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DECISION SUPPORT BASED ON THE INTERVAL RELATION

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ABSTRACT

The paper considers the method of decision support at the stage of the preliminary analysis of complex systems development projects in order to identify the most promising. Offered is a method for ranking objects (defining their weights) when specifying information about the degree of superiority of one object over another in the form of interval expert estimates. Elements of the interval relation are set in a multiply connected regions represented as a union of disjoint intervals. Unlike conventional approaches to ranking objects based on interval relation of preferences intensity the proposed method allows to process these estimates without their prior averaging procedure. It has been proven that the model has the desired properties: consistency, maintaining optimality, positive relationship with peer relations, preservation of superiority and other characteristics that enhance the legitimacy of its use in practice. The paper presents a numerical example illustrating how the method works.

Keywords: decision support, rank-ordering objects, interval relation, expert judgment.

1. INTRODUCTION

At an early stage of analysis of complex systems development projects it becomes important to quantify the importance of alternative implementation options and the importance of resources in the system, subsystem, etc. The analysis of such problems is associated with the reviewing a large number of techno-economic and social characteristics. Such indicators include the social significance of a decision, environmental factors, the amount and cost of the necessary work, competitiveness, profits and such.

An important task of decision-making support is rank-ordering the objects according to their importance. As a rule, this task is preliminary; it precedes exploration of options of complex systems projects implementations. At this stage it is important to identify the significant factors affecting the operation of the system, which will describe the mathematical models of such systems. For the decision-maker, a quantitative relation of their preference of some objects over the others can be important. In some problems, such as those associated with urban policy ore insuring environmental safety in urban districts such information is set in the form of interval relation. This is mainly because of inconsistency of some expert estimates, due to conflicting goals of different groups of individuals associated with the decision-making process (investors, district residents, the city authorities and others). Therefore, the process of rank-ordering of objects is based on the interval relation, characterized by, in some cases, the presence of disjoint segments in assessing preference of object pairs. Gathering of expert estimates occurs taking into account all object relationships towards finding a solution most correlated with initial expert information, without preliminary averaging of expert estimates.

2. THE TASK OF RANK-ORDERING OBJECTS ON THE BASIS OF EXPERT INFORMATION

Methods of ordering objects on the basis of point, interval, fuzzy relations are considering a finite set of objects $X = \{1,2,\dots,n\}$, and sets of object pairs $\mathrm{U} = \{(\mathrm{i},\mathrm{j}) | \mathrm{i},\mathrm{j} \in \mathrm{X}\}$. It should be noted that some objects are not compared with each other, i.e.$\mathrm{U} \subseteq \mathrm{X} \times \mathrm{X}$. Relation R is set on object pairs set U. Matrix is determined using expert method

$$R = \{r_{i,j}\}, i,j \in X = \{1,2,\dots,n\}. \quad (1)$$

Elements $r_{i,j}$ characterize the relation of weights of objects $x_i$and x_j:

$$r_{i,j} = x_i/x_j\,, i,j, \in X. \quad (2)$$

Values of those variables are considered to be the unknown. The task is to compute the vector of the weights $x = (x_1, x_2, \dots, x_n)$ on the basis of the relation R. Let's take a look at the concepts used for point estimates to also be used in the evaluating expert estimates of interval type. Let $r_{i,j} \epsilon R$ be point estimates, a relation R is determined for all object pairs, i.e.$U = X \times X$. The initial data in this case is recorded in a matrix of$n \times n$ size.

This problem (1-2) with point estimates (object i is preferred to object j by $r_{i,j}$ times) has been explored in several papers. For example, in papers [1-6] this problem has been successfully solved for the maximum eigenvalue search of matrix R, and determining on that basis the eigenvector, which normalization defines priority vector. In other works, the concept of proximity of pairwise comparisons matrix to the resulting matrix is determined differently [7-11]. It should be noted that the problem of objects rank-ordering based on interval binary relation was solved in the paper [11]. The direct application of this approach in the analysis of a relation defined as a system of disjoint intervals, leads to using a exhaustive search, which in some cases is an exigent computational task. Papers [12-17] discuss rank-ordering of objects, and show the use of the developed methods to solve practical problems. The proposed in this article approach builds on the methods developed in [11-13, 16].

