VOL. 12, NO. 17, SEPTEMBER 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

# FRICTIONAL EFFECT AND LOAD CARRYING CAPACITY IN POROUS INCLINED MULTI STEPPED COMPOSITE BEARINGS WITH COUPLE STRESS FLUIDS

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## ABSTRACT

This paper presents the frictional effect and load carrying capacity in porous inclined multi stepped composite bearings with couple stress fluids. The most general form of the modified Reynolds type equations derived for the squeeze film lubrication of the porous inclined multi stepped bearings and an expressions for pressure distribution, work load and frictional force is mathematically derived. These expressions can be utilized to obtain the performance characteristics of different types of geometry under consideration. The important parameters for discussing the behaviour of bearing characteristics are the couple stress parameter $l^* = \left(\frac{l}{h_3}\right)$, the percolation parameter 'β' and the permeability parameter 'Ψ'. The global system of equations was obtained for the bearing and solved using matlab iterative scheme. It is observed that there is a significant increase in the pressure distribution this will lead to an increase in load capacity for fluids with couple stress in comparison with Newtonian fluids and the coefficient of friction for inclined stepped composite bearing compared to other geometries. According to the results, the couple stress fluid as a lubricant improves the squeeze film characteristics and results in a longer bearing life.

**Keywords:** couple stress fluid, inclined multi stepped composite bearing, porous.

## INTRODUCTION

Much equipment in everyday use, such as small domestic appliances and audio equipment has porous bearings to support a rotating shaft. The bearings in this type of equipment can only be supplied with lubricant once, during manufacture, so the use of porous bearings is an obvious solution. With these bearings problems are regularly encountered such as noise, loss of lubricant, premature failure, irregular friction, and inaccurate shaft position. In a practical investigation, attempts were made to obtain a better understanding of the causes of these problems, and, where possible, to lay down guidelines for restricting or avoiding them. The application of porous bearings in mounting house power motors include vacuum cleaners, coffee grinders, hair driers, saving machines, sewing machines, water pumps, record players, generators and distributors, Kumar(1980), Murti (1974), Srinivasan (1977) have analyzed the porous slider bearing by Darcys equation to model the flow of Newtonian lubricant in the porous media with taking into consideration Newtonian fluid. The major advantage of porous bearing is that they require no exterior oil supply once the bearing with porous material impregnated with oil is installed.

Slider bearing have practical applications in machine design and in many kinds of machine elements in which rectilinear sliding motions occur. The shape of the slider bearing is one of the major geometric conditions influencing the performance of the bearings. Therefore, optimizing the shape of the slider bearing for different purposes has become an important topic for researches in recent years. Rohde analyzed a 1-D slider bearing to determine the optimum film profile, which minimizes the total friction force for a given load using a variation method.

Lin *et al* (6, 7) studied the linear stability analysis of wide inclined plane slider bearings and dynamic characteristics for wide exponential slider bearings. The experimental results suggest that, the small amount of additives can help to stabilize the flow properties and minimize the sensitivity of the lubricant to change in shear rate. Further experiment by N. M. Bujurke, H. P. patil and S. G. Bhavi (1989) on porous slider bearing with couple stress fluid analyzed that the presence of additives in the lubricant can create a significant change in the pressure distribution resulting in a gain in the load capacity of the bearing with the assumption the fluid is incompressible and non Newtonians. Several micro continuum theories have been proposed by Ariman *et al* [10] and stokes [11]. Stokes problems for an incompressible couple stress fluid by M. Devakar, T.K.V Iyengar (2008) in which they studied about stokes first and second problems for an incompressible couple stress fluids [29], for both the problems analytical solutions are obtained in Laplace transform domain, and it is found that an increase in the couple stress parameter has a decreasing influence on the velocity. Stokes [11] theory of couple stress fluid model has been widely used to study various bearings, such as considering the elasticity of the liner in journal bearing on the basis of stokes couple stress theory the main objective of this paper is to study the effects of couple stress fluids on pressure distribution, load carrying capacity and frictional force on the inclined multi stepped composite bearings.

VOL. 12, NO. 17, SEPTEMBER 2017 ISSN 1819-6608
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## MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Figure-1 shows a schematic diagram of the squeeze film geometry under consideration.

It consists of two surfaces separated by a lubricant. The x axis is taken along its length, while y axis is across lubricant film. Lower plane having velocity moving along x axis in its own plate.

