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VERIFICATION OF MATHEMATICAL MODEL OF A CRACKED CANTILEVER BEAM TO U-SHAPE CRACKS

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ABSTRACT

Structural defects may be inborn in the materials or they may develop in the materials during service period. The magnitude of defects in materials increases with service time; hence it leads to catastrophic failure. Various kinds of cracks are existing in the materials in service, i.e. V-shape, Rectangular shape and U-shape cracks. So classification, investigation and detection of cracks are of great importance in the regard of structural health monitoring. Previously, mathematical model was developed by W.M. Ostachowicz and M. Krawczuk for the cantilever beam which has two open single-sided V-shape cracks. This model is used to find the natural frequency of vibration in bending mode. The objective of this study is to verify whether the developed mathematical model can be used for U-shape cracks. Therefore, a result obtained for V-shape cracked cases is used as a reference model. Simulation is done by ANSYS software to get the fundamental natural frequencies for the different cracked cases considered by W.M. Ostachowicz and M. Krawczuk. After that the same mathematical model is used to calculate the characteristics roots of different cracked cases of a cantilever beam by keeping the similar material and geometrical properties. Through this study, it is found that the value of characteristics roots obtained for a V-shape cracked cases by W.M. Ostachowicz and M.Krawczuk and a U- shape cracked cases studied in this work has shown good agreement. So in this study, the mathematical model of W.M. Ostachowicz and M. Krawczuk of V-shape crack can be used to cantilever beam which has two open single-sided U-shape cracks. Through numerical study, close results of characteristics roots are obtained for most of the U-shape and V-shape cracked cases, and it is revealed that results are not much sensitive to the change in crack shape geometry. For various cracked cases, the effects of two open single-sided U-shape cracks on the characteristics roots are investigated as well.

Keywords: mathematical model, V-shape crack, natural frequency, ANSYS, characteristics roots, U-shape crack.

1. INTRODUCTION

Dynamic individuality of cracked and intact materials is dissimilar; it means that the reliability of both the materials is not same. So, the vibration analysis of a cracked beam or shafts is one of the most severe problems in various machinery. The investigation of cracked beams for various vibration parameters are very much needed because of its practical importance. Measurement of natural frequencies, vibration modes of cracked and un-cracked beam is used as a basic criterion in crack detection. Most of the cracks may be a of fatigue type and occurs in the beam in service due to the limited fatigue strength. In the literature many studies deal with the structural safety of beams. Kocharla *et al*. [1] studied that presence of crack in a structure changes the vibration properties of the beam like natural frequency and mode shape. These change in vibration properties used in inverse problem for the crack detection. In this study, they treated the turbine blades as a cantilever beam. Vibration analysis of a cantilever beam extended successfully to develop online crack detection methodology in turbine blade. Cantilever beam is modeled with two U-notches and observed the influence of one U-notch on the other for natural frequencies and mode shapes. By using central difference approximation, the curvature mode shapes were calculated from the displacement mode shapes. The depth and location corresponding to any peak on this curve becomes possible notch parameters. Babu *et al*. [2] treated turbine blade as a cantilever beam and a shaft as a simply supported beam. Vibration analysis of a cantilever beam and simply supported beam is extended successfully to develop online crack detection methodology in turbine blade and shaft. In this study, two transverse U-notches are considered on the turbine blade and on shafts. The finite element analysis is carried on blade and on shaft to investigate the effect of one notch on other for the natural frequency and mode shape. By using a central difference approximation, curvature mode shapes were then calculated from the displacement mode shapes. In this study, the identification procedure presented is believed to provide a useful tool for detection of medium size crack in a cantilever and simply supported beam applications. Quin *et al.* [3] derived an element stiffness matrix of a beam with a crack by using integration of stress intensity factors, and then FEM model of a cracked beam is developed. This model is applied to the cantilever beam with an edge crack, and modal frequencies are determined for different crack parameters. These results give good agreement with experimental data. For considering the effect of crack closure, the modal parameters are identified by an identification technique in the time domain. Through computation results, it is found that the difference between displacement response between the beam and the cracked beam is reduced because of the crack closure effect; the eigen frequency is considerably affected by the average value of excitation force. Lastly, a simple and direct method to determine the crack position, based on a discussion of the relationship between crack and eigen parameters of the beam, is proposed. Nahvi and Jabbari [4] established an analytical, as well as experimental approach to the crack detection in cantilever beams by vibration analysis. A cracked cantilever beam is excited by a hammer and the response is obtained using an accelerometer attached to the beam. It is assumed that crack always remain open to avoid non-linearity. Contours of the normalized frequency in terms of the normalized crack depth and location are plotted to identify the crack. To detect the crack parameters the intersection of contours with the constant modal natural frequency planes is used. A minimization approach is used for identifying the cracked element within the cantilever beam. Pandey and Biswas [5] presented an evaluation of changes in the flexibility matrix of a structure as a candidate method to identify the presence of damage. Flexibility matrix can be easily estimated from few of the lower frequency modes of vibration of the structure. First, with simple analytical beam models, the effect of damage presence in a structure on its flexibility is studied. The effectiveness of using changes in the flexibility matrix in detecting and locating damages is demonstrated. Skrinar [6] formulates the finite element of a beam with an arbitrary number of transverse cracks. Each crack is replaced by a corresponding linear rotational spring, connecting two adjacent elastic parts. Due to the presence of crack the flexural bending deformation is caused, hence the stiffness and geometrical stiffness matrices are considered. The expressions are presented in closed forms which are used to calculate the coefficients of stiffness and geometrical stiffness matrices, as well as the load vector of the element. As the equivalent interpolation functions were implemented in the derivations, transverse displacements in the finite element can also be obtained. This finite element could be efficiently implemented, not only in static and stability analysis, but also in inverse identification of cracks in beam-like structures. Binici [7] proposed a new method to obtain the eigen-frequencies and mode shapes of beams containing multiple cracks and subjected to axial force. Cracks give local flexibility and rotational spring. To determine mode shape functions the proposed method used one set of end conditions as an initial parameters. Mode shape functions of the remaining parts are determined by satisfying the continuity and jump conditions at crack locations. New set of boundary conditions give a second-order determinant that needs to be solved for its roots. For static case, the roots of the characteristic equation give the buckling load of the structure. The result of proposed method is compared against the results predicted by finite element analysis. In order to investigate the effect of cracks and axial force levels on the eigen-frequencies, a parametric study is conducted. For this, both simply supported and cantilever beam-columns are considered. It is observed that eigen-frequencies are strongly affected by crack locations, severities and axial force levels. Wang et al. [8] present a method of crack detection in a turbine blade. In wind turbines, blades are the main components. The magnitude of blade damage increases in service and lead to catastrophic failure, hence it is essential to do the structural monitoring of the turbine blade. This method includes finite element method (FEM) for dynamics analysis (modal analysis and response analysis) and the mode shape difference curvature (MSDC) information for damage detection. Wahab and Roeck [9] studied the application of the change in modal curvatures to detect damage in a prestressed concrete bridge. Narayana and Jebaraj [10] performed an analytical work to study the effect of crack at different location and depth on mode shape behavior. Kishen and Sain [11] developed a technique for damage detection using static test data.