3. PROBLEM PROPOSITION

First, let's look at point expert estimates. Their consistency or consistency of the pair comparison matrix can be described by it's super transitivity [18].

Definition. Matrix T is called super transitive, if for every$i, j, k \in X$ holds the equality [18]

$$t_{i,j} = t_{i,k} \cdot t_{k,j} \,. \tag{3}$$

Paper [18] states the condition for supertransitive matrix: the existence of numbers $x_i > 0$, set equal to the weights of objects for which hold the equations (2). Equations (3) are not always realized in the analysis of preference objects, especially under conditions of incomplete information about the properties of objects. Consequently, there are no weights of objects x_i, satisfying the system of equations (2). In cases where expert opinions are inconsistent, the following super transitive matrix is used instead of matrix R for rank-ordering (determining weights of) objects:

$$T = (t_{i,j}), i, j \in X, \tag{4}$$

which is in some way is similar to the matrix R.

Thus, it can be concluded that the relation R (matrix R) itself does not allow to order the objects. In this case, the main reason is that the ratio R is obtained from expert survey, and therefore, is not transitive, i.e., equalities (3) are not fulfilled. With such formulation of the problem the coefficients of objects importance are determined as an approximation of the initial interval relation R. On the basis of the established approximating supertransitive matrix $T = (t_{i,j})$ it is possible to determine the object importance coefficients up to a positive coefficient, and to rank order them.

In this paper, the problem of determining the weights of objects on the basis of expert judgement given in the form of a system of intervals is approached as a development of the problem for interval expert estimates [11]. The interval ratio R for interval estimates is given in the form of

$$r_{i,j} = [\alpha_{i,j}, \beta_{i,j}] \subset R_+, (i,j) \subset U \subseteq X \times X. \tag{5}$$

It is assumed that some object pairs may not be assessed by experts. Expert estimates (5) are consistent if there exists an object weight vector $x = (x_1, x_2, \dots, x_n)$, for which holds true

$$\alpha_{i,j} \le x_i / x_j \le \beta_{i,j}, \forall (i,j) \in U. \tag{6}$$

If the ratio (6) carried out for the different vectors $x = (x_1, x_2, \dots, x_n)$, then the choice of a vector closest to expert estimates should be made. The following problem is formulated for interval estimates [11]

$$\lambda \to min, \tag{7}$$

$$\alpha_{i,j}/\lambda \le x_i/x_j \le \beta_{i,j} \cdot \lambda, \forall (i,j) \in U, \tag{8}$$

$$x = (x_1, x_2, \dots, x_n) \in R_+^n. \tag{9}$$

Here the following principle is employed: expert relations are more reliable inside the intervals rather than on their boundary. The left boundaries uniformly increase when multiplied by $1 / \lambda (\lambda < 1)$, and the right ones, when multiplied by$\lambda (\lambda < 1)$, uniformly decrease. This process can be continued while there is a solution for the system (8), (9).

It should be noted that at full consistency of expert estimates, when at some value of $\lambda (\lambda < 1)$, left and right boundaries of intervals (8) will coincide and the system (8), (9) will have a unique solution. This solution will look like this

$$c_{i,j} = (\propto_{i,j} \cdot \beta_{i,j})^{1/2}. \tag{10}$$

Obtained this way matrix $= (c_{i,j})$, is supertransitive.

4. THE METHOD OF SOLVING A PROBLEM OF RANK-ORDERING OBJECTS

For solving the problems (7)-(9) it is suggested to construct a matrix$A = (a_{i,j})$ [11]. Let $i, j \in X$ be chosen at random, then there are three possibilities.

1) $(i,j), (j,i) \in U = X \times X$, expert estimates are obtained for all object pairs.

2) $(i,j) \in U, (j,i) \notin U$, this is possible when expert judgement of object pairs is partial.