With reference to Figure-1 the film thickness h(x) is defined by:

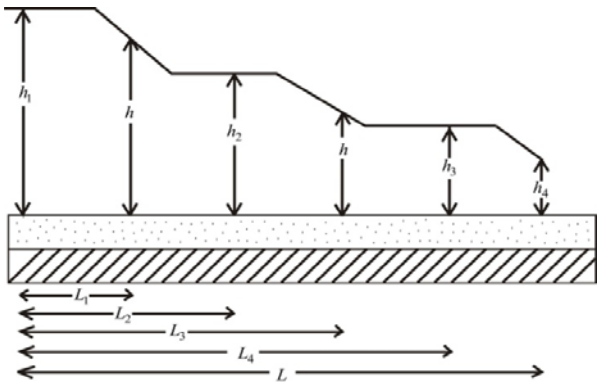


**Figure-1.**

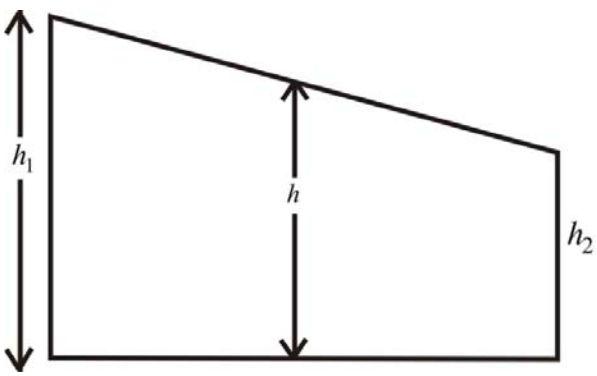


**Figure-1(a).**

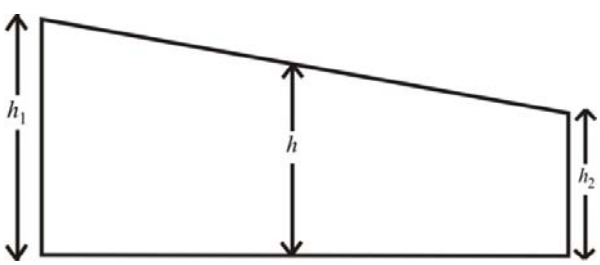


**Figure-1(b).**

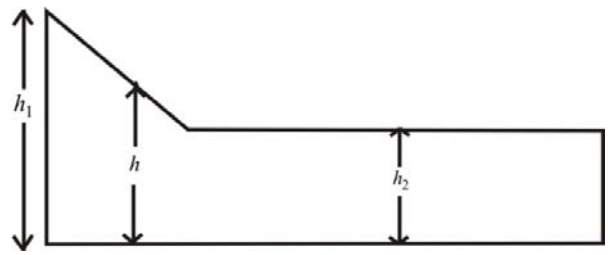


**Figure-1(c).**

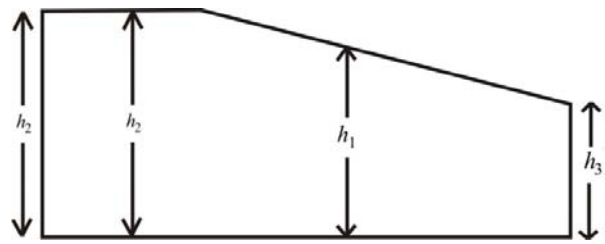


**Figure-1(d).**

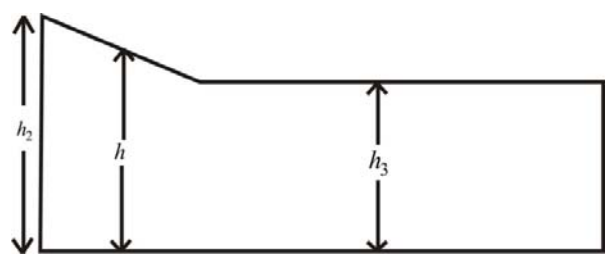


**Figure-1(e).**

$$h(x)=\begin{cases} h_1 & 0\le x\le L_1 \\ \dfrac{(h_2-h_1)(L_1-x)}{(L_2-L_1)}+h_1 & L_1\le x\le L_2 \\ h_2 & L_2\le x\le L_3 \\ \dfrac{(h_2-h_4)(L_3-x)}{(L_4-L_3)}+h_2 & L_3\le x\le L_4 \\ h_4 & L_4\le x\le L \end{cases} \quad (1)$$

Under the usual assumption of hydrodynamic lubrication:

a) The body forces and the body couples are absent and the fluid is incompressible.
b) The inertial effects are neglected in the film region.
c) The pressure is independent of z coordinate.

