OPTIMIZATION OF MAIN PARAMETERS OF GEARBOXES WITH THREE GEAR TRAINS

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ABSTRACT

Efficiency of GB design can be conveniently estimated by three relative parameters on the basis of \( n \) – the number of forward gears: 1) Coefficient of gear usage intensity equaling to ratio of the number of forward gears to the number of involved gearwheels \( = n/g \); 2) Coefficient of reduction equaling to ratio of gear numbers to total reduction \( Kb = n/\sum R \); \( \sum R \) is the total reduction of GB, it is determined using ray path plot and equals to the sum of ray projections of gear couples onto the axis of gear ratios on logarithmic scale: the sum of steps \( q \); 3) Coefficient of layout efficiency equals to the sum of the two aforementioned coefficients \( Kc = Ka + Kb \). This article discusses optimization of main parameters of GB (gearboxes) with three trains of forward gears in the case of conventional layout and in the case of loose placement of gears on shafts.

Keywords: shafts, transmissions, gearwheels, gear numbers, range, intervals.

1. INTRODUCTION

GB (gearbox) is the main unit of transmission; its parameters influence significantly operation properties of land vehicles.

The main parameters of GB are as follows: the number of shafts, \( n \) - the number of gears, \( U\text{n} \) - gear ratios, and \( U \) - gear couples, \( D \) - GB range, the ratio of low gear to high gear, \( q \) - steps (intervals), the ratios of adjacent gears, dimensions and metal intensity, \( A \) - distance between axes specifying transversal size, \( g \) - the number of gears specifying longitudinal size of GB.

2. CONVENTIONAL GEARBOXES

Figure-1 illustrates cross section of three-step three-shaft conventional GB, its kinematic diagram and ray path plot.

The idler shaft is driven by the first gear couples which is formed by driver gear of input shaft with the teeth number \( Z_{dr} = 15 \) and driven gear on the left side of gear train of idler shaft \( Z_{dn} = 29 \). The gear ratio is \( U_{1} = Z_{dn}/Z_{dr} = 29/15 = 1.933 \); \( \lg U_{1} = 0.286 \). The second gear couple of permanent mesh for the second gear: driver gear on idler shaft \( Z_{dr} = 24 \) and driven gear on secondary shaft \( Z_{dn} = 22 \); \( U_{2} = 22/24 = 0.917 \); \( \lg U_{2} = -0.038 \); it operates in increasing mode. The third gear couple for the first gear: driver gear on idler shaft \( Z_{dr} = 14 \) and driven gear–carriage on secondary shaft in the left-side position \( Z_{dn} = 29 \); \( U_{3} = 29/18 = 1.61 \); \( \lg U_{3} = 0.207 \). Reversing \( R \); driver shaft on idler shaft at the right \( Z_{dr} = 15 \) and driven gear–carriage on secondary shaft in the right position \( Z_{dn} = 29 \); idle reverse gear \( Z_{dr} = 18 \) is not shown; \( U_{R} = 29/15 = 1.933 \); \( \lg U_{R} = 0.286 \).

The first gear is formed by the first and the third gear couples: \( U_{1gc} = U_{1}U_{3} = 1.933 \times 1.61 = 3.11 \); \( \lg U_{1gc} = 0.493 \). The second gear \( U_{2gc} = U_{1}U_{2} = 1.933 \times 0.917 = 1.77 \); \( \lg U_{2gc} = 0.249 \). The third gear is direct \( U_{3gc} = 1.0 \); \( \lg U_{3gc} = 0 \). The GB range is \( D = U_{1gc} = 3.11 \). The average interval is \( q = D^{0.5} = 3.11^{0.5} = 1.764 \); \( \lg q = 0.246 \).
For the GB illustrated in Figure-1 the coefficient of gear usage intensity is not high: \( K_a = n / g = 3/6 = 0.5 \). This drawback is stipulated by the fact that each gear couple operates only in one gear.

3. GEARBOXES WITH LOOSE PLACEMENT OF GEARS ON SHAFTS

4-step GB are widely applied in up-to-date land vehicles as basic units of multi-stage GB when they are supplemented by front-mounted engine or rear supplementary gearbox duplicating the number of gears of basic GB. It is possible to apply both approaches; in this case we obtain \( 2 \times 4 \times 2 = 16 \) gears.

In order to improve GB layout efficiency, mainly by means of gear usage intensity, it would be reasonable to apply loose placement of gears on shafts [1-3]. Modified gearboxes (MGB), based on loose placement of gears on shafts, provide significant increase of the aforementioned estimates. For instance, for 4-step MGB \( K_a = 4/6 = 0.67 \); for 8-step \( K_a = 8/8 = 1.0 \); for 16-step \( K_a = 16/10 = 1.6 \); and so on. Reduction coefficient can vary in wide range depending on arrangement of operation of gears and clutches. For instance, it can reach \( K_b = 4/2 = 2.0 \) for 4-step MGB; 8-step MGB can provide \( K_b = 8/5 = 1.6 \); and for 16-step MGB \( K_b = 16/10 = 1.6 \).