3) $(i,j) \notin U, (j,i) \notin U$. In this case an relation of preference of two objects i, j was not made. For example, they have different characteristics, and therefore are not comparable.

Let's make calculations according to those possibilities:

1) $a_{i,j} = \max(\alpha_{i,j}, 1/\beta_{j,i})$,
$a_{j,i} = \max(\alpha_{j,i}, 1/\beta_{i,j})$.

The interval in this case can narrow.

2) $a_{i,j} = \propto_{i,j}, a_{j,i} = 1 / \beta_{i,j}$. Expert estimation for (j,i) is defined in such a way that it correlates with estimation for (i,j).

3) $a_{i,j} = a_{j,i} = 0$, objects i and j are not compared to each other.

For possibility 1) inconsistency is possible when for some$(i,j), (j,i) \in U$ is carried out the following ratio

$$1/\alpha_{j,i} < \alpha_{i,j} \; or \; 1/\beta_{j,i} > \beta_{i,j}. \tag{11}$$

This possibility, described in (11), was not explored in [11]. Such situation may occur when expert evaluation is conducted by two groups of persons who hold opposing views on the issue under investigation. In this case expert judgment can be considered as

$$r_{i,j} = [\alpha_{i,j}, \beta_{i,j}] \cup [\gamma_{i,j}, \delta_{i,j}] \subset R_+,$$

$$(i,j) \subset U \subseteq X \times X, \tag{13}$$

Where

$[\alpha_{i,j}, \beta_{i,j}] \cap [\gamma_{i,j}, \delta_{i,j}] = \emptyset,$

$$(i,j) \subset U \subseteq X \times X. \quad (14)$$

For expert estimates that do not satisfy (13), (14), the elements of matrix $A = (a_{i,j})$ are calculated following one of the examples described above. For object pairs expert estimates for which are subject (13), (14), calculations are made as follows. Let's calculate those elements as$a_{i,j} = \alpha_{i,j}, a_{j,i} = 1 / \beta_{i,j}$. Further, let's introduce matrix$B = (b_{i,j})$, and for that pair or elements determine$b_{i,j} = \gamma_{i,j}, b_{j,i} = 1 / \delta_{i,j}$. For expert estimates not fulfilling (13), (14), let's set $b_{i,j} = a_{i,j}$.

For a more general case of a problem of expert judgment, fulfilling (13) and (14), the elements of matrix $A = (a_{i,j})$ are calculated on the basis of information about intervals$[\alpha_{i,j}, \beta_{i,j}]$, elements of matrix$B = (b_{i,j})$ - $[\gamma_{i,j}, \delta_{i,j}]$, according to possibilities explored above. Thus, here in after $U = X \times X$. It should be noted that described in the article case of two intervals is not restrictive. The method described below allows considering any system of intervals to represent expert information. Taking into account those observations, the restrictions forx_i/x_j, similarly (8), written as

$$a_{i,j}/\lambda \le x_i/x_j \le \lambda/a_{j,i} \cup b_{i,j}/\lambda \le x_i/x_j \le \lambda/b_{j,i},$$

$$(i,j) \in U. \quad (15)$$

Direct application of the results of work [11] for solving problem (7), (9), (15) is not feasible due to computational complexity.

For the numerical implementation of the algorithm the search for solutions will be conducted in the neighborhood of the points $c_{i,j} = (a_{i,j} / a_{j,i})^{1/2}$ and $d_{i,j} = (b_{i,j} / b_{j,i})^{1/2}$. Then the restrictions (15) can be recorded as

$$c_{i,j}/\lambda \le t_{i,j} \le \lambda/c_{j,i} \cup d_{i,j}/\lambda \le t_{i,j} \le \lambda/d_{j,i},$$