The continuity equation is written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2)$$

The basic equation governing the motion of an incompressible couple stress fluid in the absence of body forces and the body couple derived by stoke is

$$\rho\frac{D\vec{v}}{Dt} = -\nabla p + \mu\nabla^2\vec{v} - \eta\nabla^4\vec{v} \tag{3}$$

$$\mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 v}{\partial y^4} = \frac{\partial p}{\partial x} \tag{4}$$

Where u, v, w are the fluid velocity components along x, y, z direction respectively, p is the pressure in the film region.

**Boundary condition**

At upper surface y=h

$$u = 0, v = 0, \frac{\partial^2 u}{\partial y^2} = 0 \tag{5a}$$

At lower surface y=0

$$u = U, v = -v^*, \frac{\partial^2 u}{\partial y^2} = 0 \tag{5b}$$

Where v* is the modified Darcy velocity component in the y direction for the flow of couple stress in porous region.

The flow of couple stress fluid in the porous matrix is governed by the modified form of Darcy's law which account for the polar effects is:

$$q^* = \frac{-K}{\mu(1-\beta)}V_p^*$$

Where $q^* = \left(u^*, v^*\right)$ where $u^*, v^*$ are the Darcy velocity along x,y direction consider $p^*$ the pressure in the porous region and 'K' is the isotropic permeability of the porous matrix.

**Solution of the problem**

$$u = U\left(1-\frac{y}{h}\right) + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[y^2 - yh + 2l^2\left\{1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right\}\right] \tag{6}$$

Where $l^*$ is the couple stress parameter?

$$\frac{\partial}{\partial x}\left[h^3 f(h,l)\frac{\partial p}{\partial x}\right]_{y=0} = 6\mu U\frac{\partial h}{\partial x} + \frac{12K}{(1-\beta)}\frac{\partial p^*}{\partial x} \tag{7}$$

Where

$$f(h,l) = \left[1 - 12\frac{l^2}{h^2} + 24\frac{l^3}{h^2}\tanh\left(\frac{h}{2l}\right)\right]$$

Let '$\delta$' to be small (porous layer thickness), the Morgan Cameron approximation gives:

$$\left.\frac{\partial p^*}{\partial y}\right|_{y=0} = -\delta\frac{\partial^2 p}{\partial x^2} \tag{8}$$

We get the modified Reynolds equation in the form, after using (11) in (10)

$$\frac{\partial}{\partial x}\left[h^3 f(h,l) + \frac{12K\delta}{(1-\beta)}\frac{\partial}{\partial x}\right] = 6\mu U\frac{\partial h}{\partial x} \tag{9}$$

Introducing the non dimensional quantities:

$$\left.\begin{aligned} x^* &= \frac{x}{L}, P = \frac{2ph_3^2}{\mu UL}, H = \frac{h}{h_4}, l^* = \frac{l}{h_4}, \\ L^* &= \frac{h_4}{L}, L_1^* = \frac{L_2}{L}, L_2^* = \frac{L_2}{L}, \psi = \frac{K\delta}{h_4^3} \end{aligned}\right\} \tag{10}$$

Equations (9) becomes

$$\frac{\partial}{\partial x^*}\left[\left(H^3 f(H,l^*) + \frac{\psi}{1-\beta}\right)\frac{\partial p}{\partial x^*}\right] = \frac{\partial H}{\partial x^*} \tag{11}$$

$$f(H,l^*) = \left[1 - \frac{12l^{*2}}{H^2} + \frac{24l^{*3}}{H^3}\tanh\left(\frac{H}{2l^*}\right)\right] \tag{12}$$

On integrating Eq (9) with respect to x*

$$\frac{\partial p}{\partial x^*} = \left[\frac{H - H_0}{H^3 f(H,l^*) + \frac{\psi}{1-\beta}}\right] \tag{13}$$

Where $h_0$ is the film thickness when $\frac{\partial p}{\partial x^*} = 0$

The relevant boundary condition for pressure is

P=0 at x*=0, 1 (14)

Pressure is continuous at $x^*=L_1^*$, $x^*=L_2^*$, $x^*=L_3^*$, $x^*=L_4^*$

Integrating Equation (9) by using (7) gives

$$P=\frac{(H_1-H_0)x^*}{H_1^3 f(H,l^*)+\frac{\psi}{1-\beta}} \quad 0\le x^*\le L_1^* \tag{15}$$

$$P=P_c+\int_{L_1}^{x^*}\frac{(H-H_0)\,\mathrm{d}x^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}},\ L_1^*\le x^*\le L_2^* \tag{16}$$