The number of layout variants and arrangement of operation of MGB gears equals to the product of the number of gears and the number of clutch states. The following is possible for 4-step MGB: \( 4 \times 2 = 8 \) variants of layout, for 8-step \( 8 \times 3 \times 2 = 48 \) variants, for 16-step MGB \( 16 \times 4 \times 3 \times 2 = 384 \) variants, for 32-step MGB \( 32 \times 5 \times 4 \times 3 \times 2 = 3840 \) variants; and so on.

It is necessary to determine approach to selection of optimum variant. Let us attempt to select optimum variant for simple case - 4-step MGB.

Figure-2 illustrates kinematic diagrams of 4-step MGB and fragment of ray path plots: on the left - two-shaft non-coaxial; on the right - three-shaft coaxial. The gear couples (1, 2, 3), positioned loosely on shafts, and two shifting clutches (A, B) provide four forward gears. Some gears can be combined into trains as in Figure-2, b. If reverse gears are positioned according to the layout in Fig., we obtain the number of reverse gears by two times less than the number of forward gears.

![Figure-2. 4-step modified GB: on the left - two-shaft non-coaxial; on the right - three-shaft coaxial; a,b -kinematic diagrams; c,e,g,i,g,f,h,j- fragment of ray path diagrams; 1, 2, 3 - couples of forward gears; R- train of reverse gears; A, B- shifting clutches.](image-url)
4. RAY PATH PLOTS

Ray path plots visually illustrate the GB layout and operation: gear ratios, operation sequence of gear couples and shifting clutches. Above upper horizontal line the forward gears 1-4 are highlighted, below lower horizontal line - reverse gears 1R, 2R. The input torque from primary shaft is indicated by 0: on the left - on middle line, on the right - on upper line.

The plot is arranged on logarithmic scale as a function of gear ratios in horizontal direction, herewith, the ray value and its slope for each gear couple are constant at each fragment of the plot. The numbers of gear couples are shown on the rays. The ray right-hand slope of ray determines reducing operation mode. The flatter is the ray the higher is the gear ratio, for instance, in Figure-2, \( U_3 > U_1 \). The vertical ray corresponds to gear ratio equaling to unity, \( U_2 = 1, \lg U_2 = 0 \). The ray left-hand slope of determines increasing operation mode of gear couples, for instance, the left-side ray 2 in Figure-2,e.g. Gear ratios in \( \lg q \) are shown in parentheses and under the parentheses cumulative reductions are shown, for instance, in Figure-2, c (1, 0, 2), \( P_c = 3 \).

For two-shaft MGB the top 4-th gear is direct for the variant c - vertical ray 2, increasing for variants e, g - ray 2 is inclined to the left, i - ray 1. The 3-rd gear: ray 1 of variants c, e; ray 3 - variant g, and ray 2 - variant i. The 2-nd gear – ray 3 of variants c, e, i,ray 1 of variant g.

An important advantage of MGB is multi-pair contact in low gears, enabling moderate gear ratios in gear couples with significant MGB range while reducing distance between axes and MGB dimensions. For instance, the first gear of variants c, e, g is formed by three consecutive rays: 1, 2 and 3. In the first gear the clutch A is located in left-side position (L) and the clutch B in the right-side position (R). The torque of primary shaft is transferred by the clutch A to the first gear couple, then by gear train of secondary shaft, then by the second gear couple to the gear train of primary shaft, and the third gear couple and the clutch B to the secondary shaft.

Figure-3 illustrates eight ray path plots of 4-step MGB, corresponding to Figure-2, a, b; common both for two-shaft and for three-shaft layouts. Table of states of shifting clutches is shown above: L – left-side position, R - right-side position. In the left column the clutch A operates in the mode L,R,L,R or R,L,R,L, and the clutch B - L,L,R,R, or R,R,L,L. And vice versa in the right column: the clutch B in the mode L,R,L,R or R,L,R,L, and the clutch A - L,L,R,R or R,R,L,L.