From the literature, it is found that there is no existence of the mathematical model for the U-shape cracked cantilever beam for evaluating its natural frequency and on the other hand U-shape cracks found to be most practical in the dynamic structures. So structures carry U-shape cracks needs attention. Similarly, two open single-sided cracks are mostly found in the application where fluctuating load acts on the structure. Previously, for natural frequency mathematical model was developed by W. M. Ostachowicz and M. Krawczuk for the cantilever beam which has two open single-sided V-shape cracks. In this study, mathematical model of W. M. Ostachowicz and M. Krawczuk is taken as a reference model and applied to the cantilever beam which has two open single-sided U-shape cracks to see its adaptability for U-shape cracks. Moreover, the effect of two open single-sided U-shape cracks on the characteristics roots is studied.

2. MATHEMATICAL MODEL

W.M. Ostachowicz and M. Krawczuk have presented a method of analysis of the effect of two open cracks on the bending mode natural frequencies in a cantilever beam. W. M. Ostachowicz and M. Krawczuk considered two types of cracks in their study, double sided and single sided cracks. Double sided cracks occur in the beam due to the action of cyclic loading and on the other hand, single sided cracks occur in the beam due to fluctuating loads. When fluctuating loads acts on the smooth beam, then beam gets bent, due to which two open single-sided cracks occur on the beam due to less fatigue strength of the material or due to some localized manufacturing defects in the beam such as corrosion, corrosion fatigue and erosion in the beam.

Calculation of natural vibration frequencies of a beam with two cracks: One can take the natural vibration frequencies equation of the beam in the well known form.

$$EJ\ \delta^4 y(x,t)/\delta x^4\ + \rho F \delta^2 y(x,t)/\delta t^2 = 0 \tag{1}$$

where ρ is the material density, F is the cross sectional area of the beam, $y(x,t)$ is the deflection of the beam and J is the geometrical moment of inertia of the beam cross section. By introducing elasticity elements in crack locations one obtains a system of three beams. The equivalent stiffness of the elasticity element is calculated in section 2 [12].

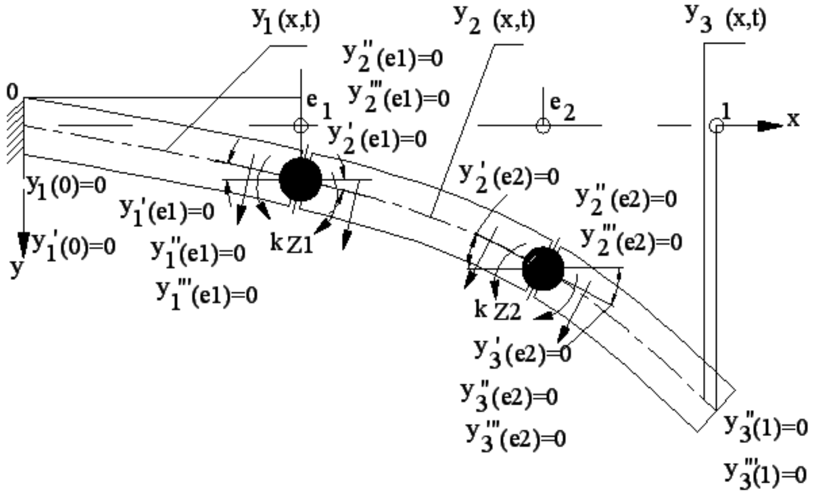

Figure-1. Clamped cantilever beam with two open cracks [12].