$$(i,j) \in U = X \times X. \quad (16)$$

$$\lambda \ge 1. \quad (17)$$

The type of restrictions (16) allows to consider elements $t_{i,j}$ with $j > i, i,j \in X$. The process of searching for supertransitive matrix $T = (t_{i,j})$ most closely correlated to expert estimates is based on the approach developed in [12]. Each supertransitive matrix is determined by any of its column (row). Using the ratio (16), let's examine the elements of supertransitive matrix $T = (t_{i,j})$, for which condition $i < j, i,j \in X$ holds. For a fixed value $\lambda \ge 1$ let's define the permissible values of $t_{1,2}$ (satisfying (16)). Further, when a permissible value of the element $t_{2,3}$ using the ratio (3) feasible set of columns is calculated $\{(t_{1,3}, t_{2,3}, 1)^T\}$($T$–transposition sign). Continuing the calculation process in a similar way, one can define a set of columns with the number$n$: $\{(t_{1,n}, t_{2,n}, \ldots, t_{n-1,n}, 1)^T\}$. The scheme of calculations to solve the problem (7), (9), (16), (17) implies an increase in value $\lambda = 1$ until the solution (16) is achieved for $T = (t_{i,j})$. Let's designate this value of λ asλ_0. From the thus obtained sets of super transitive matrices the ones for elements of which the following ratio holds true are isolated: $t_{i,j}$- it belongs to the left (right) boundary of one of the corresponding interval (16), and $t_{i,k}, t_{k,j}$–belong to both right (left) boundaries. If the above condition is satisfied for several intervals describing the intensity of preferences for any pair of objects, then they segregate a set of matrices into disjoint classes, which are then examined independently of each other. For simply connected intervals of expert estimates, these elements will be the only ones on the respective intervals. Identified in such a way elements thus further considered constants, which can be considered as either the left or the right boundaries of the corresponding expert estimates. The computation process continues the same way until all elements of the matrices (one matrix only for simply connected intervals) are determined. The corresponding sequence of values λ is subject to the condition $\lambda_0 > \lambda_1 > \cdots > \lambda_l$.

5. PROPERTIES OF THE MODEL OF OBJECTS RANK-ORDERING

5.1. Consistency of the model when using a scale of the logarithmic relationship

The theory of intentionality [19] states that a property of a model is meaningful (intentional) if its truth is not affected by any permissible transformation of the measurement scale. A scale of logarithmic relationships is chosen as such scale. Let $x = (x_1, \ldots, x_n)$ and $x' = (x'_1, \ldots, x'_n)$ are weights of objects for relations R and R', accordingly. Relation R' is obtained from the relation R using a valid conversion $\Psi = \{\psi | \psi(x) = x^\beta, \beta > 0\}$. The relation R' converted using $\psi \in \Psi$ looks like this:

$$R' = \{((i,j), r'_{i,j}) | (i,j) \in X \times X\},$$
$$r'_{i,j} = r^\beta_{i,j}, \beta > 0, \forall (i,j) \in X \times X.$$

In order to show the intentionality of the method, it is necessary to make sure that the vectors x and x' establish the same order on the initial set of objects X:

$$x_i > x_j \iff x'_i > x'_j, \forall\, i.j \in X, i \ne j$$

In matrix form this relationship can be represented as

$$t_{i,j} > 1 \Leftrightarrow t'_{i,j} > 1, \forall\, i.j \in X, i \ne j. \quad (18)$$

Proposition 1. The method has the property of consistency, that is, the ratio is true (18).

Proof. For the interval relation R' the method begins to work at points $c'_{i,j} = c^\beta_{i,j}$. Elements of the matrix are determined from the relations

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$$t'_{i,j} = t'_{i,k} \cdot t'_{k,j}. \tag{19}$$

In this equation $t'_{i,j}$ belongs to the left (right) boundary, and $t'_{i,k}, t'_{k,j}$ belong to the right (left) boundaries of the corresponding intervals. The equality (19) for the interval relation R'can be obtained from the equations (3) for interval relation R when raised to β power. At the same time the following relation holds true $\lambda'_k = \lambda_k^{\beta}, k = 0,1, \dots, M$, where M- the number of iterations in defining elements of supertransitive matrices for the interval relations R and R'.Therefore, $t'_{i,j} = t_{i,j}^{\beta}$, which proves the inequality (18) and the intentionality of the rank-ordering model.

