



A COMPARATIVE STUDY OF PROBABILITY OF FAILURE OF CONCRETE IN BENDING AND DIRECT COMPRESSION AS PER IS 456:2000 AND AS PER IS 456:1978

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ABSTRACT

The paper aims at ascertaining the reliability index of different grades of concrete in bending compression and direct compression both in limit state method and working stress method by considering the permissible stresses in both the methods by Level I reliability method. Thus probability of failure of concrete in bending compression and direct compression both in limit state method and working stress method are compared for different grades of concrete.

Keywords: bending compression, direct compression, partial safety factor, standard deviation, coefficient of variation, reliability index, probability of failure.

INTRODUCTION

Indian Standard Code of practice for Plain and Reinforced Concrete IS456:2000 is the fourth revision after the publication of the IS456. Several clauses were changed and incorporated afresh to emphasis quality control and control assurance measures of concrete. It has been recommended that minimum grade of concrete shall not be less than M20 in reinforced concrete work and the suggested standard deviations of concrete have been changed. The impact on probability of failure of concrete in bending and direct compression both as per IS456:2000 and IS456:1978 in R.C.C members were compared by Level I reliability method and revived with permissible stresses both in limit state method and working stress method.

OBJECTIVE

The objective of the present research work is to ascertain the reliability index of the various grades of concrete and finally to ascertain the probability of failure of concrete both in bending compression and direct compression for the R.C.C members designed as per the Indian code IS456:2000 and to compare with IS456:1978.

LITERATURE REVIEW

American National Standard A58-1980 assumes normal distribution for strength of concrete with a coefficient of variation of 18% for concretes of 3000psi and 4000psi and a coefficient of variation of 15% for concretes 5000psi (35N/mm²). Nowak, A.S., Rakoczy, A.M reported a coefficient of variation of 17% to 12% for ordinary concretes varying from 3000psi to 6500psi with a bias factor varying from 1.31 to 1.14.

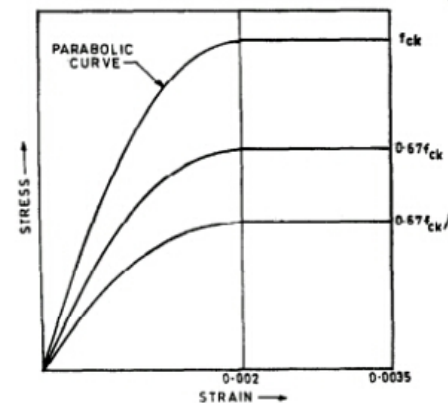
Ranganathan. R reported a coefficient of variation of 24% for M15 grade and 21% for M20 grade for nominal mix and 18% for M15 grade and 15% for M20 grade under design mix category.

IS 456: 2000 has strongly advocated the design of R.C.C members as per limit state method and as an alternative to a lesser degree in working stress method also.

The various postulates incorporated for both in limit state method and working stress method are as follows:

Limit state of collapse: Flexure (Clause 38.1)

Assumptions:



Design for the limit state of collapse in flexure shall be based on the assumption given below:

a. Plane section normal to the axis remains plane after bending.

The maximum strain in concrete at the outermost compression fibre is taken as 0.035 in bending.

The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.

The tensile strength of the concrete is ignored.

**Limit state of collapse: Flexure (Clause 39)****Assumptions:**

In addition to the above assumptions for flexure the following shall be assumed:

- The maximum compressive strain in concrete in axial compression is taken as 0.002
- The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre
- Partial safety factors γ_m for Material Strength When assessing the strength of a structure of structural member for the limit state of collapse, the values of partial safety factor, γ_m should be taken as 1.5 for concrete and 1.15 for steel.
- When assessing the deflection, the material properties such as modulus of elasticity should be taken as those associated with the characteristic strength of concrete.

Materials

The design strength of the materials, f_d is given by $f_d = f / \gamma_m$

Where

f = characteristic strength of the material and
 γ_m = partial safety factor appropriate to the material and the limit state being considered.

Partial safety factors γ_m for Material Strength

When assessing the strength of a structure of structural member for the limit state of collapse, the values of partial safety factor, γ_m should be taken as 1.5 for concrete and 1.15 for steel.