The model of the problem is shown in Figure-1. The boundary conditions, in terms of the non dimensional beam length ξ = x/L, can be expressed as follows: $y_1(0) = 0$, zero displacement of the beam at the restraint point; $y_1'(0) = 0$, zero angle of rotation of the beam at the restraint point; $y_1(e_1) = y_2(e_1)$, compatibility of the displacement of the beam at the location of the first crack; $y_2'(e_1) - y_1'(e_1) = \theta_1 y_2''(e_1)$, total change of the rotation angle of the beam at the location of the first crack; $y_1''(e_1) = y_2''(e_1)$,compatibility of the bending moments at the location of the first crack; $y_1'''(e_1) = y_2'''(e_1)$, compatibility of the shearing forces at the location of the first crack; $y_2(e_2) = y_3(e_2)$, compatibility of the displacements of the beam at the location of the second crack; $y_3'(e_2) - y_2'(e_2) = \theta_2 y_3''(e_2)$, total change of the beam rotation angle at the location of the second crack; $y_2''(e_2) = y_3''(e_2)$, compatibility of the bending moments at the location of the second crack ;$y_2'''(e_2) = y_3'''(e_2)$, compatibility of the shearing forces at the location of the second crack; $y_3''(1) = 0$, zero bending moment at the end of the beam;$y_3'''(1) = 0$, zero shearing force at the end of the beam. Here e_1 and e_2 are the distances between the end of the cantilever beam and the crack locations. The solution of (1) is sought in the form

$$y(\xi, t) = y(\xi) \sin \omega\, t \tag{2}$$

Substituting this solution into (1), after simple algebraic transformation, one has

$$y^{iv}(\xi) - \beta^4 y(\xi) = 0 \tag{3}$$

where $\beta^4 = \omega^2 \rho A / L^4 EJ$.Taking the function $y(\xi)$ in the form of a sum of three functions,

$$y_1(\xi) = A_1 \cosh\ (\beta\xi) + B_1 \sinh\ (\beta\xi) + C_1 \cos(\beta\xi) + D_1 \sin(\beta\xi)$$

$, \xi \varepsilon [0, e_1)$

$$y_2(\xi) = A_2 \cosh\ (\beta\xi) + B_2 \sinh\ (\beta\xi) + C_2 \cos(\beta\xi) + D_2 \sin(\beta\xi)$$

$, \xi \varepsilon (e_1, e_2)$

$$y_3(\xi) = A_3 \cosh\ (\beta\xi) + B_3 \sinh\ (\beta\xi) + C_3 \cos(\beta\xi) + D_3 \sin(\beta\xi)$$

$, \xi \varepsilon\ (e_2, 1]$ (4)

$$\omega_i = \left(\frac{\beta_i}{L}\right)^2 \sqrt{EJ/\rho F}\ , \text{i=1, 2,……..,n,} \tag{5}$$

where ω_i is the i^{th} natural vibration frequency of the beam and β_i is the i^{th} characteristic root.

3. SIMULATED CRACK CONFIGURATIONS

In this study, free vibrations of a cantilever beam having U-shape edge cracked cases are studied. The same geometric and material of reference model are taken to compare the results of cantilever beam with U- shape edge crack cases with V-shape edge crack cases.

Geometric properties: The Length and cross sectional area of the beam are1m and 0.1 * 0.1 m^2, respectively.

Material properties: Modulus of elasticity (E) is $2.1*10^{11}$ N/m^2, the density (ρ) is 7860 kg/m^3. The value of poisons ratio is not given in the reference model so it is assumed as 0.3.

In present work, total number of specimens considered is 63 and it is same as that of cracked cases of the reference model to study the amount of deviation given by the cantilever beam for V-shape edge cracked cases and U-shape edge cracked cases for the characteristics roots. Two separate cases are considered, in case 1 each specimen has a single crack and each specimen have two cracks in case 2. The details of each case are given below.

Case 1: Three specimens are considered in this case. Transverse cracks are taken on each specimen by keeping crack location constant at 100 mm from the beam

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fixed end. At this location, crack depth is taken as 30 mm for one specimen, similarly for next two specimens it is taken as 50 mm, and 70 mm respectively.

Case 2: In this case, 60 specimens are considered. Two transverse cracks are taken on each specimen. This case is divided into 3 sub cases. Each sub case has 20 specimens. In the first sub case, location of the first crack is at 100 mm and crack depth is taken as 30 mm as mentioned in Figure-3, then the location of the second crack is varied as 110 mm, 200 mm, 400 mm, 600 mm and 800 mm from the cantilever end and at each location crack depth is taken as 10 mm, 30 mm, 50 mm and 70 mm respectively. Similar configuration is used for next 2 sub cases, but instead of 30 mm crack depth at the first location, 50 mm and 70 mm crack depth is taken for the second and third sub cases, respectively.

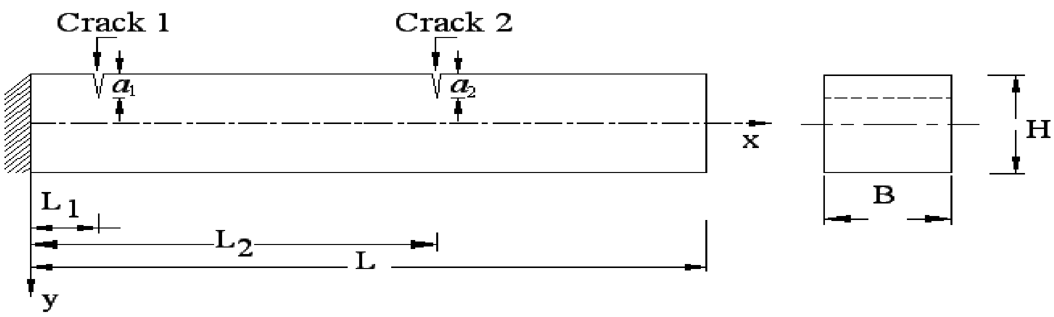

Figure-2. Diagram of a cantilever beam with two open V-shape crack [12].