Where

$$P_c=\frac{(H_1-H_0)L_1^*}{H_1^3 f(H,l^*)+\frac{\psi}{1-\beta}} \tag{17}$$

$$P=P_c+\int_{L_2}^{x^*}\frac{(H_1+H)-H_0\,\mathrm{d}x^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}},\ L_2^*\le x^*\le L_3^* \tag{18}$$

Where

$$P_c=\frac{(H_2-H_0)L_2^*}{H_2^3 f(H,l^*)+\frac{\psi}{1-\beta}} \tag{19}$$

$$P=P_c+\int_{L_3}^{x^*}\frac{(H_1+H+H_2)-H_0\,\mathrm{d}x^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}},\ L_3^*\le x^*\le L_4^* \tag{20}$$

Where

$$P_c=\frac{(H_3-H_0)L_3^*}{H_3^3 f(H,l^*)+\frac{\psi}{1-\beta}} \tag{21}$$

$$P=\frac{(1-H_0)(x^*-1)}{f(1,l^*)+\frac{\psi}{1-\beta}},\ L_4^*\le x^*\le 1 \tag{22}$$

And film thickness is given by $h_0=\frac{a}{b}$

$$a=\frac{H_1L_1^*}{H_1^3 f(H_1,l^*)+\frac{\psi}{1-\beta}}-\frac{L_2^*}{f(1,l^*)+\frac{\psi}{1-\beta}}+\int_{L_1^*}^{L_2^*}\frac{H\,dx^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}+\int_{L_2^*}^{L_3^*}\frac{(H_1+H)dx^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}+\int_{L_3^*}^{L_4^*}\frac{(H_1+H+H_2)dx^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}$$

$$b=\frac{L_1^*}{H_1^3 f(H_1,l^*)+\frac{\psi}{1-\beta}}-\frac{(L_2^*-1)}{f(1,l^*)+\frac{\psi}{1-\beta}}+\int_{L_1^*}^{L_2^*}\frac{dx^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}+\int_{L_2^*}^{L_3^*}\frac{dx^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}+\int_{L_3^*}^{L_4^*}\frac{dx^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}$$

## LOAD CARRYING CAPACITY AND FRICTIONAL FORCE

The load carrying capacity W per unit width is

$$\omega=\int_0^1 p\,dx \tag{23}$$

$$W=\frac{2\omega h_3^2}{\mu U L^2} \tag{24}$$

$$=\left\{\frac{1}{2}\frac{(1-H_0)(L_4^*-1)}{f(1,l^*)+\frac{\psi}{1-\beta}}+\frac{1}{2}\frac{(H_1-H_0)L_1^*}{H_1^3 f(H,l^*)+\frac{\psi}{1-\beta}}+\int_{L_1^*}^{L_2^*}\left[\frac{(H_1-H_0)x^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}\right]dx^*+\int_{L_2^*}^{L_3^*}\left[\frac{(H_1+H-H_0)x^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}\right]dx^*+\int_{L_3^*}^{L_4^*}\left[\frac{(H_1+H+H_2-H_0)x^*}{H^3 f(H,l^*)+\frac{\psi}{1-\beta}}\right]dx^*\right\} \tag{25}$$

The frictional force 'f' per unit width on the bearing surface is given by:

$$f=\int_0^L\left(\tau_{yx}\right)_{y=0}dx \tag{26}$$

Where $\left(\tau_{yx}\right)=\mu\frac{\partial u}{\partial y}-\eta\frac{\partial^3 u}{\partial y^3}$ (27)

On using the expression (6) in equation (27) and substituting in equation (26), gives the frictional force f and $F = -\frac{2fh_4}{\mu UL}$ Which gives

$$= \frac{1}{H_1} + \frac{H_1(H_1 - H_0)}{2H_1^3 f\left(H, l^*\right) + \frac{\psi}{(1-\beta)}} + \frac{\left(1 - L_1^*\right)(1 - H_0)}{2f\left(1, l^*\right) + \frac{\psi}{(1-\beta)}} + \int_{L_1^*}^{L_2^*} \left[ \frac{(H_1 - H_0)x^*}{H^3 f\left(H, l^*\right) + \frac{\psi}{(1-\beta)}} \right] dx^* + \frac{\left(1 - L_2^*\right)(1 - H_0)}{2f\left(1, l^*\right) + \frac{\psi}{(1-\beta)}} +$$