Assumptions in working stress method for design of members (Clause B1.3)

In the methods based on elastic theory, the following assumptions shall be made:

- At any cross-section, plane sections before bending remain plain after bending.
- All tensile stresses are taken up by reinforcement and none by concrete, except as otherwise specifically permitted.
- The stress-strain relationship of steel and concrete, under working loads, is a straight line.
- The modular ratio m has the value $280 / 3\sigma_{cbc}$ where σ_{cbc} is permissible compressive stress due to bending in concrete in N/mm² as specified in Table.
- The stress-strain relationship of steel and concrete, under working loads, is a straight line.
- The modular ratio m has the value $280 / 3\sigma_{cbc}$ where σ_{cbc} is permissible compressive stress due to bending in concrete in N/mm² as specified in Table.

Permissible stresses in concrete (Working Stress Method) IS 456: 2000

(Clause B-1.3, B-2.1, B-2.1.2, B-2.3 and B-4.2) Permissible Stress in Compression (MPa)

Grade of concrete	Bending	Direct
(1)	(2)	(3)
M10	3.0	2.5
M15	5.0	4.0
M20	7.0	5.0
M25	8.5	6.0
M30	10.0	8.0
M35	11.5	9.0
M40	13.0	10.0
M45	14.5	11.0
M50	16.0	12.0

**Table-1.** Assumed standard deviation
(Clause 9.2.4.2), IS-456:2000

Grade of concrete	Assumed standard deviation
M20	4.0
M25	4.0
M30	5.0
M35	5.0
M40	5.0
M45	5.0
M50	5.0

Table-2. Assumed standard deviation
clause 14.5.3), IS-456:1978

Grade of concrete	Assumed standard deviation(MPa)
M15	3.5
M20	4.6
M25	5.3
M30	6.0
M35	6.3
M40	6.6

GRADE OF CONCRETE
(Clause 6.1, 9.2.2, 15.1.1 & 36.1) IS456-2000

Group	Grade Designation	Specified Characteristics compressive strength of 150mm cube at 28 days in N/mm ²
Ordinary concrete	M10	10
	M15	15
	M20	20
Standard concrete	M25	25
	M30	30
	M35	35
	M40	40
	M45	45
	M50	50
	M55	55

Factors contributing to variations in concrete strength

It is found that the strength of concrete varies from batch to batch over a period of time. The sources of variability in the strength of concrete may be considered to be due to the following factors:

- Variation in the quality of constituent materials used,
- Variation in the mix proportions due to batching process,
- Variations in the quality of batching and mixing equipment available,
- The quality of supervision and workmanship, and

- Variation due to sampling and testing of concrete specimens.

The above variations are inevitable during production to varying degrees. For example, different strengths and the variability is more when cement from different sources is involved. The grading and shape of widely and it is not economically feasible to eliminate such variations particularly when the aggregates are not factory made. Considerable variations occur in the mix proportions from batch to batch irrespective of whether the batching is by weight or volume. These can be attributed partly to the quality of plant available and partly due to the efficiency of operation.

Some of the variations in the test results are due to variations in the sampling, making, curing, and testing the specimen even when carried out in terms of the relevant Indian Standard specifications.

Probability of failure of material

When the stress developed in the material is greater than the allowable stress, it is defined as failure. Hence the probability of failure of material, P_f can be written as:

$$P_f = P(X < P_f)$$

Where X is the random variable, namely the strength of the material.

If X follows normal distribution, Probability of failure of material $p_f = \Phi(f_a - \mu_x / \sigma_x)$

It is now generally recognized that the variations in concrete strength follow normal distribution. The failure of an under-reinforced beam is termed as tension failure, so called because the primary cause of failure is the yielding in tension of the steel. The onset of failure is gradual, giving ample prior warning of the impending collapse. Hence, such a mode of failure is highly preferred in design practice. The actual collapse, although triggered by the yielding of steel, occurs by means of the eventual crushing of concrete in compression ('secondary compression failure').

Reinforcement will yield first in under-reinforced concrete structure. The percentage of elongation of mild steel is of the order of 24% and that of high yield strength deformed (HYSD) bars is about 15% while the ultimate tensile strain in plain concrete is of the order of 0.015% and the crushing strain of concrete is only 0.35%. Invariably concrete fails before the fracture of the steel. This type of failure is called secondary compression failure. It is not compression failure but secondary compression failure.