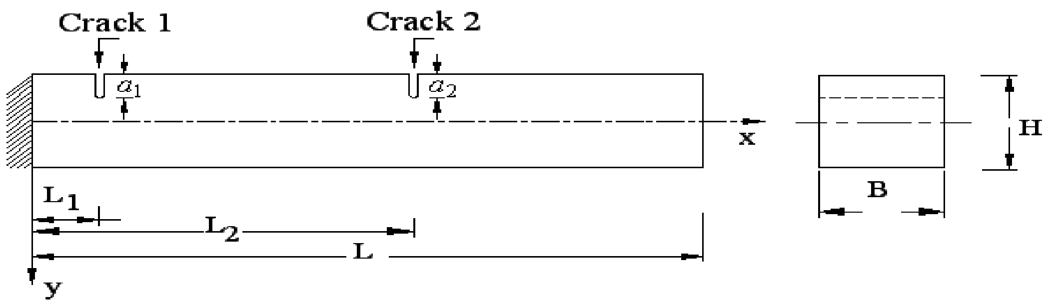

Figure-3. Design of a cantilever beam with two open U- shape cracks. First crack details: $L_1/L= 0.1; a_1/H=0.3;$ Second crack details: $L_2/L= 0.6; a_2/H=0.3.$

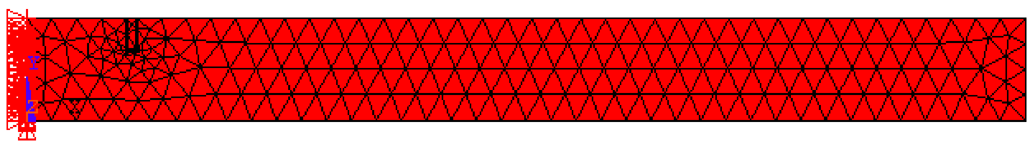

Figure-4. Finite element modelling of the cracked beam.

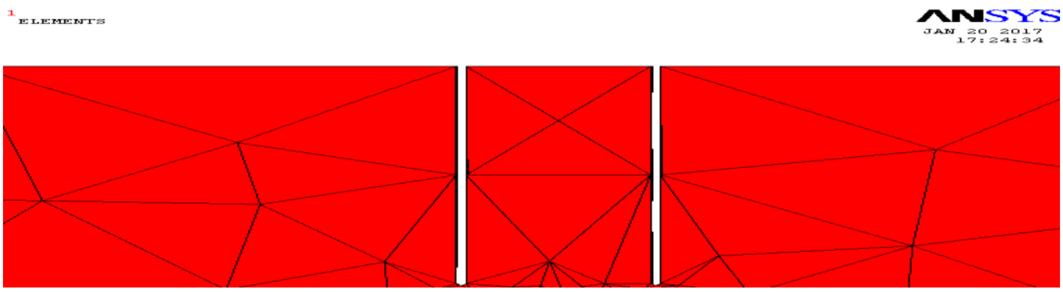

Figure-5. Crack zone details of a FEA model.

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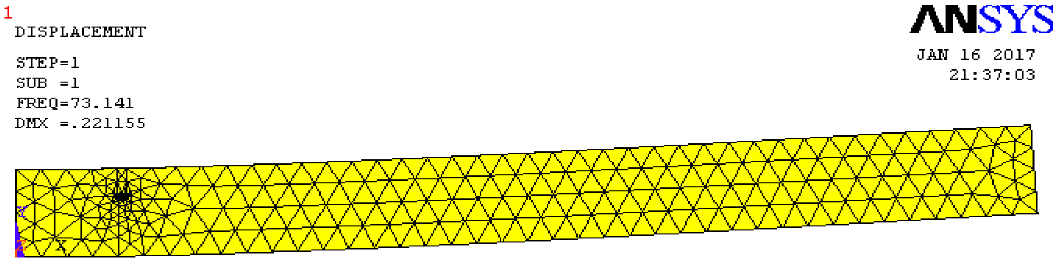

Figure-6. Natural frequency plot, First crack details: $L_1/L= 0.1$; $a_1/H=0.3$; Second crack details: $L_2/L= 0.11$; $a_2/H=0.3$.

4. FINITE ELEMENT MODELLING AND ANALYSIS

ANSYS 12.1 [13] finite element program is used to determine natural frequencies of the undamaged and cracked cases of beams. For this purpose, block of required dimensions is created by volume command. Then at the required locations on the surface of the beam model, two small U shape areas of required dimensions are created and extruded. Then small volumes are subtracted from large volume of cantilever beam model to obtain three dimensional models with two open U-shape cracks. A 20 node structural solid element (solid 186) is selected for the analysis, because of several special features like stress stiffening, large strain, and large deflection. Finite element boundary conditions are applied on the beam to constrain all degrees of freedom of the extreme left hand end of the beam. The Block Lanczos eigen value solver is used to calculate the natural frequencies of the beams.

By finite element analysis, the natural frequencies of the cracked cases of the beams are determined as shown in Table-1. The values of natural frequencies obtained by FEA for different damaged cases are substituted in (5) to get the values of characteristics root as shown in Table-2.

5. RESULTS

Table-1. The first fundamental natural frequency for cracked cases of the beam.

a_2/h	$L_2/L = 0.11$	$L_2/L = 0.2$	$L_2/L = 0.4$	$L_2/L = 0.6$	$L_2/L = 0.8$
$L_1/L=0.1$; $a_1/H=0.3$; $f_1=74.216$ Hz					
f_1 (Hz)					
0.1	74.119	73.708	74.013	74.129	74.2
0.3	73.141	69.414	72.123	73.694	74.198
0.5	59.961	60.105	67.314	72.303	74.114
0.7	40.044	43.619	55.521	67.871	73.679
$L_1/L=0.1$; $a_1/H=0.5$; $f_1=59.669$ Hz					
f_1 (Hz)					
0.1	59.688	59.397	59.551	59.619	59.711
0.3	59.722	57.141	58.613	59.459	59.674
0.5	57.741	51.581	56.018	58.766	59.528
0.7	39.909	39.968	48.731	56.225	59.448
$L_1/L=0.1$; $a_1/H=0.7$; $f_1=39.832$ Hz					
f_1 (Hz)					
0.1	39.555	39.534	39.627	39.69	39.569
0.3	39.586	38.763	39.419	39.42	39.582
0.5	39.411	36.903	38.542	39.365	39.636
0.7	36.974	31.961	35.958	38.662	39.604

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Table-2. Single-sided crack: roots of the characteristics equation.

