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FOR THE ISSUE OF STATISTICAL VERIFICATION OF DATA FOR BENEFICIATION OF ORES WITH VARIOUS GENESES

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ABSTRACT

The article gives review of statistical criteria for multiple comparison and examples of their use for processing of experimental data in the field of beneficiation of mineral products. The most widely used criteria of Student have been envisaged with the adjustments of Bonferroni, Newman-Keuls, Tukey and Dunnett. Based on the example of the results of flotation beneficiation of non-traditional gold-containing ore, it was shown that the result of data interpretation depends on selection of a certain criterion. Comparing Student's modified criterion with Bonferroni's adjustment and Dunnett's criterion, is can be noted that Dunnett's criterion is more sensitive, and it gives a more precise result with comparatively few samples.

Keywords: student's criterion, Newman-Keuls' criterion, tukey's criterion, dunnett's criterion, minerals processing, statistics.

INTRODUCTION

Gradual depletion of fields under development requires refilling of the emerging deficit as soon as possible through adding to processing such refractory ores the processing of which is ineffective with "classic" methods. Due to this fact, improvement of beneficiation efficiency, using new processing methods, intensification through the use of physical and chemical impacts and introduction of new reagents into processes etc. are priority tasks for beneficiation researchers (Aleksandrova T.N. *et al.*, 2013; Rasskazova A.V. *et al*., 2014; Mamaev, Yu.A. *et al*., 2009; Gurman, M.A, 2017)

Such research usually has a supplementary nature, and in the outcome of any measures, a comparison whether they are effective or can be used for a certain type of raw materials is carried out. It is often the case that obtained results is not verified statistically, which leads to methodological errors and distorted interpretation of results obtained. For example, instead of recognizing several flotation agents as equally effective (or ineffective), one of them is declared as "the best one". Meanwhile, using appropriate criteria will considerably assist in studying the influence of a certain factor on the beneficiation process, namely it will make possible to answer the question whether the variance in the obtained results is statistically significant (with a certain probability), or whether the obtained result is only a random one.

METHOD

One of the most widely used criteria is Student's t-criterion which is a special case of the variance analysis. The criterion is designed for comparison of only *two* series, but in practice it is often wrongly used to assess variance in a large number of groups through their pairwise comparison. For example, in selection of a more efficient reagent, x number of reagents are tried, and then the 1 reagent is compared with the 2, the 2 with the 3 etc. Upon obtaining quite a high value of the t criterion in any comparison, a conclusion is made that the probability of erroneous statement on existence of a variance does not exceed 5%. This judgment is erroneous, and the probability of an error is considerably higher, and it can be calculated by the formula:

$$P = 1-(1-0{,}05)^n \text{ or } P = 0{,}05n$$

where n is the number of comparisons. It is not difficult to make sure that even in comparison of 3 different types of reagents; the error in using t will be equal to at least 15%. In literature, this property has obtained the name of a multiple comparison effect.

For studying series the number of which exceeds two, methods of multiple comparison should be used. The simplest of them is to use Bonferroni's inequation:

$$\alpha' < n\alpha \quad (1)$$

where α' is the probability of erroneous detection of variances for at least one time. I.e., if a criterion with the α significance level is to be used k times, then the probability to find (at least once) a variance where it is absent does not exceed the product of k by α.

It follows from Bonferroni's inequation that to provide the probability of an error α' (normally for technical studies this value is 5%), then in each comparison it is necessary to adopt the significance level α'/n. For example, to compare 3 reagents, the significance level should be 0.05/3=1.7%. In a general case, it is rational to use Bonferroni's inequation with the number n<8, because an increased number of comparisons enlarges the chance for even significant variances to be recognised as statistically insignificant.

Newman-Keuls' criterion is a more precise but less strict one (q criterion). It gives a more precise probability assessment, and its sensibility is higher than that of Student's criterion with Bonferroni's adjustment. When Newman-Keuls' criterion is used, it is primarily necessary to check whether the average values are equal by using the variance analysis, and, if it is not confirmed, all average values are to be ordered in an ascending order and compared in pairs, each time calculating the value of the *q* criterion:

$$q = \frac{\overline{X_A} - \overline{X_B}}{\sqrt{\frac{s_{ins}^2}{2}\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \qquad (2)$$

where $\overline{X_A}, \overline{X_B}$ are the average values compared, s_{ins}^2 - variance within the series, n_A and n_B is the number of elements in the series.

And to optimize the calculation, the minimum average value should be first compared with the maximum one, then with the 2nd one by the value and so on. Then the second to the last one is compared in the same order. Sorting out of all pairs can be rather time-consuming and not always necessary, because if some average values do not vary, all average values lying between then will not vary.

The calculated value of the criterion should be compared directly with the one in the table, and based on the comparison, a conclusion should be made whether to assume or deny the zero hypothesis. The tabular value should be selected based on the assumed probability, the number of degrees of freedom $v = N - m$ (where N is the number of elements in all series, m is the number of samples) and the size of the interval under comparison which should be determined as the variance between the sequence numbers increased by one in the range of average values ordered from the smallest to the largest.