In under-reinforced concrete beams, the reinforcement yield first. Even though the failure is initiated by the yielding of steel, concrete reaches the crushing strain faster than the fracture strain of steel, the collapse of the beam will be by crushing of concrete. This is called secondary compression failure and it is different from the primary compression failure. Ultimately, the RC



beams collapse by crushing of concrete unless there is reasonable compression steel.

Hence the concrete in bending compression is important in collapse of reinforced concrete flexural members like beams and the concrete in direct compression is vital in collapse of columns as the concrete in columns carry in order of 75% of the total load coming on the column. In the present study the probability of failure of concrete both in direct compression and bending compression is considered. All the grades of concrete from mix M20 to M50 is considered along with standard deviations proposed by the IS-456: 2000 and IS-456: 1978. The probability of failure of concrete for each grade of concrete is calculated as per the limit state method and compared with the working stress method in both IS-456 : 2000 and IS-456 : 1978.

Design strength of axially loaded short columns (Clause 39.3 IS-456:2000) (Limit State Method)

The maximum compressive strain in concrete under axial loading at the limit state of collapse in compression is specified as $\epsilon_c = 0.002$ by the code (Cl. 38.1a). corresponding to this (some what conservative) limiting strain of 0.002, the design stress in the concrete is $0.67 f_{ck} / 1.5 = 0.447 f_{ck}$. Accordingly, under 'pure' axial loading conditions, the design strength of a short column is obtained by from

$$P_{uo} = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} \dots (\text{Clause 39.3})$$

Where

P_u = axial load on the member,

f_{ck} = characteristic compressive strength of the concrete

A_c = Area of concrete

f_y = characteristic strength of the compression reinforcement

A_{sc} = area of longitudinal reinforcement for the columns

PERMISSIBLE LOADS IN COMPRESSION MEMBERS (Working stress method) (Clause B-3 IS-456:2000)

Pedestals and short columns with lateral ties (Clause B-3.1 IS-456:2000)

The axial load P permissible on a pedestal or short column reinforced with longitudinal bars and lateral ties shall not exceed that given by the following equation:

$$P = \sigma_{cc} A_c + \sigma_{sc} A_{sc}$$

Where

σ_{cc} = Permissible stress in concrete in direct compression,

A_c = Cross-sectional area of concrete excluding any finishing material and reinforcing steel,

σ_{sc} = Permissible compressive stress for column bars, and

A_{sc} = Cross-section area of the longitudinal steel.

RESULTS AND DISCUSSIONS

As the grade of concrete increased from M20 to M50 the reliability index increased and probability of failure of concrete decreased in limit state method as per IS456:2000 under direct compression but the trend is not uniform which is clearly visible in graph particularly for M25. (Figure-4) As the grade of concrete increased from M20 to M50 the reliability index increased and probability of failure of concrete decreased in working stress method as per IS456:2000 under direct compression but the trend is not uniform which is again clearly visible in graph particularly for M25 grade of concrete. (Figure-8)

As the grade of concrete increased from M20 to M50 the reliability index increased and probability of failure of concrete decreased in limit state method as per IS456:1978 under direct compression but the trend is uniform. (Figure-7) A nonlinear correlation was observed between reliability index and grade of concrete. As the grade of concrete increased from M20 to M50 the reliability index increased and probability of failure of concrete decreased in working stress method as per IS456:1978 under direct compression but the trend is uniform. (Figure-6) A nonlinear correlation was observed between reliability index and grade of concrete.

The ratio of probability of failure of concrete as per limit state method IS456:2000 under direct compression to working stress method IS456:2000 varies from 144 to 18851661 but the correlation between the ratio and grade of concrete has changed abruptly after M45 grade of concrete. (Figure-10).

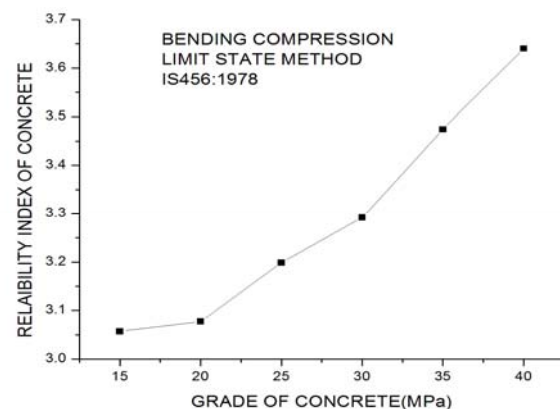


Figure-1. Bending compression limit state.