Crack parameters	***L_2/L* →** ***a_2/h* ↓**	***0.11***	***0.2***	***0.4***	***0.6***	***0.8***
$L_1/L=0.1$; $a_1/H=0.3$; $\beta_1=1.767$						
U-Shape	0.1	1.766	1.761	1.765	1.766	1.767
V-Shape		1.758	1.760	1.764	1.766	1.766
Deviation (%)		0.453%	0.056%	0.056%	0%	0.056%
U-Shape	0.3	1.754	1.709	1.742	1.761	1.767
V-Shape		1.690	1.706	1.741	1.760	1.766
Deviation (%)		3.648%	0.175%	0.057%	0.056%	0.056%
U-Shape	0.5	1.588	1.59	1.683	1.744	1.766
V-Shape		1.490	1.658	1.658	1.740	1.762
Deviation (%)		6.171%	-4.276%	1.485%	0.229%	0.226%
U-Shape	0.7	1.298	1.355	1.529	1.69	1.761
V-Shape		1.181	1.256	1.450	1.658	1.756
Deviation (%)		9.013%	7.306%	5.166%	1.893%	0.283%
$L_1/L=0.1$; $a_1/H=0.5$; $\beta_1=1.585$						
U-Shape	0.1	1.585	1.581	1.583	1.584	1.585
V-Shape		1.521	1.523	1.525	1.525	1.526
Deviation (%)		4.037%	3.668%	3.663%	3.724%	3.722%
U-Shape	0.3	1.585	1.551	1.571	1.582	1.585
V-Shape		1.485	1.486	1.513	1.523	1.525
Deviation (%)		6.309%	4.19%	3.691%	3.729%	3.785%
U-Shape	0.5	1.559	1.473	1.535	1.573	1.583
V-Shape		1.370	1.403	1.480	1.511	1.523
Deviation (%)		12.123%	4.752%	3.583%	3.941%	3.79%
U-Shape	0.7	1.296	1.297	1.432	1.538	1.582
V-Shape		1.138	1.199	1.345	1.470	1.521
Deviation (%)		12.191%	7.555%	6.075%	4.421%	3.855%
$L_1/L=0.1$; $a_1/H=0.7$; $\beta_1=1.295$						
U-Shape	0.1	1.29	1.29	1.291	1.292	1.29
V-Shape		1.184	1.185	1.185	1.186	1.186
Deviation (%)		8.217%	8.139%	8.21%	8.204%	8.062%
U-Shape	0.3	1.291	1.277	1.288	1.288	1.291
V-Shape		1.174	1.177	1.182	1.184	1.185
Deviation (%)		9.062%	7.83%	8.229%	8.074%	8.21%
U-Shape	0.5	1.288	1.246	1.273	1.287	1.291
V-Shape		1.136	1.146	1.169	1.183	1.184
Deviation (%)		11.8%	8.025%	8.169%	8.08%	8.288%
U-Shape	0.7	1.247	1.16	1.23	1.275	1.291
V-Shape		1.021	1.056	1.125	1.181	1.182
Deviation (%)		18.123%	8.965%	8.536%	7.372%	8.443%

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The comparison of the results of characteristics roots for V-shape and U-shape cracked cases are shown in Figures 7-9. Total 12 critical cracked cases are shown for the characteristics roots. From Figures 7 and 8, it is seen that the value of characteristics roots for V-shape and U-shape cracked cases gives good agreement. From Figure-9, it is noticed that V-shape and U-shape cracked cases gives some error for characteristics roots from 8.21% to 18.09%. The main advantage of the existing mathematical model is that it gives outstanding results for characteristics roots at critical region of the beam.

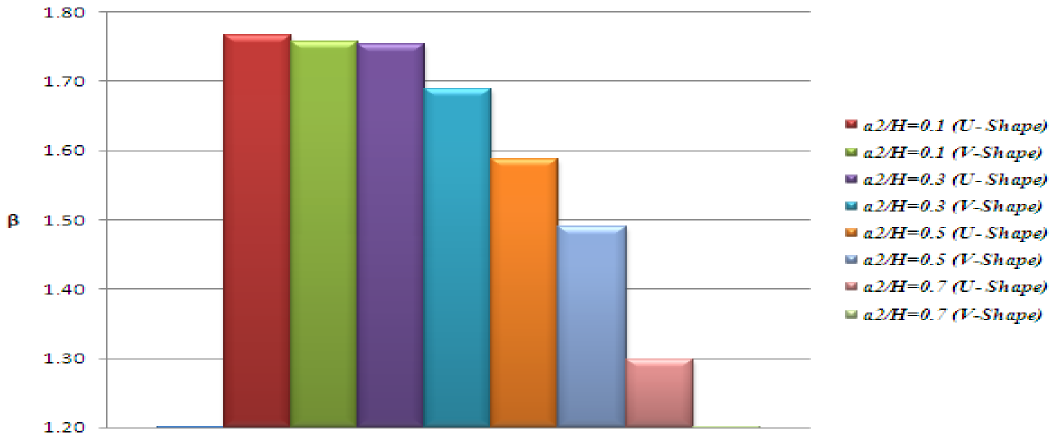

Figure-7. Characteristics roots for Rec. shape and V- shape cracks. The first crack: Location L_1/L= 0.1; Size a_1/H= 0.3; The second crack: Location L_2/L= 0.11.