5.2. Preservation of optimality

Let's construct the interval relation R' on the basis of interval relation R by replacing certain values of $C_{i,j}$ for resulting values $t_{i,j}, (i,j) \epsilon\, U \subset X \times X$. The resulting solution for the interval relation R' is denoted as T'. The model has a property of preserving optimality (PO), if the equation $T = T'$ holds.

Proposition 2. The model has the property of (PO).

Proof. Let for $(s,d) \in X \times X, (s,d) \notin U$ for interval solution R' with $\lambda = \lambda'$ the value $t'_{s,d}$ be calculated. For the interval relation R with $\lambda = \lambda^0$ for$(s,d) \in X \times X, (s,d) \notin U$ the value $t^0_{s,d} \in [C_{s,d}/\lambda^0, C_{s,d} \cdot \lambda^0]$ was obtained. In the first instance$t'_{s,d} \notin [C_{s,d}/\lambda^0, C_{s,d} \cdot \lambda^0]$. This case cannot be realized, because the method will stop at the left or right boundary of the interval $[C_{s,d}/\lambda^0, C_{s,d} \cdot \lambda^0]$, and this interval will not expand. In the second instance,

$$t'_{s,d} \in \left[C_{s,d}/\lambda^0, C_{s,d} \cdot \lambda^0\right] \text{ and } t'_{s,d} \neq t^0_{s,d}.$$

In other words, $t'_{s,d} \in \left[C_{s,d}/\lambda', C_{s,d} \cdot \lambda'\right]and\lambda' < \lambda^0$. In this case the method for the relation R should've calculated the value of $t^0_{s,d}$ with $\lambda = \lambda'$. This is a contradiction. Therefore, $\lambda' = \lambda^0$. According to the algorithm, the values $t'_{s,d}$ and $t^0_{s,d}$ belong simultaneously to both left or right boundaries$\left[C_{s,d}/\lambda^0, C_{s,d} \cdot \lambda^0\right]$. Thus, $t'_{s,d} = t_{s,d}$.

5.3. The property of transposable

Transposable ratio R^T in relation to the ratio R is determined this way:
$R^T = \{(i,j), r^T_{i,j} | r^T_{i,j} = r_{j,i}\}$.

Here transposable ratio means the equalities $C^T_{i,j} = C_{j,i}, (i,j) \in X \times X$ hold true.

Proposition 3. The model has the property of transposable, that is the matrix T, calculated for the ratio R, and the matrix T', calculated for the ratio R^T, are related in the following manner: $T' = T^T$(superscript T is a sign of the transposition).

Proof. Since $C^T_{i,j} = C_{j,i}$ the method calculates values $t'_{i,j} = t_{j,i}, (i,j) \in X \times X$. Proposition 3 is proved.

5.4. The property of positive relation (PR) with expert estimates

This property will be explored for simply connected interval estimates.

By definition, the method of rank-ordering of objects has a property of positive relation, if a new judgment of pairwise comparison between objects k and l happened in favor of k, and it does not change its relative weight in the new rank-ordering. It is assumed here that other estimates have not changed their value. This means constructing new interval relation R', different from$R$ only for the pair $(k,l) \in X \times X$: $C'_{k,l} > C_{k,l}$, where inequality $t'_{k,l} \geq t_{k,l}$ holds true.

Proposition 4. The proposed method of rank-ordering objects possesses the property PR.

Proof. For the interval relation R the value of the element of the supertransitive matrix $t_{k,l}$ was calculated for $\lambda = \tilde{\lambda}$,$\{T(\tilde{\lambda})\}$ is a set of matrices containing that element. Let's note that if the value of element $t_{k,l}$was determined at the last step then the set $\{T(\tilde{\lambda})\}$ consists of one matrix. Thus,

$$t_{k,l} \in \left[C_{k,l}/\tilde{\lambda}, \quad C_{k,l} \cdot \tilde{\lambda}\right]. \tag{20}$$

ForR'– the value of $t'_{k,l}$ was determined with$\lambda = \lambda'$, and

$$t'_{k,l} \in [C'_{k,l}/\lambda', C'_{k,l} \cdot \lambda'] \tag{21}$$

For the relation R' let's denote the set of matrices, containing $t'_{k,l}$, and satisfying (21), by$\{T'(\lambda')\}$. Notice that $C'_{k,l}/\lambda' \geq C_{k,l}/\tilde{\lambda}$, because the method does not expand the corresponding intervals if solution exists.