$$\int_{L_2^*}^{L_3^*} \left[ \frac{(H_1 + H - H_0)x^*}{H^3 f\left(H, l^*\right) + \frac{\psi}{(1-\beta)}} \right] dx^* + \frac{\left(1 - L_3^*\right)(1 - H_0)}{2f\left(1, l^*\right) + \frac{\psi}{(1-\beta)}} + \int_{L_3^*}^{L_4^*} \left[ \frac{(H_1 + H + H_2 - H_0)x^*}{H^3 f\left(H, l^*\right) + \frac{\psi}{(1-\beta)}} \right] dx^* + \frac{\left(1 - L_4^*\right)(1 - H_0)}{2f\left(1, l^*\right) + \frac{\psi}{(1-\beta)}}$$

$$+2\left(1 - L_4^*\right)$$

The coefficient of Friction is given by C=F/W

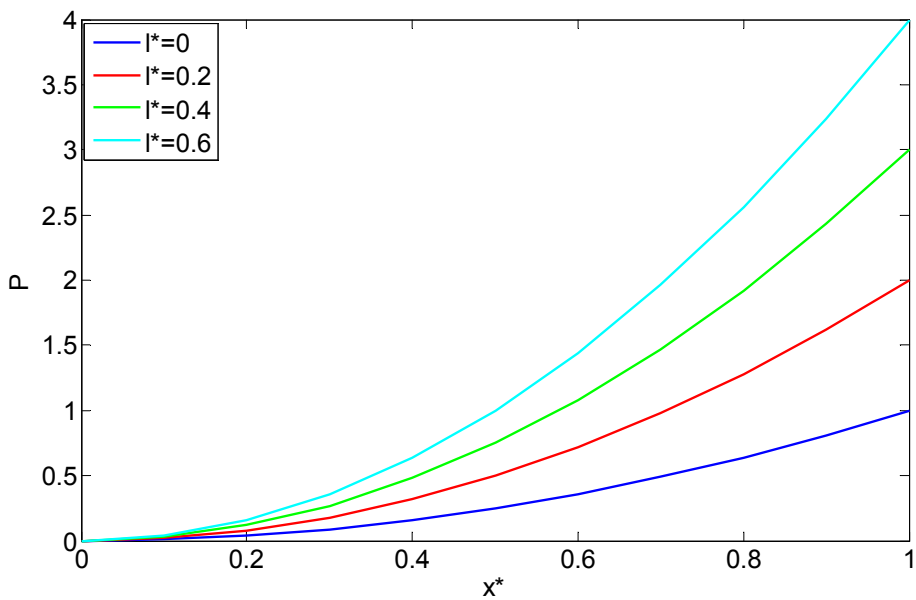


**Figure-2.**Variation of non dimensional Pressure with dimensionless x* coordinate for different values of $l^*$ with $\psi$ =0.001 and $\beta$ =0.3

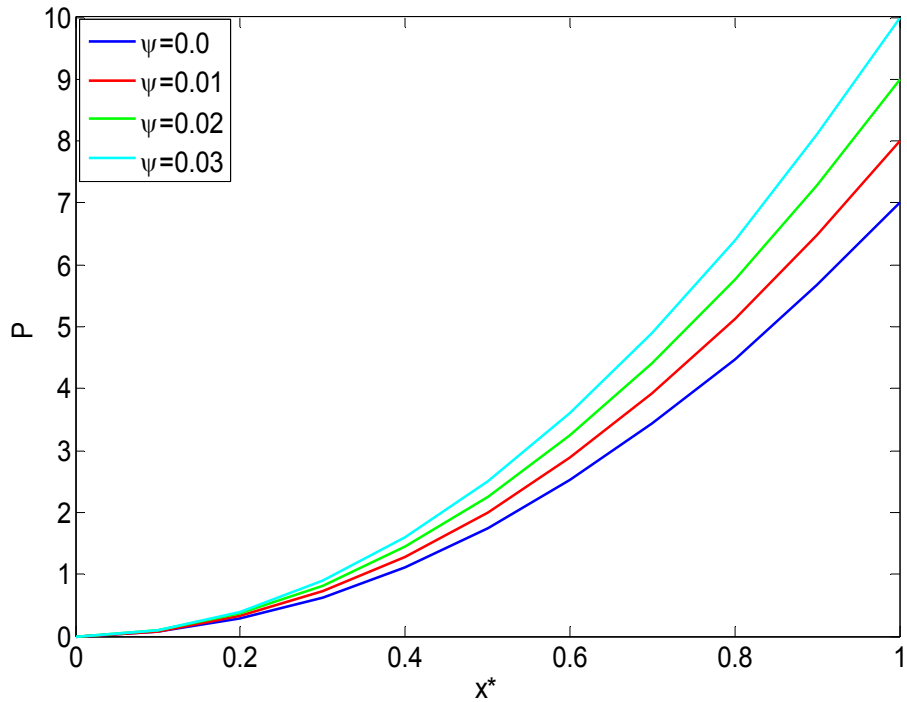


**Figure-3.** Variation of non dimensional pressure with dimensionless x* coordinate for different values of $\psi$ with $l^*$ =0.2 and $\beta$ =0.35.