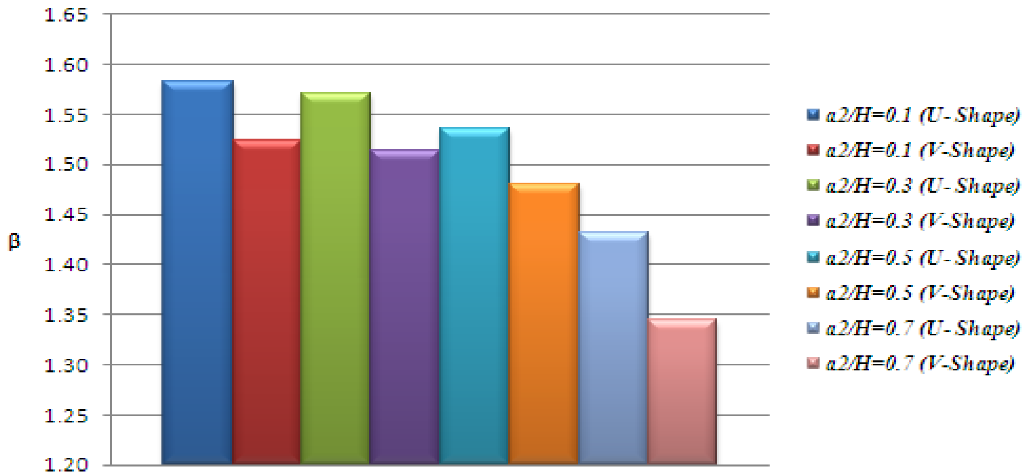

Figure-8. Characteristics roots for Rec. shape and V- shape cracks. The first crack: Location L_1/L= 0.1; Size a_1/H= 0.5; The second crack: Location L_2/L= 0.4

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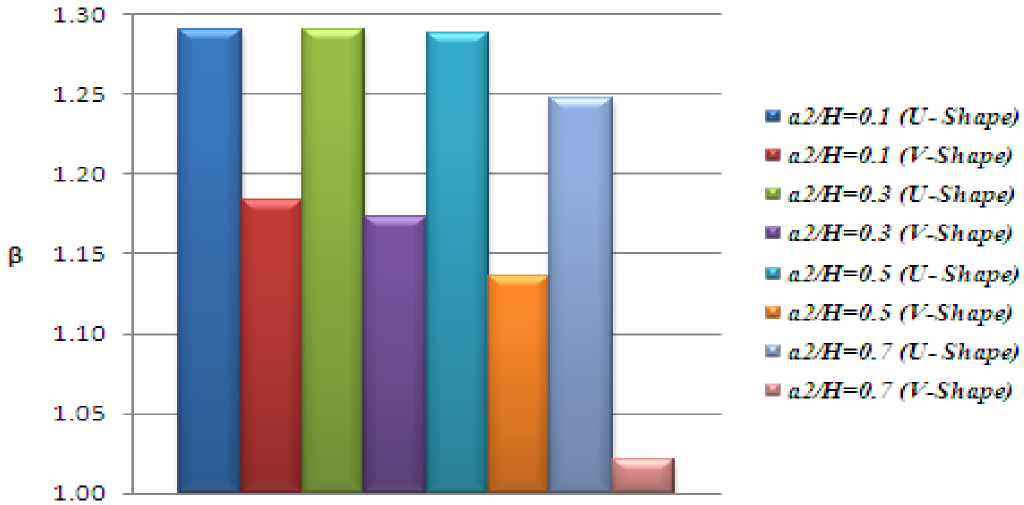

Figure-9. Characteristics roots for Rec. shape and V- shape cracks. The first crack: Location L_1/L= 0.1; Size a_1/H= 0.7; The second crack: Location L_2/L= 0.11

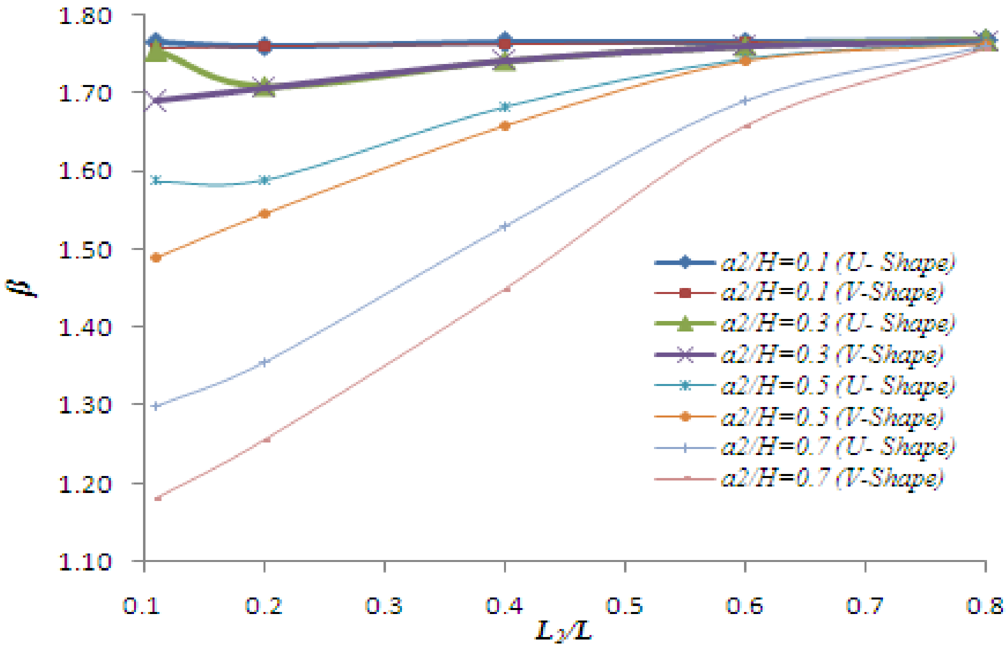

Figure-10. Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location $L_1/L=0.1$; Size= a_1/H= 0.3; β_1 =1.767

From Figure-10, it is found that when the location of the second crack increases from the first crack location by keeping the same depth of the second crack then value of characteristics root almost remains constant for V-shape as well as U-shape cracked cases of the beam. This is true when the depth of the second crack is less than 30 % of the total depth of the beam. Constant value of characteristics root means almost constant value of stiffness of the beam.