According to the algorithm, the value $t_{k,l}$ for the relation R can belong either to the left boundary of the interval (20)–$t_{k,l} = C_{k,l} / \tilde{\lambda}$–the first case, or to the right boundary – $t_{k,l} = C_{k,l} \cdot \tilde{\lambda}$– the second case. For the first case lets select the value λ'' in such a way, that the equality $C'_{k,l}/\lambda'' = t_{k,l}$ is true, with that $\lambda'' > \tilde{\lambda}$, and $\lambda' \leq \lambda''$. In this case the following relation holds true
$\left[C'_{i,j}/\lambda', C'_{i,j} \cdot \lambda'\right] \subseteq \left[C'_{i,j}/\lambda'', C'_{i,j} \cdot \lambda''\right]$,
$(i,j) \in X \times X, i \neq j$.

Thus, the set of supertransitive matrices calculated for relations $R'$with $\lambda = \lambda'$, is a subset of the set of supertransitive matrices obtained for the ratio R' with $\lambda = \lambda''$:$\{T'(\lambda')\} \subseteq \{T'(\lambda'')\}$. All matrices$T'(\lambda'')$, belonging to the$\{T'(\lambda'')\}$, contain elements$t'_{k,l} \geq t_{k,l}$, since the algorithm does not expand the boundaries of corresponding intervals for the relation R' if at least one permissible matrix exists. The first case is proven.

For the second case the inequality $t'_{k,l} < t_{k,l}$ can not be performed, since for the interval relation R for the pair (k,l) the corresponding value of $t'_{k,l}$ is impermissible. Consequently, $t'_{k,l} \geq t_{k,l}$. Thus, Proposition 4 is proven.

5.5. Preservation of dominance

The object i dominates over object j in relation to$R$, if

$r_{i,j} \succcurlyeq r_{j,i}, r_{i,k} \succcurlyeq r_{j,k}, r_{k,i} \preccurlyeq r_{k,j}, \forall\, k \in X.$ (22)

Here the sign $\succcurlyeq$ ($\preccurlyeq$) means "not worse than" ("no better than"). The condition (22), as in the case of studying a positive relation, will be presented in the form of

$C_{i,j} \geq C_{j,i}, C_{i,k} \geq C_{j,k}, C_{k,i} \leq C_{k,j}, \forall\, k \in X.$ (23)

The method has a property of dominance (DP), if the inequality (23) results in domination (nonstrict) of object i over object j in the final rank ordering of objects, that is $\hat{t}_{i,j} \geq 1$.

Proposition 5. The proposed method has the DP property.

Proof. Proof, like in the case of property PR, will be conducted for simply connected expert estimates. From the inequality $C_{i,j} \geq C_{j,i}$ follows that $C_{i,j} \geq 1$.

Let's solve the problem of rank-ordering objects for the relation R. Matrix $\hat{T}$ is obtained as a result.

The element of the matrix of the solution $\hat{t}_{i,j} \in \hat{T}$ on the basis of (3) is presented as

$\hat{t}_{i,j} = \hat{t}_{i,l} \cdot \hat{t}_{l,j} = \hat{t}_{i,l} \,/\, \hat{t}_{j,l}, l \in X \times X, l \neq i, l \neq j$

Therefore, further rows i and j are examined.