Conventional symbols are listed above the tables which estimate MGB and make it possible to arrange ray path plot. For instance, at the upper left 4a) 2,0. \( AB \). 3: 4,3,1; 1+0+2=3; at the upper right 4b) 2,0. \( BA \). 2: 4,2,1; 2+0+1=3. The letters "a" and "b" denote variants of operation modes of shifting clutches. The numbers denote the value of coefficient of layout efficiency \( K_c = 4/6+4/3 = 2.0 \). The letters \( AB \) and \( BA \) characterize the operation sequence of shifting clutches. The first clutch is activated at each shifting and the second every other shifting.
The number after letters indicates the gear against which the common point is located, highlighted by circle on the lower horizontal line. Initial rays originate from the common point highlighted by bold lines, the number of gear couple is indicated on them. These rays are directed to the numbers of gears on the upper horizontal line: ray 1 to point 4, ray 2 to point 3, and ray 3 to point 1. Termination of ray 1 always corresponds to the clutch state L, L; ray 2 - R, L; ray 3 - R, R. Operation of MGB with three activated gear couples (consecutively rays 1, 2, 3) is provided at the clutch state L, R. Summation at the end of conventional symbol illustrates reduction of each gear couple and total reduction indicated under the lower horizontal line for various positions of common point. The sum of minimum values of total reduction $P_c = 3 + 2 + 2 + 3 = 10$ is the same for both columns. The value of steps between gear couples is interrelated with operation sequence of shifting gears - one step, if the clutch operates upon each shifting, two steps ($2q$) if the clutch operates every other shifting.

If we assume the range $D = 4$, then $\log D = 0.6$; the step between gears in logarithmic form is $\log q = 0.6/3 = 0.2$ and in natural form $q = 1.59$. In comparison with conventional GB illustrated in Figure-1, MGB on the basis of three gear couples and increased range from 3.11 to 4 provides better performances: step reduction of from 1.764 to 1.59; which simplifies conditions of gear shifting; gear usage intensity increases from 0.5 to 0.67.

For the variant 4a initial rays, characterizing gear ratios of gear couples and highlighted by bold lines in the diagram, originates from common point positioned on lower horizontal line at the level of the third gear. Ray 1 is double-sided - for three-shaft MGB it is directed downward to the right. For two-shaft MGB the ray of gear couple 1 is directed upward to the left to point 4 (4-th gear) and determines increasing operation mode of
this gear couple with the value of one step \( g = U_{1gc} = U_1 = 1/1.59 = 0.629 \). Ray 2 is directed vertically upward to point 3 (3-rd gear) and determines operation mode with gear ratio \( U_{2gc} = U_2 = 1.0 \). Ray 3 is directed upward to the right to point 1 (1-st gear) and determines decreasing operation mode of this gear couples with the value of two steps \( 2g = U_{3gc} = U_3 = 1.59^2 = 2.528 \). Total reduction in this case is \( P_2 = 1+0+2 = 3 \) (for the sake of simplicity the signs of \( q \) are not mentioned), coefficient of reduction \( Kb = 4/3 = 1.33 \); coefficient of layout efficiency \( Kc = Ka+Kb = 0.67+1.33 = 2.0 \). The 2-nd gear is provided upon operation of all three gear couples - \( U_{2gc} = U_1U_2U_3 = 0.629\times1.0\times2.528=1.59 \). Let us check the MGB range \( D = U_{1bg}/U_{4gc} = 2.528/0.629 = 4 \).

For three-shaft MGB the 4-th gear is direct, the 3-rd gear is provided by the first and the second gear couples - \( U_{3gc} = U_1U_2 = 1.59 \); the 2-nd gear– the second and the third gear couples - \( U_{2gc} = U_2U_3 = 2.528 \); the 1-st gear – the first and the third gear couples - \( U_{1gc} = U_1U_3 = 1.59\times2.528 = 4.0 \).

5. OPTIMIZATION OF MAIN PARAMETERS OF GEARBOXES

Displacement of ray output point influences significantly on total reduction \( P_2 \), it is shown below the bottom horizontal line, and on coefficients of reduction and layout efficiency (Figure-4). For instance, if for the variant 4a with total reduction \( P_2 = 1+0+2 = 3 \) this point is displaced by one step to the left, then we have \( P_2 = 0+1+3 = 4 \), \( Kc = 0.67+4/4 = 1.67 \). Herewith, we obtain the gear ratios: \( U_{4gc} = U_1 = 1.0 \); \( U_{3gc} = U_2 = 1.59 \); \( U_{2gc} = U_1U_2U_3 = 1.0\times(1/1.59)\times4.0 = 2.528 \); \( U_{1gc} = U_1 = 1.59^2 = 4 \). Upon displacement by two steps to the left we obtain the top reducing gear in one step, total reduction \( P_2 = 1+2+4 = 7 \). \( Kc = 0.67+4/7 = 1.24 \). Herewith, we obtain the following gear ratios: \( U_{4gc} = U_1 = 1.59 \); \( U_{3gc} = U_2 = 2.528 \); \( U_{2gc} = U_1U_2U_3 = 1.59\times(1/2.528)\times6.39 = 4.0 \); \( U_{1gc} = U_1 = 1.59^2 = 6.39 \). The gear ratio \( U_5 = 6.39 \) is high, generally maximum gear ratio in one shear couple is limited by 4. It is required to reduce the step Ig \( q = (lg4)/4 = 0.602/4 = 0.15 \); \( q = 1.414 \); which will result in the following parameters: \( U_{4gc} = U_1 = 1.414 \); \( U_{3gc} = U_2 = 2.0 \); \( U_{2gc} = U_1U_2U_3 = 1.413\times(1/2.0)\times4.0 = 2.83 \); \( U_{1gc} = U_1 = 1.413^2 = 4.0 \) and significantly reduce the MGB range \( D = 4.0/1.414 = 2.83 \). Figure-4 combines the results of Figures 2 and 3. The numbers of gears from upper horizontal line of Figure-3 are indicated on horizontal axis; total reduction from lower horizontal line in Figure-3 are indicated on left vertical axis; generalized estimates are shown on right vertical axis.