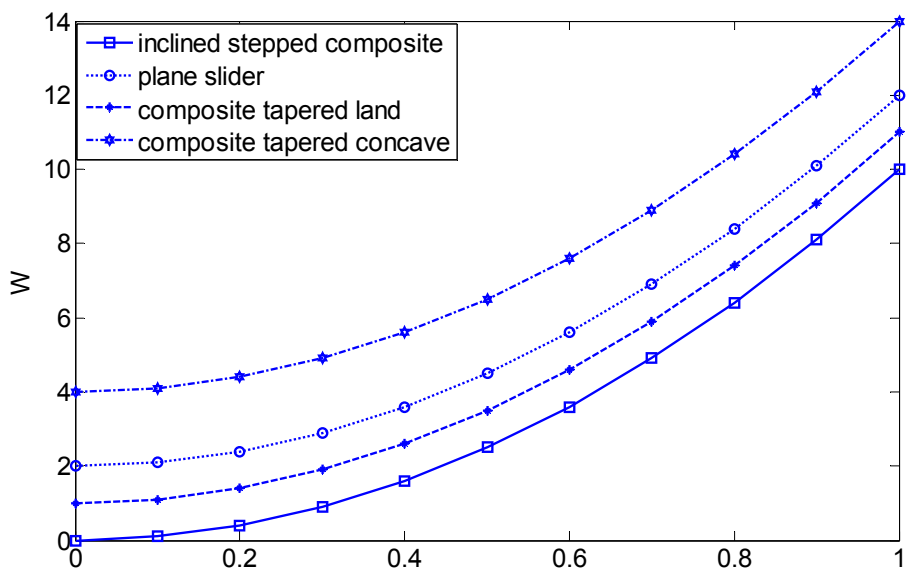


**Figure-4.** Variation of non dimensional load W with $l^*$ for different values of bearings with beta=0.35.

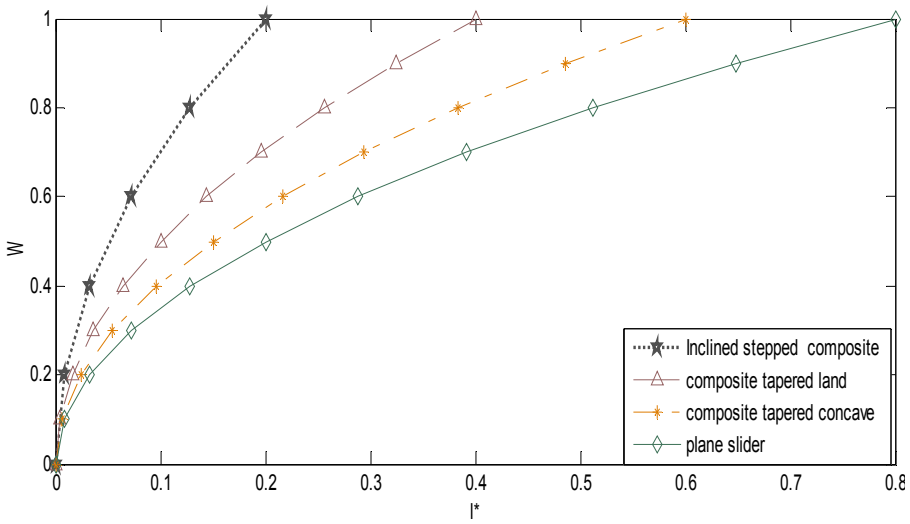


**Figure-5.** Variation of non dimensional load W with $l^*$ for different values of bearings with $\beta$=0.35.

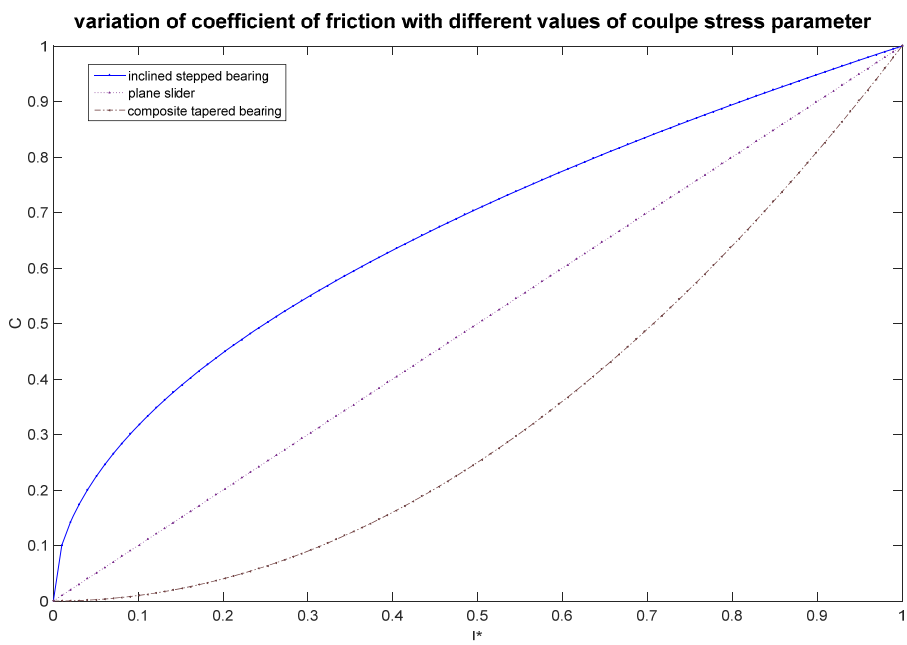


**Figure-6.** Variation of coefficient of friction with different values of Couple Stress Parameter.

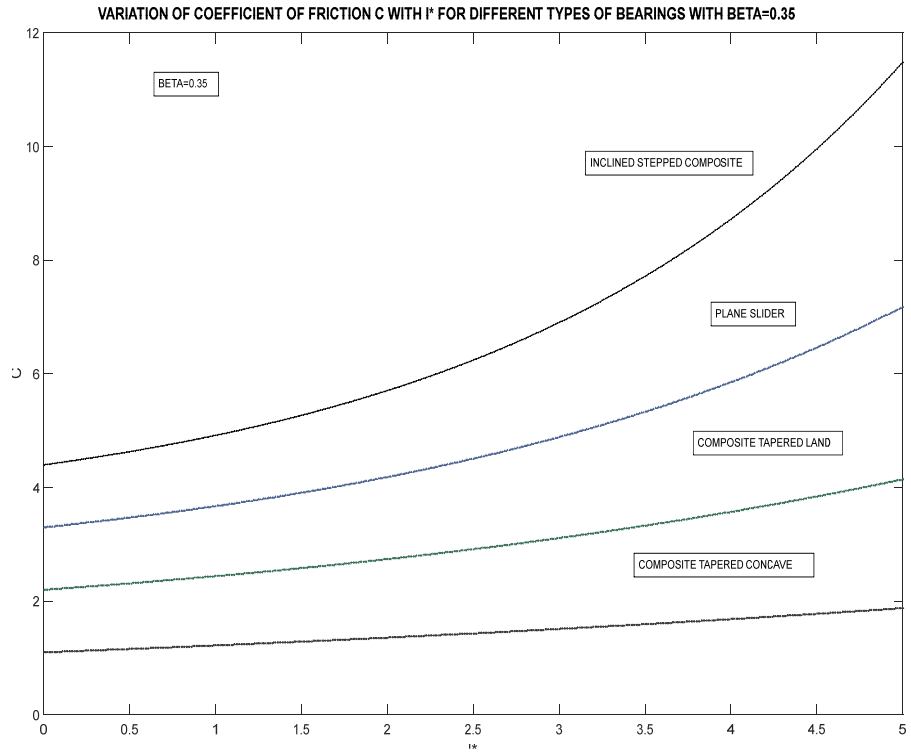


**Figure-7.**Variation of coefficient of friction with Couple Stress Parameter for different types of bearings with $\beta$=0.35

## RESULTS AND DISCUSSIONS

This paper predicts the influence of couple stresses on the lubrication of porous inclined multi stepped composite bearings. According to the Stokes theory, the couple stress parameter characterizes the effects of couple stresses on the bearing characteristics of the system. If we assume $l^* \to 0$ then the dimensionless Reynolds equation reduces to the newtonian lubricant case. When the value of $l^*$ is large, the couple stress effects are known to be more. The permeability parameter $\psi = \frac{k\delta}{h_4^{\,3}}$ is responsible for the ability of a porous material to allow fluids to pass through it. The permeability of a medium is related to the porosity, but also to the shapes of the pores in the medium and their level of connectedness.