The other criterion which may be used in comparison of several series is named Tukey's criterion. Newman-Keuls' criterion was developed as an improvement of Tukey's criterion and, respectively, they are almost identical by calculation, except for the method of determining the critical value. In Newman-Keuls' *q* criterion, the critical value depends on the comparison interval. For calculation by Tukey's criterion, in all comparisons, the number of series used in the analysis is used instead of the comparison interval; therefore, the critical value *q* will always be the same. Since in Tukey's criterion the maximum critical value is used in all comparisons, variances will be detected more seldom than with the use of Newman-Keuls' criterion.

RESULTS

Practical benefit of these criteria can be used in various studies (Khanchuk A.I. *et al*., 2014; Aleksandrova T.N. *et al*., 2015). For example, in the thesis [Aleksandrova T.N. *et al*., 2015), the influence of foaming agents on the decrease in the content of organic carbon in the concentrate during flotation of gold-containing ore from the "Mayskoe" field was studied.

The main indicator of efficiency of the process was the ratio of the gold content in the material being processed, which was expressed in units g/t to the content of the organic portion of carbonaceous ore matter, and expressed in kg/t (Au/C_{org}). OP F-597, Flotanol 7196, Flotanol 7197 was used as reagents. The experiment was repeated 3 times, the results are summarized in Table-1.

The standard deviation will then be equal to: OP F-597 - 0.2; Flotanol 7197 - 0.3; Flotanol 7196 - 0.5. On the basis of the variance analysis, the estimated F criterion is equal to 14.59, and the tabular criterion (with the significance level of P=95 %) is equal to 5.14. Since $F>F_{tab}$, then the zero hypothesis on equality of all average values is rejected.

Table-1. Conditions and results of the experiment.

Reagent	**Value Au/C_{org} in the experiment**			**Average content of Au/C_{org}**
	1	2	3	
OP F-597	4.88	4.58	4.61	4.69
Flotanol 7197	5.86	6.19	5.56	5.87
Flotanol 7196	6.57	6.18	5.61	6.12

We calculate Student's criterion with adjustment for Bonferroni's inequation (1). Evaluation of variance within the group: $s_{grupp}^2 = \left(0{,}2^2 + 0{,}3^2 + 0{,}5^2\right)\cdot 1/3 = 0{,}12$. The number of series 3, the number in each series is 3. Therefore, the number of degrees of freedom ν=3(3–1)=6. Then, comparing all series by pairs, we obtain the following:

$$t_1 = \frac{\overline{X_2} - \overline{X_1}}{\sqrt{\frac{2s_{ins}^2}{n}}} = \frac{5{,}87 - 4{,}69}{\sqrt{\frac{2\cdot 0{,}12}{3}}} = 4{,}173,$$

$$t_2 = \frac{6{,}12 - 4{,}69}{\sqrt{\frac{2\cdot 0{,}12}{3}}} = 5{,}02;$$

$$t_3 = \frac{5{,}87 - 6{,}12}{\sqrt{\frac{2\cdot 0{,}12}{3}}} = -0{,}884$$

where $\overline{X_i}$ is the average content of Au/C_{org}.

Since only 3 comparisons were made, then, with adjustment for Bonferroni's inequation, the significance level should be at least α′=0.05/3=0.016. With such significance level, the tabular criterion of Student is 5.959 (Zar J. H., 1984). Therefore, no statistic variance between the experiments is noted.

We calculate Newman-Keuls' criterion and Tukey's criterion for the data of Table-1.

The ordered values of the average results obtained: 4.69(X_{1av})-5, 87(X_{2av})-6, 12(X_{3av}), and respectively, the comparison is made in the following

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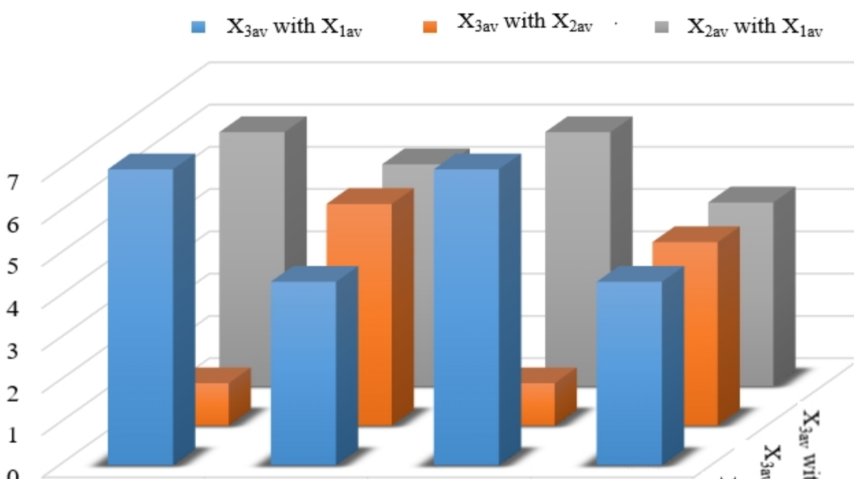
order: We compare X_{3av} c X_{1av}, X_{3av} with X_{2av}, X_{2av} with X_{1av}

The tabular and estimated values of Newman-Keuls' and Tukey's criteria by (2) are summarized in Figure-1.



