of standing wave is a series of points corresponding to the resonant frequencies of the decelerating system, equal to the number of resonators in the decelerating structure. Figure-1 shows the dispersion curve of the BDS of a continuous LEA model.

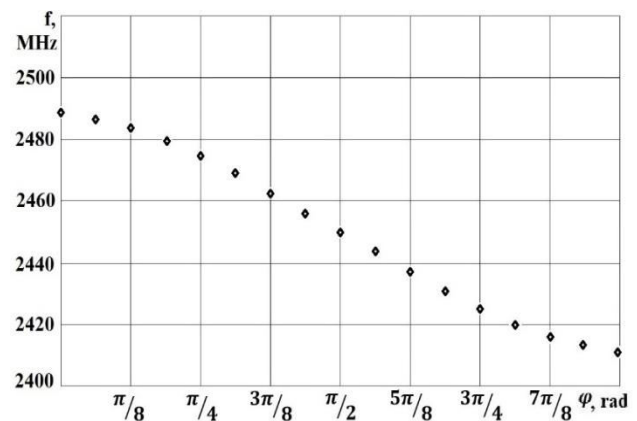
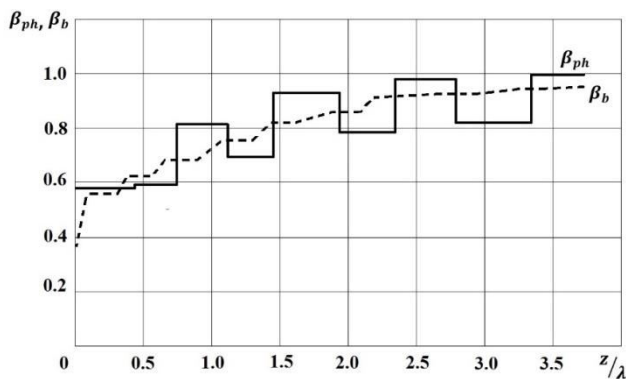


Figure-1. The dispersion curve of the BDS of a continuous LEA model.

It is shown theoretically and experimentally [10] that the farther is the working mode of oscillations from the neighboring oscillation modes, the smaller is the sensitivity of BDS to the current load and other types of detuning. Therefore, to determine the BDS sensitivity to perturbing factors, one needs to know the dispersion relation.

In the general case, the dispersion curve is split into two branches, which are separated by the stop band [11]. The presence of the stop band leads to uneven distribution of the electric field in the accelerating cells and reduces the effective shunt resistance due to the appearance of electric field in the connection cells.

The process of the BDS tuning is to eliminate the stop band by way of changing some sizes of cells and satisfying the condition  $\omega_g = \omega_{\pi/2}$ , where  $\omega_g$  is the frequency of the supply generator,  $\omega_{\pi/2}$  is the frequency of the  $\pi/2$ -type of oscillations. Figure-2 shows the dependence of the relative phase velocity  $\beta_{ph}$  and the relative velocity  $\beta_b$  of bunches on the longitudinal coordinate  $z/\lambda$ .



**Figure-2.** The dependence of the relative phase velocity  $\beta_{ph}$  and the relative velocity  $\beta_b$  of bunches on the longitudinal coordinate  $z/\lambda$ .

As a result of the performed calculations, we have succeeded in obtaining a family of parametric graphs and tables for the basic electrodynamic characteristics of the accelerating cells of BDS, using which it is possible to assess the effectiveness of this or that construction of the decelerating system.

All the main electrodynamic characteristics and geometric dimensions are normalized by the wavelength, which allows constructing, using these relations, the graphs and tables that can be used for any frequency range. These relationships allow calculating the basic electrodynamic characteristics of BDS, the distribution of the electromagnetic field along the length of the decelerating system depending on the input power and taking into account the effect of the geometry of the connection gap on the dispersion characteristic of BDS.

The obtained results can be used for engineering calculation of BDS and its tuning to the operating frequency.

#### 4. CONCLUSIONS

In conclusion, we can make some general comments about the approach to the choice of the dimensions of the accelerating cells and the BDS connection cells: the frequency of the supply generator is usually given; the shape of the peripheral border of the accelerating cells is determined by the adopted manufacturing technology; the shapes and dimensions of connection cells are determined depending on the requirements for the value of the necessary effective shunt resistance, the longitudinal and transverse dimensions of the BDS, and the conditions of heat removal and the design of the cooling system and the output parameters of the selected HF energy source.

The period of the BDS structure is defined as follows: if the connection cells are taken out in the design from the axis of the decelerating system, then the following relation holds for the length of the accelerating cell  $L_{ac} = \frac{\beta_f \lambda_v}{4}$ ; if the connection cells are located on the axis of the decelerating system, then the BDS period equals  $L_{bs} = L_{ac} + L_{cc}$ .

One of the most important dimensions of the accelerating cells, which determines the efficiency of the accelerating structure as a whole, is the parameter  $\frac{a}{L_{ac}}$ , where  $a$  is the radius of the drift channel opening. This value is selected from the condition of obtaining the maximum shunt resistance under the given phase velocity of the accelerating wave. No less important quantity, the radius of the drift channel opening, is determined by the longitudinal and radial dynamics of electrons. The value of  $a$  should be as small as possible, since the value of the effective shunt resistance greatly increases with the decreasing of  $\frac{a}{\lambda_v}$ .

The thickness of the drift tube is chosen taking into account several factors, since with the increase in the tube thickness, the electric strength of the resonator increases, and the value of the effective shunt resistance decreases. Therefore, in the BDS design one should always be aware of the possibility of breakdown in the accelerating cell. The thickness of the wall between the resonators is selected from the condition of heat removal, a given connection coefficient and the tuning method.

The studies demonstrated the possibility of creating a continuous LEA with the following characteristics: the length of the decelerating structure is  $\sim 0.5$  m, the energy of the accelerated electrons is  $\sim 0.5-0.8$  MeV; the current of the accelerated electrons is  $\sim 20$  mA; the operating frequency of the generator is  $\sim 2450$  MHz; the power of the HF generator is  $\sim 40$  kW; the injection voltage is  $\sim 20$  kV.

The authors will continue to study the optimization of the thermal regime in the continuous accelerators and the opportunities for grouping and focusing of the accelerated electrons by the high-frequency fields of BDS with a standing wave [12,13].

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