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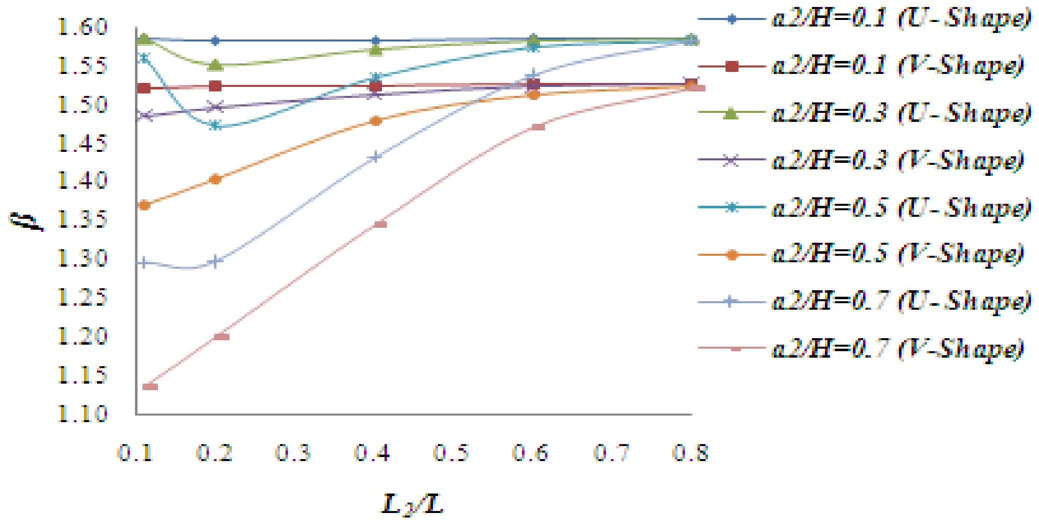

Figure-11. Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location $L_1/L=0.1$; Size= a_1/H= 0.5; β_1 =1.585

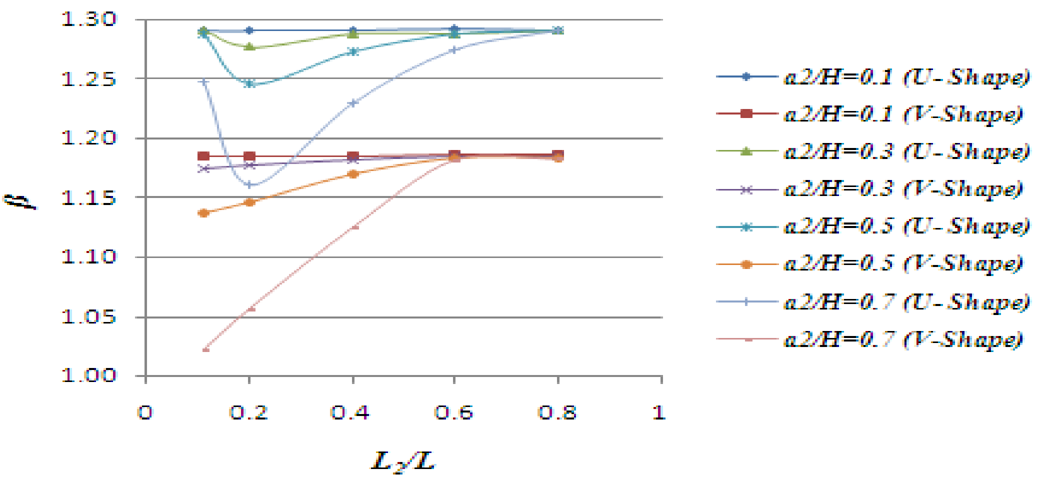

Figure-12. Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location $L_1/L=0.1;$ Size= a_1/H= 0.7; β_1 =1.295.

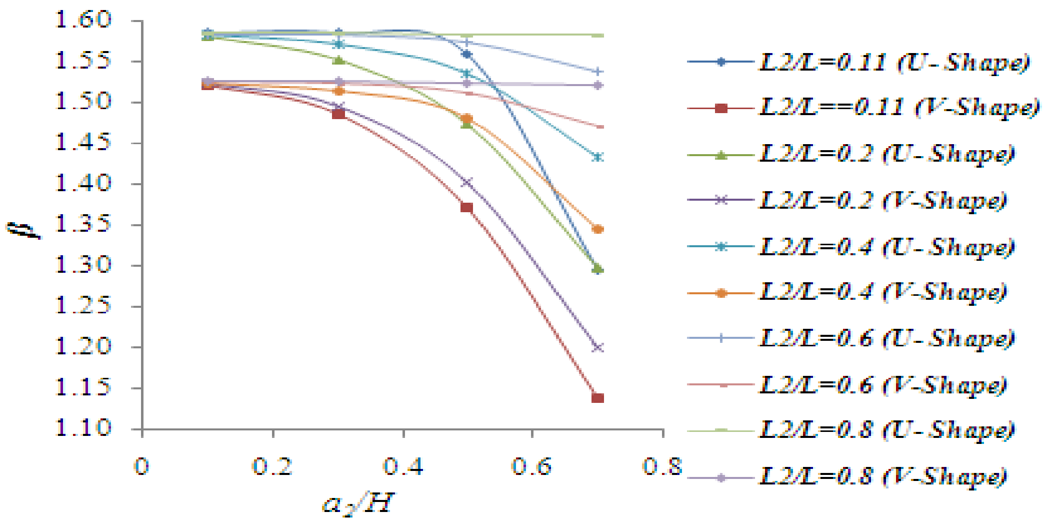

Figure-13. Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location L_1/L=0.1; Size= a_1/H= 0.5; β_1 = 1.585.