	Newman-Keuls' estimated criterion	Newman-Keuls' tabular criterion	Tukey's estimated criterion	Tukey's tabular criterion
X_{3av} with X_{1av}	7,002	4,339	7,002	4,339
X_{3av} with X_{2av}	1	5,243	1	4,339
X_{2av} with X_{1av}	6,002	5,243	6,002	4,339

Figure-1. The tabular and estimated values of Newman-Keuls' and Tukey's criteria.

As can be seen, the Dunnett's criterion analysis of statistical processing by using Newman-Keuls' and Tukey's criteria produced other results. In both tests, the results of experiments with Flotanol 7197 and Flotanol 7196 are different from the results with OP F-597, which is confirmed by the further study. The obtained results show that the strictness of Student's criterion with adjustment for Bonferroni's inequation is excessive, and it rejects any variances much more often than it is necessary. In this connection, for studies in the field of beneficiation, the use of Newman-Keuls' q criterion is more preferable.

Quite often, for inspection purposes, a comparison with only one reference (initial) group is needed rather than a comparison of all obtained results by pairs. For example, such need may occur for modernization of a beneficiation system, when it is necessary to compare performance indicators of the operating technology with those after such modernization. Such inspection is necessary upon detection of variances upon replacement of a reagent in a flotation system for finding any statistically significant variance etc.

For such purposes, it is possible to use the above methods, but it results in a considerable increase in process complexity due to the need to compare all series. At the same time, the risk of omission of actual variances and of erroneous assumption of the zero hypothesis sharply increases. In such studies, using specially designed methods will be more correct. Student's modified criterion with adjustment for Bonferroni's and Dunnett's criterion have become the most wide-spread ones.

The use of Student's modified criterion with Bonferroni's adjustment is effected by the same algorithm as in multiple comparison, but with the number of comparisons decreased by one. Correspondingly, the significance level will be calculated as $\alpha'/(n\text{-}1)$, which will lead to liberalization of the criterion. Due to a decrease in the critical level t the method's sensitivity increases, and it is correct to make conclusions only about variance (or coincidence) of series with the reference series. The conclusion about variance between non-reference series is impossible when this method is used.

Dunnett's criterion is a modification of Newman-Keuls' criterion. The calculation algorithm is as follows. The average values are ranged by ascension of their absolute variance from the average value of the reference series. Then the procedure of comparison between the reference series with the other ones, starting from the one having the most different value from the reference one, is started. If no variances with a subsequent series are found, the calculations are stopped, because the other series enclosed between them will not have any variance. The comparison interval is constant and equal to the number of series, including the reference one. The number of degrees of freedom is calculated as in Newman-Keuls' criterion. Dunnett's criterion itself is calculated as follows:

$$q' = \frac{\overline{X_{contl}} - \overline{X_A}}{\sqrt{s^2_{group}\left(\frac{1}{n_{contl}} + \frac{1}{n_A}\right)}} \quad (3)$$

and is compared with the tabular value.

Figure-2 gives a calculation of Student's modified criterion with Bonferroni's adjustment and of Dunnett's criterion by the formula (3). The experiments with the foaming agent OP F-597 are assumed as the reference series (Harter H.I., 1970; Dunnett. C.W., 1964.).

CONCLUSIONS

The analysis has shown that variance is absent by Student's modified criterion with Bonferroni's adjustment, and meanwhile, Dunnett's criterion has shown a directly opposite result.

Unfortunately, many researchers neglect statistic processing, considering it to be insignificant and having no influence on research. Meanwhile, using statistic criteria significantly improves accuracy in evaluation of data obtained and their correct interpretation, and directions of further research with positive results directly depend on it.

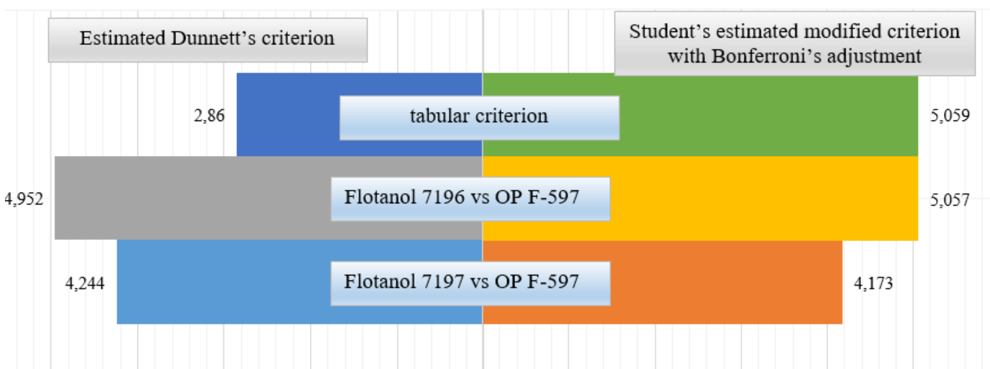


Figure-2. Tabular and estimated values of Student's modified criterion with Bonferroni's adjustment and Dunnett's criterion.

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