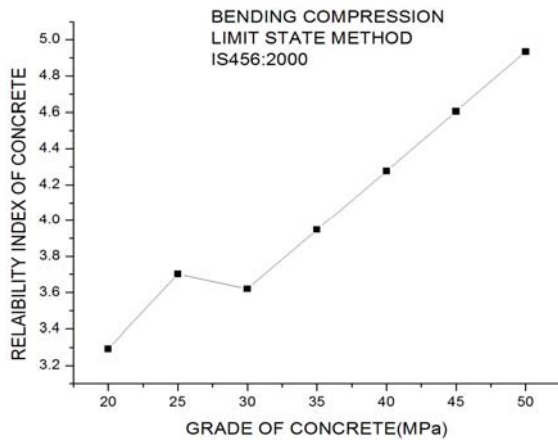


Figure-2.

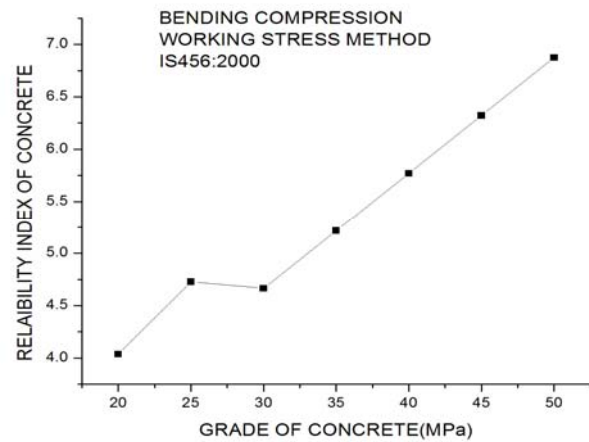


Figure-5.

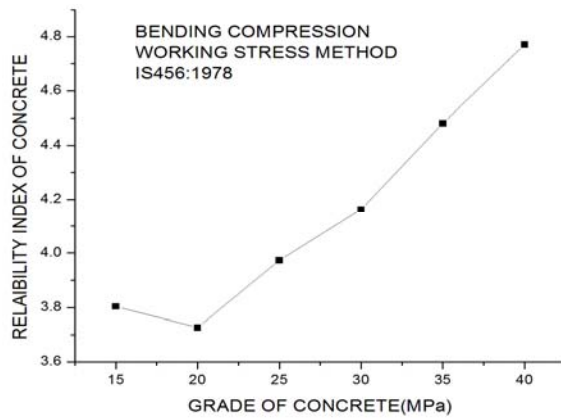


Figure-3.

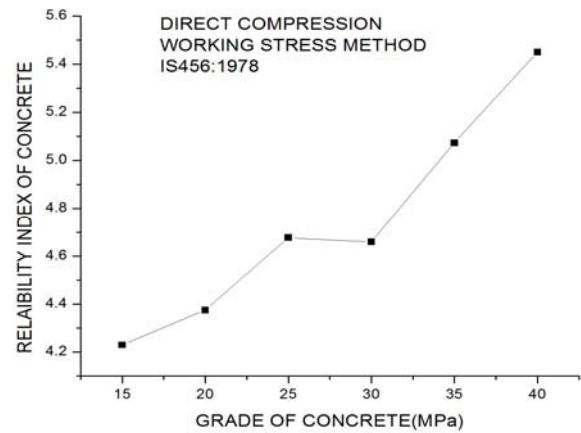


Figure-6.

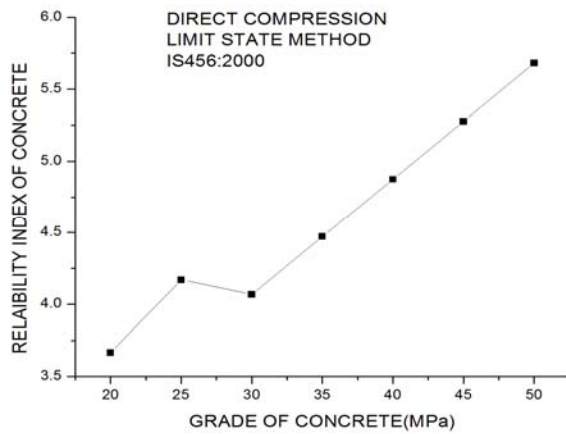


Figure-4.