For instance, variant 3a in Figure-4 is an analog of the variant in Figure-2 upon displacement of common point by one step. This value is located at the cross point of horizontal axis 4 and left vertical axis 3, which gives \( Kb = 4/3 = 1.33 \) and \( Kc = 0.67 +1.33 = 2.0 \).

\[ \text{Figure 4. Main performances of modified GB as a function parameters of common point position: in horizontal position - number of gear with regard to which the common point is placed; } P_2 \text{ is the total reduction; } Kb \text{ is the coefficient of reduction efficiency; } Kc \text{ is the coefficient of layout efficiency.} \]

While analyzing the data in Figures 3 and 4, it is possible to substantiate an optimum variant of MGB. The variants of MGB, where initial ray approaches low gear, are as follows: 4a, 3b, 2a, 1a, b; they exclude advantage of multi-pair contacts. Maximum value \( Kc = 2.67 \) was obtained for the variants 3a, 3b and 2a, 2b; upon three-shaft layout one increasing gear is provided by the variants 3a (rays 1,2), 3 b (rays 2,3), and the variants 2 a, 2b- two gears (ray 1 in point 2), but the use of top increasing gears in GB leads to increase in gear ratio, dimensions and metal intensity of main gear. Fir three-shaft GB is would be reasonable to apply top direct gear (\( U=1.0 \)) with high efficiency - variants 4a, 4b; in this case we have \( Kc = 2.0 \).

GB design efficiency strongly depends on conditions of gear shifting. It is required to provide low step between top gear couples since they are used more frequently. This requirement is met by the variants 4a, 3a, as well as all variants b. Besides, it is necessary to provide high efficiency for the most frequently used gears, this requirement is met by the variants 3a and 2b, where three top gears are implemented either by one-pair contact in the case of two-shaft MGB or by direct gear and two-pair contacts in the case of three-shaft MGB.
6. TECHNICAL AND ECONOMICAL ASSESSMENT OF OPTIMIZATION OF MAIN PARAMETERS OF GEARBOXES

The proposed method makes it possible to justify optimum variant of GB with minimum dimensions and weight. For instance, for the variant 3a at the range of $D = 6$, $\lg D = 0.778$; we obtain the step between gears in logarithmic form $\lg q = 0.778/3 = 0.259$ and in natural form $q = 1.817$. For 2-shaft modified GB: $U_{1gc} = U_1 U_2 U_3 = 1.0 \times 1.817 \times 1.817 = 3.3$; $U_{2gc} = U_3 = 1.817$; $U_{3gc} = U_1 = 1.0$; $U_{4gc} = 1/U_2 = 1/1.817 = 0.55$. It is preferred to have $U_1 \approx 1.0$.

For 3-shaft modified GB: $U_{1gc} = U_2 U_3 = 1.817 \times 1.817 = 3.3$; $U_{2gc} = U_3 = 1.0 \times 1.817$; $U_{3gc} = 1.0$ – direct gear; $U_{4gc} = U_1 (1/U_2) = 1.0 \times (1/1.817) = 0.55$. $D = U_{1gc} / U_{4gc} = 3.3/0.5 = 1.817^3 = 6$.

7. CONCLUSIONS

The optimum variant of 4-step modified GB is 3a, which meets the requirement of minimum weight, high efficiency, and convenience of gear shifts. Its identification code is: 2, 6, 7. AB. 3: 3, 4, 2; 0+1+1=2.

The proposed approach can be used upon selection of optimum variants of 6-, 8-, 12-, 16-step and higher modified GB, aimed for development of compact units with low metal intensity with Russian patent pending.

REFERENCES