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For the same configuration, When the depth of the second crack increases above 30% then value of characteristics root increases due to increase in stiffness of the beam materials. It means that as the location of the second crack increases from the first crack location then beam stiffness increases more rapidly.

The values of characteristics roots for V-shape and U-shape cracked cases gives good agreement when depth of the cracks are less than 50% of the total depth of the beam.

For larger depth, the found error in the values of characteristics roots for V-shape and U-shape cracked cases is somewhat more, this may be due to lack of width specifications of the V-shape crack profile as mentioned in the reference model [12].

The decrease in the value of characteristics root is largest, if cracks are nearer to each other as well as closer to the cantilever end. When the value of characteristics roots are compared for Figures 10-12, it is found that the value of characteristics roots are minimum due to the presence of largest crack depth of the first crack at the first location as shown in Figure-12. The significant reduction in characteristics roots value is only due to the presence of largest crack depth at the first location.

From Figure-13, it is found that when the depth of the second crack at any location (apart from last location, L_2/L=0.8) increases then the value of characteristics root decreases. But decrease in the value of characteristics root is rapid, when second crack location is nearer to first crack location because the effect of damping remains largest in such cracked case. For the larger crack depths, as the location of the second crack increases from the first crack location then rigidity of the beam increases significantly or vice versa. From Figure-13, it is also found that the values of characteristics roots for V-shape and for U-shape cracked cases are close to each other. The last location of the second crack (L_2/L= 0.8) is very nearer to the free end, so at this location even though crack depth increases the value of characteristics roots almost remain constant due to presence of negligible amount of damping effect in the beam.

6. CONCLUSIONS

Analysis focuses on free vibration only. In the FEA part of this study, the effect of the crack depth and location on modal properties of the beam was investigated. The following conclusions can be drawn from the analyses:

a) The theory presented by W.M. Ostachowicz and M. Krawczuk for the cantilever beam which has two open single-sided V-shape cracks is appropriate to the same beam which has two open single-sided U-shape cracks.

b) Up to 50% crack depth, the values of characteristics roots for V-shape and U-shape cracked cases gives good agreement.

c) Found error in the values of characteristics roots for V-shape and U-shape cracked cases is slightly more when crack depth increases above 50% of the depth of the beam.

d) 4. When the depth of the second crack is kept constant and second crack location is varied from the cantilever end of the beam, then characteristics root of the beam increases.

e) When the location of the second crack is kept constant and crack depth increases then characteristics root of the beam decreases.

f) Characteristics root of the beam decreases considerably, when second crack location is nearer to the first crack location.

g) When the depth of the first crack increases, then value of characteristics roots decreases rapidly.

REFERENCES

[1] R.P. Kocharala, R.K. Bandlam and M.R. Kuchibotla. 2016. Finite element modeling of a turbine blade to study the effect of multiple cracks using modal parameters. Journal of Engineering Science and Technology. 11(12): 1758-1770.

[2] K.R.P. Babu, B.R. Kumar, K.L. Narayana, and K.M. Rao. 2015. Multiple crack detection in beams from the differences in curvature mode shapes. ARPN Journal of Engineering and Applied Sciences. ARPN Journal of Engineering and Applied Sciences. 10(4): 1701-1710.

[3] G.L. Quin, S.N. Gu, and J.S. Jiang. 1990. The dynamic behaviour and crack detection of a beam with a crack. Journal of Sound and Vibration. 138(2): 233-243.

[4] H. Nahvi and M. Jabbari. 2005. Crack detection in beams using experimental modal data and finite element model. International Journal of Mechanical Sciences. 47(10): 1477-1497.

[5] A.K. Pandey and M. Biswas. 1994. Damage Detection in Structures Using Changes in Flexibility. Journal of Sound and Vibration. 169(1): 3-17.

[6] M. Skrinar. 2009. Elastic beam finite element with an arbitrary number of transverse cracks. Finite Elements in Analysis and Design. 45(3): 181-189.

[7] B. Binici. 2005. Vibration of beams with multiple open cracks subjected to axial force. Journal of Sound and Vibration. 287(1-2): 277-295.

[8] Y. Wang, M. Liang and J. Xiang. 2014. Damage detection method for wind turbine blades based on dynamic analysis and mode shape difference curvature information. Mechanical Systems and Signal Processing. 48(1-2): 351-367.

[9] M. Wahab and G. Roeck. 1999. Damage detection in bridges using modal curvatures: application to real damage scenario. Journal of Sound and Vibration. 226(2): 217-235.

[10] K. Lakshminarayana and C. Jebaraj. 1999. Sensitivity analysis of local/global modal parameters for identification of a crack in a beam. Journal of Sound and Vibration. 228(5): 977-994.

[11] J.M.C. Kishen and T. Sain. 2004. Damage detection using static test data. Journal of Structural Engineering. India. 31(1): 15-21.

[12] W.M. Ostachowicz and M. Krawczuk. 1991. Analysis of the effect of cracks on the natural frequencies of a cantilever beam. Journal of Sound and Vibration. 150(2): 191-201.

[13] ANSYS Release 12.1, ANSYS Inc.