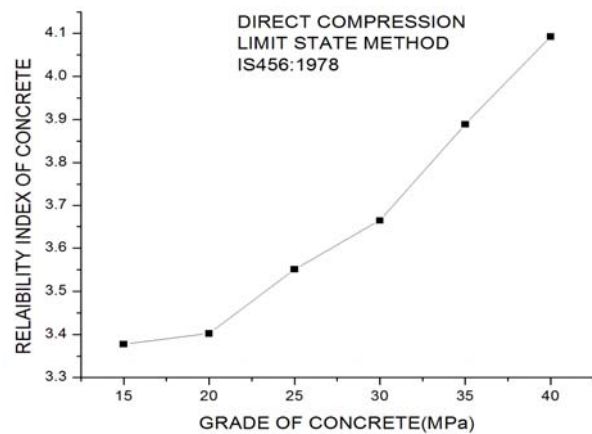


Figure-7.

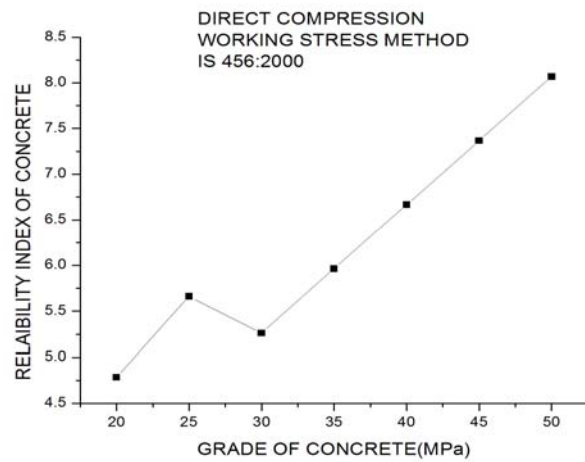


Figure-8.

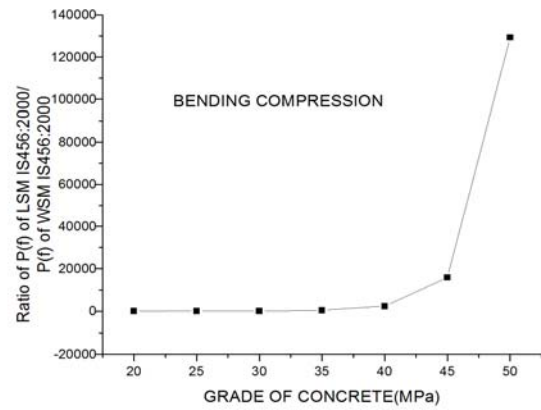


Figure-11.

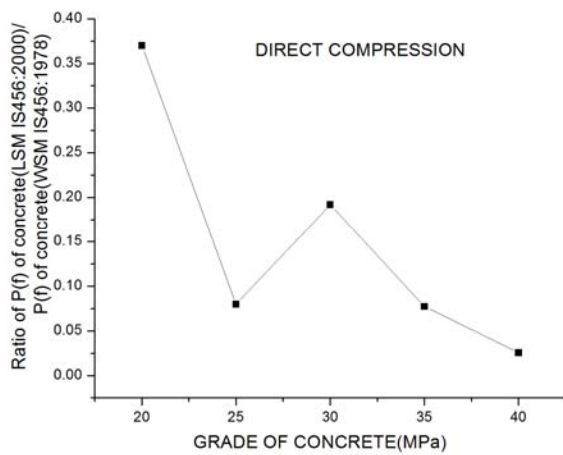


Figure-9.

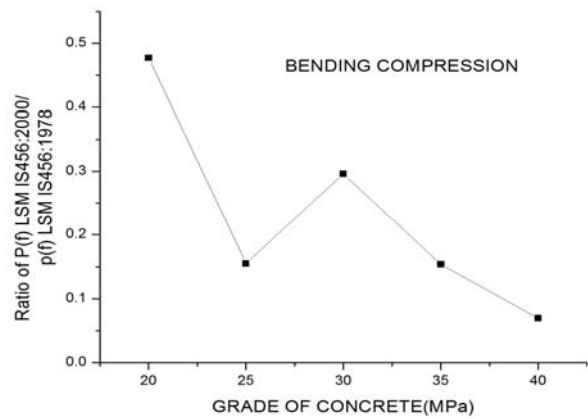


Figure-12.