First, let's examine the case, when values of elements $t_{i,j}, t_{i,l}, t_{j,l}$ are calculated simultaneously, in other words, vertices i, j, l lie on the single contour line, and the arcs $(i,j), (j,l), (l,i)$ form that contour line. There are two possible case. The first case: $C_{i,j} \cdot C_{j,l} \leq C_{i,l}$, the value $\lambda \geq 1$is determined from the equation $C_{i,j} \cdot \lambda \cdot C_{j,l} \cdot \lambda = C_{i,l}/\lambda$, with that the value $\hat{t}_{i,j} = C_{i,j} \cdot \lambda \geq 1$. The second case: $C_{i,j} \cdot C_{j,l} > C_{i,l}$, value $\lambda > 1$is determined from the equality $C_{i,j}/\lambda \cdot C_{j,l}/\lambda = C_{i,l} \cdot \lambda$, value $\hat{t}_{i,j} = C_{i,j}/\lambda = C_{i,l} \,/\, C_{j,l} \cdot \lambda^2 > 1$.

In what follows let's set $n \geq 4$. Note, that the case where $n = 3$, was explored above.

Below it is assumed that the values of elements $t_{i,j}, t_{i,l}, t_{j,l}$ were calculated at different values of λ.

Let element $\hat{t}_{j.l} \in \hat{T}$ be calculated first in rows i and j with $\lambda = \lambda_1$, the rest of the elements in those rows were calculated not prior to it, and element $\hat{t}_{i,l}$ was determined with $\lambda = \lambda_2,\ \lambda_1 \geq \lambda_2$. Let $\hat{t}_{j,l} = C_{j,l} \,/\, \lambda_1$, then for this value case $\hat{t}_{i.l} = C_{i,l}/\lambda_2$ or $\hat{t}_{i.l} = C_{i,l} \cdot \lambda_2$ are possible. For the first case the equation $\hat{t}_{i,j} \cdot C_{j,l}/\lambda_1 = C_{i,l} \,/\, \lambda_2$ holds, from which $\hat{t}_{i,j} \geq 1$ follows. For the second case the $\hat{t}_{i,j} \cdot C_{j,l}/\lambda_1 = C_{i,l} \cdot \lambda_2$ holds, from which $\hat{t}_{i,j} \geq 1$ follows. Let's consider the value $\hat{t}_{j,l} = C_{j,l} \cdot \lambda_1$. To calculate it, according to the method, there must be an object with a number m for which the equation $C_{j,l} \cdot \lambda_1 \cdot C_{l,m} \cdot \lambda_1 = C_{j,m}/\lambda_1 = \hat{t}'_{j,m}$ holds. The case $\hat{t}'_{j,m} = C_{j,m} \,/\, \lambda_1$ was explored above, hence, $\hat{t}_{i,j} \geq 1$.

Now suppose that the element $\hat{t}_{i,l} \in \hat{T}$ was calculated first in the rows i and j with $\lambda = \lambda_1$, the rest of the elements in those lines were calculated not prior to it. Element $\hat{t}_{j,l}$ is calculated at $\lambda = \lambda_2,\ \lambda_1 \geq \lambda_2$.

Let $\hat{t}_{i,l} = C_{i,l} \cdot \lambda_1$, then the following case are possible: $\hat{t}_{j.l} = C_{j,l}/\lambda_2$ or $\hat{t}_{j.l} = C_{j,l} \cdot \lambda_2$. For the first case equation $\hat{t}_{i,j} \cdot C_{j,l}/\lambda_2 = C_{i,l} \cdot \lambda_1$ holds, from which $\hat{t}_{i,j} \geq 1$ follows. For the second case equation $\hat{t}_{i,j} \cdot C_{j,l} \cdot \lambda_2 = C_{i,l} \cdot \lambda_1$ holds, from which $\hat{t}_{i,j} \geq 1$ follows. Consider value $\hat{t}_{i,l} = C_{i,l}/\lambda_1$. Then, according to the method for its calculation shall be subject to the number m for which the equation $\frac{C_{i,l}}{\lambda_1} \cdot \frac{C_{l,m}}{\lambda_1} = C_{i,m} \cdot \lambda_1 = \hat{t}_{i,m}$ holds. The case of $\hat{t}_{i,m} = C_{i,m} \cdot \lambda_1$ was explored above, therefore $\hat{t}_{i,j} \geq 1$. Thus, Proposition 5 is proven.