In the present paper, the values of couple stress parameter and permeability parameter are chosen as

$$l^* = 0.0, 0.2, 0.4, 0.6$$

$$\psi = 1\text{x}10^{-5}, 1\text{x}10^{-4}, 1\text{x}10^{-3}, 1\text{x}10^{-2}, 1\text{x}10^{-1}$$

In Figure-3, depicts the variation of non dimensionless pressure P with x* for different values of permeability parameter. The effect of permeability parameter is to decrease the pressure P as compared to the solid case.

Figure-4 presents the variation of non dimensional load carrying capacity with couple stress parameter for different types of bearing.

Figure-5 shows the variation of non dimensional load carrying capacity with couple stress having constant $\beta = 0.35$.

Figure-6 Shows for different values of permeability parameter $(\psi = 0, \psi = 0.01)$ the relationship between the force of friction between two object and the normal force between the forces is constant. It shows that no gradual change which indicates that it has more frictional value (>1).

Figure-7 it is shown that the variation of coefficient of friction C with $l^*$ for different types of bearings with $\beta = 0.35$.

## CONCLUSIONS

On the basis of micro- continuum theory this paper predicts the performance of squeeze film characteristics of porous inclined multi-stepped composite bearings with the help of various non- dimensional parameters

The following conclusions are drawn on the basis of result presented in the last section.

The inclined stepped bearing has the largest film pressure than the other geometries. With the increase in steps, the multi-stepped bearings seem to have the largest pressure distribution than any other geometry. The porous consistency on the bearing surfaces reduces the Work load and increases the frictional force and to minimize the effect of porous facing is to select the lubricant according to the bearing characteristics. We conclude that the

coefficient of friction depends on the objects that are causing friction; it can be changed by the mass and speed of the moving object. In the lubrication process, the chosen starting point of the lubricant film has a considerable influence on the determination of the load capacity. Increase in Load capacity indicates if a bearing suffers a decrease in load carrying capacity, the fluid pressure will increase to compensate and support the load which characterizes the bearing.

Numerical explanation of porous inclined multi-stepped composite bearings with couple stress fluids.

| Physical parameter | Notations | Range of values taken |
|---|---|---|
| Length of the bearings | L | 10mm |
| Minimum film thickness | $h_4$ | 0.05mm |
| Maximum film thickness | $h_1$ | 0.05mm |
| viscosity | $\mu$ | 580cp |
| Porous layer thickness | $\delta$ | 10mm |
| permeability | k | $12.5x10^{-12}m^2, 12.5x10^{-16}m^2\ 12.5x10^{-15}m^2,$ $12.5x10^{-14}m^2, 12.5x10^{-12}m^2, 12.5x10^{-11}m^2,$ |
| Material constant | $\eta$ | $5x10^{-15}Ns, 1.5x10^{-15}Ns$ |

**Nomenclature**

h = Film thickness given in (1)
$h_0$ = Film thickness when =0
$h_1$ = Inlet film thickness
$h_2$ = Inlet film thickness
$h_3$ = Outlet film thickness
$h_4$ = Minimum film thickness
H = Dimensionless film thickness (=$h/h_4$)
$H_0$ = Dimensionless film thickness when =0
$H_1$ = Dimensionless inlet film thickness ($h_1/h_2$)
$H_2$ = Dimensionless inlet film thickness ($h_1/h_3$)
K = Permeability of the porous bearing
L = Total length of the sliding surface along the direction of Motion
$L_1$ = Length of the leading parallel portion
$L_2$ = Sum of the length of the leading parallel portion and the inclined portion
$L_3$ = Sum of the length of the leading parallel portion, the Inclined and the last leading parallel portion.
$L_4$ = Sum of $L_1$, $L_2$, $L_3$ and the length of the inclined portion.
$l$ = Couple stress parameter
$l^*$ = Dimensionless couple stress parameter $\left(\frac{l}{h_3}\right)$
$L_1$* = Dimensionless form of $L_1$= ($L_1$/L)
$L_2$* = Dimensionless form of $L_2$= ($L_2$/L)
$L_3$* = Dimensionless form of $L_3$= ($L_3$/L)
$L_4$* = Dimensionless form of $L_4$= ($L_4$/L)
L = Length ratio = ($h_3$/L)
P = Dimensionless hydrodynamic pressure
p = Hydrodynamic pressure
u,v = Fluid velocity components in the x & y direction, respectively
w = Load carrying capacity/ width
W = Dimensionless load carrying capacity per unit width
x,y = Cartesian coordinates
x* = Dimensionless length coordinate=(x/L)
$\mu$ = Classical viscosity
$\Psi$ = Permeability parameter
C = Coefficient of frction
F = Frictional force per unit width on the surface y=0
F =Dimensionless force corresponding to $\left(=\frac{2fh_4}{\mu UL}\right)$

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