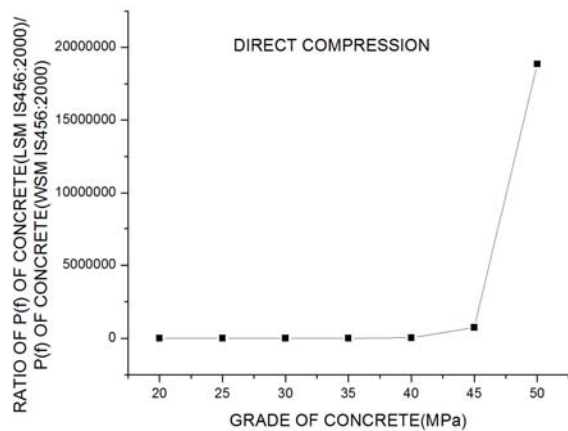


Figure-10.

**Table-3.** Grade of Concrete versus Probability of Failure of concrete in Direct Compression.

Sl.No	GRADE OF CONCRETE	PROBABILITY OF FAILURE OF CONCRETE IN DIRECT COMPRESSION AS PER LIMIT STATE METHOD IS 456: 2000	PROBABILITY OF FAILURE OF CONCRETE IN DIRECT COMPRESSION AS PER WORKING STRESS METHOD IS 456:2000	PROBABILITY OF FAILURE OF CONCRETE IN DIRECT COMPRESSION AS PER LIMIT STATE METHOD IS 456: 1978	PROBABILITY OF FAILURE OF CONCRETE IN DIRECT COMPRESSION AS PER WORKING STRESS METHOD IS 456:1978
1	M15	-	-	0.0003663	1.16800E-05
2	M20	0.00012371	8.57800E-07	0.00033434	6.05760E-06
3	M25	1.5320E-05	7.51620E-09	0.00019198	1.45330E-06
4	M30	2.3718E-05	7.12720E-08	0.00012371	1.58140E-06
5	M35	3.8946E-06	1.23500E-09	0.000050369	1.95430E-07
6	M40	5.4714E-07	1.32400E-11	0.000021353	2.53000E-08
7	M45	6.5706E-08	8.76370E-14	-	-
8	M50	6.7406E-09	3.57560E-16	-	-

NOTE: As per IS 456:1978 M15 Grade of concrete is the minimum grade allowed in R.C.C. members whereas

IS 456:2000 M20 is the minimum Grade of concrete allowed in R.C.C. members.

Table-4. Grade of Concrete versus Reliability Index of concrete in Direct Compression.

Sl.No	GRADE OF CONCRETE	RELAIBILITY INDEX OF CONCRETE IN DIRECT COMPRESSION AS PER LIMIT STATE METHOD IS 456: 2000	RELAIBILITY INDEX OF CONCRETE IN DIRECT COMPRESSION AS PER WORKING STRESS METHOD IS 456:2000	RELAIBILITY INDEX OF CONCRETE IN DIRECT COMPRESSION AS PER LIMIT STATE METHOD IS 456: 1978	RELAIBILITY INDEX OF CONCRETE IN DIRECT COMPRESSION AS PER WORKING STRESS METHOD IS 456:1978
1	M15	-	-	3.37707889	4.22995736
2	M20	3.66492537	4.78432836	3.40210902	4.37550292
3	M25	4.16865672	5.66119403	3.55087299	4.67731625
4	M30	4.06791045	5.2619403	3.66492537	4.65995025
5	M35	4.47089552	5.96343284	3.88880597	5.07335939
6	M40	4.8738806	6.66492537	4.09233379	5.44918589
7	M45	5.27686567	7.36641791	-	-
8	M50	5.67985075	8.06791045	-	-

NOTE: As per IS 456:1978 M15 Grade of concrete is the minimum grade allowed in R.C.C. members whereas

IS 456:2000 M20 is the minimum Grade of concrete allowed in R.C.C. members.