6. EXAMPLE OF CALCULATIONS

Paper [6] gives an example of describing distances from Philadelphia to six cities in the form of point estimates. That example presents the expansion of this problem for multi-connected interval estimates. Experts determined the range of changes of elements of the pairwise comparisons matrix $s_i \,/\, s_j$, where s_i, s_j- are distances from Philadelphia to cities i and j, respectively. Table 1 presents information, defined as system of interval expert estimates. The same paper presents normalized relations of factual distances and priority vector calculated using the developed method.

On the first stage value$\lambda_0 = 1{,}37$ was determined. With that value the following values of super transitive matrix$T = (t_{i,j})$: $t_{2,5} = 2{,}0646$; $t_{2,6} = 30{,}1293$; $t_{5,6} = 14{,}5985$. The next value is $\lambda_1 = 1{,}2388$, with it $t_{1,5} = 1{,}427$; $t_{1,6} = 20{,}8321$ were calculated. With $\lambda_2 = 1{,}1649 -$ $t_{1,4} = 2{,}3509$; $t_{2,4} = 3{,}401$; $t_{4,5} = 0{,}607$; $t_{4,6} = 8{,}8615$.

Further, $\lambda_3 = 1{,}1437$, with that value $t_{1,2} = 0{,}6912$ was calculated. The last value is $\lambda_4 = 1{,}1164$, the rest of the elements calculated with that value are $t_{1,3} = 9{,}3861$; $t_{2,3} = 13{,}5787$; $t_{3,4} = 0{,}2504$; $t_{3,5} = 0{,}152$; $t_{3,6} = 2{,}2192$.

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Table-1.

Comparison	Cairo	Tokyo	Chicago	San Francisco	London	Montreal
Cairo	1	[0,5;1,25] ∪[2;3]	[5;10] ∪[12,15]	[1,5;5]∪ [8,10]	[1,25;2,5] ∪[5;6]	[2;5] ∪ [10;20]
Tokyo	[0,33;0,5] ∪[0,8;2]	1	[10;20] ∪[25;30]	[2;5] ∪ [6;7]	[2;4] ∪ [5;10]	[10;12] ∪ [15;25]
Chicago	[0,07,0,83] ∪ [0,1;0,2]	[0,03;0,04] ∪ [0,05;0,1]	1	[0,1;0,5] ∪[0,75;0,8]	[0,1;0,25] ∪[0,67;0,5]	[1;2,5] ∪ [4;5]
San Francisco	[0,125;0,1] ∪ [0,2;0,67]	[0,14;0,17] ∪ [0,2;0,5]	[1,25;1,33] ∪ [2;10]	1	[0,5;1] ∪ [1,25;2]	[2;5] ∪ [6;10]
London	[0,17;0,2] ∪ [0,4;0,8]	[0,1;0,2] ∪ [0,25;0,5]	[2;3] ∪ [4;10]	[0,5;0,8] ∪ [1;2]	1	[4;10] ∪ [11;15]
Montreal	[0,05;0,1] ∪ [0,2;0,5]	[0,04;0,67] ∪ [0,05;0,1]	[0,2;0,25] ∪ [0,4;1]	[0,1;0,17] ∪[0,2;0,5]	[0,91;0,067] ∪ [0,1;0,25]	1
normalized distance	0,278	0,361	0,032	0,132	0,177	0,019
priority vector	0,2683	0,3881	0,0286	0,1141	0,1880	0,0129

7. CONCLUSIONS

Research of various types of processes in the environmental, economic and technical sphere, as a rule, use opinions of specialists in a given subject area- experts. The developed method of processing such information is effective at the stage of express analysis of these processes (relation of the significance of the project, the means to achieve goals and such.). Under the conditions of incomplete information or its qualitative description it is necessary to solve an important problem of quantification (quantitative relation) of the studied objects. The proposed approach to the determining weights of objects will allow to aggregate interval estimates, given in the form of a system of non-overlapping intervals, characteristic to non-consensual expert estimates.

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