**Table-5.** Grade of Concrete versus Probability of Failure of concrete in Bending Compression.

Sl.No	GRADE OF CONCRETE	PROBABILITY OF FAILURE OF CONCRETE IN BENDING COMPRESSION AS PER LIMIT STATE METHOD IS 456: 2000	PROBABILITY OF FAILURE OF CONCRETE IN BENDING COMPRESSION AS PER WORKING STRESS METHOD IS 456:2000	PROBABILITY OF FAILURE OF CONCRETE IN BENDING COMPRESSION AS PER LIMIT STATE METHOD IS 456: 1978	PROBABILITY OF FAILURE OF CONCRETE IN BENDING COMPRESSION AS PER WORKING STRESS METHOD IS 456:1978
1	M15	-	-	0.001116891	7.13278E-05
2	M20	0.000497758	2.69476E-05	0.001043219	9.70502E-05
3	M25	0.000106853	1.13171E-06	0.000689862	3.54434E-05
4	M30	1.47217E-04	1.54365E-06	0.000497758	1.57434E-05
5	M35	3.93200E-05	9.08416E-08	0.000256178	3.71327E-06
6	M40	9.47715E-06	3.97764E-09	0.000136293	9.17661E-07
7	M45	2.06011E-06	1.29399E-10	-	-
8	M50	4.03675E-07	3.12393E-12	-	-

NOTE: As per IS 456:1978 M15 Grade of concrete is the minimum grade allowed in R.C.C. members

whereas IS 456:2000 M20 is the minimum Grade of concrete allowed in R.C.C. members.

Table-6. Grade of Concrete versus Reliability Index of concrete in Bending Compression.

Sl.No	GRADE OF CONCRETE	RELAIBILITY INDEX OF CONCRETE IN BENDING COMPRESSION AS PER LIMIT STATE METHOD IS 456: 2000	RELAIBILITY INDEX OF CONCRETE IN BENDING COMPRESSION AS PER WORKING STRESS METHOD IS 456:2000	RELAIBILITY INDEX OF CONCRETE IN BENDING COMPRESSION AS PER LIMIT STATE METHOD IS 456: 1978	RELAIBILITY INDEX OF CONCRETE IN BENDING COMPRESSION AS PER WORKING STRESS METHOD IS 456:1978
1	M15	-	-	3.05724947	3.80351812
2	M20	3.29179104	4.0380597	3.07764439	3.72657365
3	M25	3.70223881	4.72835821	3.19885948	3.97328921
4	M30	3.62014925	4.66492537	3.29179104	4.16243781
5	M35	3.94850746	5.21716418	3.47421227	4.48108268
6	M40	4.27686567	5.76940299	3.64004975	4.77075984
7	M45	4.60522388	6.32164179	-	-
8	M50	4.93358209	6.8738806	-	-

NOTE: As per IS 456:1978 M15 Grade of concrete is the minimum grade allowed in R.C.C. members

whereas IS 456:2000 M20 is the minimum Grade of concrete allowed in R.C.C. members.

The ratio of probability of failure of concrete as per limit state method IS456:2000 under direct compression to limit state method IS456:1978 varies from 37% to 2.5% but the correlation between the ratio and grade of concrete has changed abruptly at M25 grade of concrete. (Figure-9)

A similar trend in relationship between grade of concrete and reliability index, probability of failure of concrete is observed for permissible stresses under bending compression when compared with present IS456:2000 and earlier code IS456:1978.

The ratio of probability of failure of concrete as per limit state method IS456:2000 under bending

compression to working stress method IS456:2000 varies from 18 to 129220 but the correlation between the ratio and grade of concrete has changed abruptly for M50 grade of concrete. (Figure-11)

The ratio of probability of failure of concrete as per limit state method IS456:2000 under bending compression to limit state method IS456:1978 varies from 48% to 7% but the correlation between the ratio and grade of concrete has changed abruptly at M25 grade of concrete. (Figure-12)



CONCLUSIONS

Based on the results of this study the following conclusions are drawn:

The present code IS456:2000 has changed the standard deviations of the concrete which are quite arbitrary and empirical which does not reflect the sound relationship between probability of failure of concrete and grade of concrete;

There is no uniform correlation between grade of concrete and reliability index;

There is no consistency in reliability of concrete for different grades;

The structures designed as per IS456:2000 are safer than the earlier code IS456:1978

Hence code authorities should incorporate appropriate standard deviations of the concrete to ensure uniform quality control taking all the relevant experimental and statistical parameters into consideration when extending it to higher grades of concrete so as to ensure consistent reliability of concrete.

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